ENHANCEMENT OF THE ACCELERATING GRADIENT IN SUPERCONDUCTING MICROWAVE RESONATORS*

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Abstract

The accelerating gradient of superconducting resonators can be enhanced by engineering the thickness of a dirty layer grown at the cavity's rf surface. In this paper the description of the physics behind the accelerating gradient enhancement by meaning of the dirty layer is carried out by solving numerically the the Ginzburg-Landau (GL) equations for the layered system. The calculation shows that the presence of the dirty layer stabilizes the Meissner state up to the lower critical field of the bulk, increasing the maximum accelerating gradient.

INTRODUCTION

The possible enhancement of the accelerating gradient by meaning of layered structures in accelerating cavities was introduced by A. Gurevich [1]. He showed that high κ (GL parameter) superconducting layers separated by insulating layers (SIS structure) deposited at the rf surface can in principle enhance the superheating field, and allow for higher gradients.

In the same direction T. Kubo [2, 3] and S. Posen *et al.* [4] explored thoughtfully the behavior of the SIS structure. T. Kubo in particular, described also the SS structure [3, 5], i.e. a high κ (dirty) superconducting layer on top of a low κ (clean) bulk superconductor. He approached the problem in the high κ approximation by meaning of the London equations as done for the SIS structure, showing that the dirty layer can in principle enhance the superheating field even if no insulating layer is present.

In the present paper an alternative description of the SS structure is presented. The approach here is different since the calculation is carried out numerically from the GL equations, where the dirty layer is assumed to behave as a perturbation on the magnetic induction profile in the material. We show that the dirty layer stabilizes the superconductor against the vortex nucleation, and shifts the lower critical field of the whole structure up to the bulk's value, increasing the magnetic field range in which the Meissner state is stable.

THE BEAN-LIVINGSTON BARRIER FOR NON-CONSTANT κ

Let us assume a semi-infinite superconductor, where the normal to the surface directed towards the material bulk is $\hat{x}$, the external magnetic field is applied along the z direction and the screening currents are flowing along the y direction. On top of such superconductor we assume the presence of a thin superconducting dirty layer with higher κ than the bulk (grown by diffusion of impurities for example), so that the κ profile of the system can be described by the analytic form:

$$\kappa(x) = -\frac{\kappa_s - \kappa_b}{1 + e^{-(x-x_0)/c}} + \kappa_s \,, \tag{1}$$

which corresponds to a sigmoidal function, where κ_s and κ_b are the superficial and bulk GL parameters, c is a constant that defines the steepness of the function (normally $c = 0.018$) and x_0 corresponds to the inflection point assumed as the thickness of the dirty layer.

Since we are dealing with dimensionless units $x = \text{depth}/\lambda$, with λ the penetration depth $\lambda = \lambda_0\sqrt{1 + \xi_0/l}$, l the mean free path, $x_0 = d/\lambda$, $\xi_0 = 38$ nm and $\lambda_0 = 39$ nm [6].

Forces Acting on the Vortex

The forces acting on a vortex penetrating from the surface can be calculated in first approximation by implementing the same description of C. P. Bean and J. D. Livingston [7]. The repulsive force (with respect the surface) due to the interaction of the vortex with the magnetic induction profile in the material is:

$$\mathbf{f_f}(x) = -\frac{4\pi}{\kappa(x)}\frac{\partial\, b_f(x)}{\partial x}\,\hat{\mathbf{x}} \,, \tag{2}$$

where $b_f(x) = a'(x)$ is calculated numerically from the GL equations modified in order to account also for the variation of κ with x:

$$\begin{aligned} &\frac{1}{\kappa^2(x)} f'' - a^2 f + f - f^3 = 0 \\ &\mathbf{a}'' - f^2\mathbf{a} = 0 \,, \end{aligned} \tag{3}$$

where f and $\mathbf{a}$ are respectively the dimensionless order parameter and vector potential as defined in [8], while the boundary conditions are the same assumed in [9].

The attractive force is instead calculated as:

$$\mathbf{f_v}(x) = \frac{4\pi}{\kappa(x)}\frac{\partial\, b_v(2x)}{\partial x}\,\hat{\mathbf{x}} \,, \tag{4}$$

where $b_v(r) = a'(r) + (1/r)a(r)$ is the magnetic induction of the image-vortex.

In order to maintain the description mono-dimensional, we calculate the magnetic induction profile from the cylindrically symmetric GL equations. Such approach is carried

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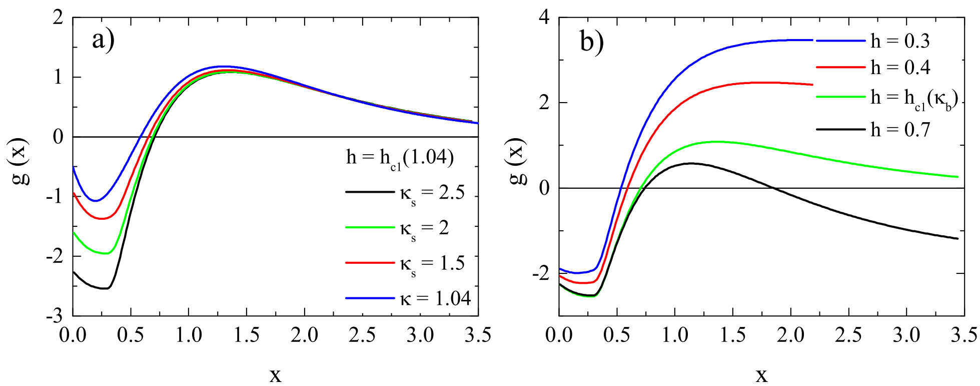

Figure 1: Gibbs free energy density as a function of x a) for different values of κ_s calculated at $h = h_{c1}(\kappa_b)$, b) for increasing applied fields, with $\kappa_s = 2.5$ and $d = 15$ nm. In both cases $\kappa_b = 1.04$.

out by meaning of some approximations: i) both vortex and anti-vortex are assumed to be point-like objects, ii) the interaction between vortex and anti-vortex is defined only along the conjunction segment between them, iii) because of (ii) the magnetic induction profile along the line of interaction is assumed to be not affected by the non-cylindrical symmetry of the κ profile.

Under such assumptions the cylindrical symmetric GL equations that describes the vortex are:

$$f'' + \frac{1}{r} f' - \Gamma^2(r,x)\, f \left[f^2 - 1 + \left(a - \frac{1}{\Gamma(r,x)\, r} \right)^2 \right] = 0$$
$$\mathbf{a}'' + \frac{1}{r}\mathbf{a}' - \frac{1}{r^2}\mathbf{a} - f^2 \left(\mathbf{a} - \frac{1}{\Gamma(r,x)\, r}\, \hat{\theta} \right) = 0\,, \tag{5}$$

where $\Gamma(r,x) = \kappa(x-r) + \kappa(r-x) - \kappa_s$ is the κ profile as seen by the image-vortex and x is the vortex position with respect the surface. The boundary conditions used are the same assumed in [9] where the constant κ is substituted by $\Gamma(R,x)$, with R the domain extension.

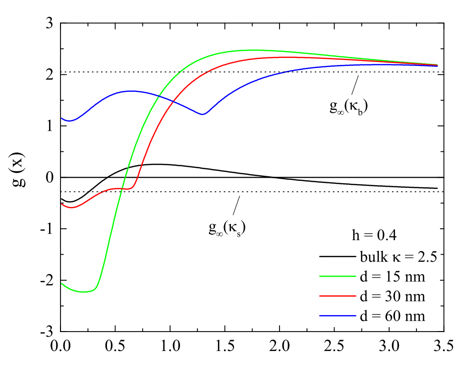

Figure 2: Comparison between the Gibbs free energy density calculated for a constant $\kappa = 2.5$ profile and a for non-constant profiles with layers having $\kappa_s = 2.5$, $\kappa_b = 1.04$ and various thicknesses, when $h = 0.4 > h_{c1}(\kappa_s)$.

Gibbs Free Energy Density

Now that the forces acting on the vortex are known, the Gibbs free energy density can be calculated using the integral relation:

$$\begin{aligned} g(x) &= -\int f_v(x) + f_f(x)\, dx \\ &= -\int \frac{4\pi}{\kappa(x)} \left(\frac{\partial\, b_v(2x)}{\partial x} - \frac{\partial\, b_f(x)}{\partial x} \right) dx \\ &= g_s^*(x) + g_\infty\,. \end{aligned} \tag{6}$$

where $g_s^*(x)$ and g_∞ are the Gibbs free energy densities associated to the interaction with the surface and at infinite respectively.

Assuming a dirty layer thickness d lower than the penetration depth at the surface ($d < \lambda_s$), the dirty layer can be treated as a perturbation to the magnetic induction profiles. If $x \to \infty$, then both $f_v(x) + f_f(x)$ and $g_s(x)$ go to zero, and g_∞ can be calculated as dependent only on κ_b:

$$g_\infty = \epsilon - \frac{4\pi}{\kappa_b} h = \frac{4\pi}{\kappa_b} (h_{c1} - h)\,, \tag{7}$$

where $h_{c1}(\kappa_b) = \kappa_b \epsilon / 4\pi$ is the bulk lower critical field, with ϵ the energy of a single vortex line as defined in [10].

In order to assure that the Gibbs free energy density associated to the interaction with the surface is zero for $x \to \infty$, we need to define a new constant C such that:

$$g_s^*(x) = g_s(x) + C\,. \tag{8}$$

The constant C can be calculated similarly to g_∞. Since $g_s(x)$ is associated to the total force of interaction with the surface $f(x)$, then when $x \to \infty$, $f(x) \to 0$ and $g_s(x) \to 0$. Thus, if the solutions domain X is larger enough, the force value is so small that $g_s(x) \approx f(x)$ and the integration constant is:

$$C = g_s^*(X) - f(X) \tag{9}$$

It follows that the Gibbs free energy density for the SS structure is calculated as:

$$g(x) = [g_s^*(x) - g_s^*(X) + f(X)] + g_\infty\,. \tag{10}$$

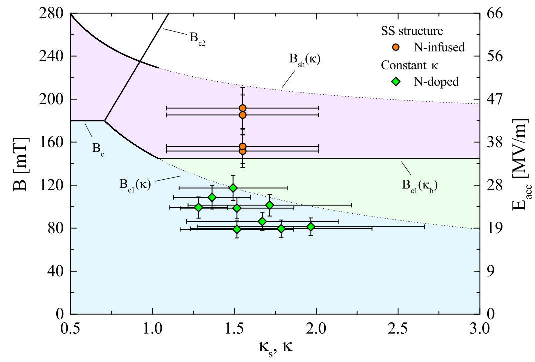

Figure 3: Experimental quench fields data plotted along with the numerical calculations as a function of the GL parameter.

In Fig. 1 the Gibbs free energy density calculated by meaning of Eq. (10) is reported. The graph in Fig. 1a shows how the energy barrier evolves as a function of the increasing κ_s. The blue curve is calculated in the cleanest limit with no layer at the surface. As the dirtiness of the layer is increased the barrier height with respect to zero is slightly decreased. On the opposite, the presence of the dirty layer increases the absolute height of the barrier: at the very surface the Gibbs free energy density assumes larger negative values for increasing κ_s values. The stability of the meta-stable Meissner state is then increased since larger Δg are needed for the vortex to penetrate. Since all the curves in Fig. 1a are calculated for $h = h_{c1}(\kappa_b)$ and $d < \lambda_s$ the Gibbs free energy density tends to zero in the bulk of the material as expected.

In Fig. 1b the evolution of g as a function of the applied field for a 15 nm layer with $\kappa_s = 2.5$ and $\kappa_b = 1.04$ is reported. As expected from the constant κ calculations [7] the barrier is lowered by the presence of the field.

As showed in Fig. 2 for the SS structure, the Gibbs free energy density at infinite corresponds to $g_\infty(\kappa_b)$. If we compare the results obtained for constant $\kappa = 2.5$ (black curve) to those obtained for layers of thickness $d = 15$, 30 and 60 nm the practical outcome of such finding is clear. When κ is constant and $h > h_{c1}(\kappa)$ vortices are stable in the bulk ($g_\infty < 0$) and the superconductor is in the meta-stable Meissner state. If instead the dirty layer is present, vortices are not stable in the bulk ($g_\infty > 0$) even if $h > h_{c1}(\kappa_s)$. Indeed the Meissner state is preserved up to $h = h_{c1}(\kappa_b)$ and the meta-stable Meissner state shifted to fields larger than $h_{c1}(\kappa_b)$ independently on k_s.

In practical words, the probability of quenching the cavity at $h_{c1}(\kappa_s)$ because of flux penetration is decreased when the dirty layer is present at the surface. In real surfaces where defects are always present (let us assume only geometric superconducting defects, thermal breakdown is not considered) the Bean-Livingston barrier is weakened or totally suppressed, hence we should expect a generous part of constant κ cavities (e.g. nitrogen-doped) to quench at the lower critical field or below. On the other hand, assuming same type of defects, cavities that possess a dirty layer with higher κ than the bulk (e.g. 120 °C baked [11]) should have more probability of reaching fields above $h_{c1}(\kappa_s)$, since their Meissner state will survive up to $h_{c1}(\kappa_b) > h_{c1}(\kappa_s)$.

EXPERIMENTAL DATA

Several niobium single-cell TESLA cavities [12] where used in this study. The first batch was prepared with standard N-doping [13]. The second batch was instead prepared with the N-infusion treatment: low temperature bake at 120-160 °C in 25 mTorr of nitrogen for 48 hours, without breaking the vacuum after the 800 °C bake for 3 hours [14, 15]. The quench field of such cavities was measured at the FNAL's vertical test facility. The data is reported in Fig. 3 along with the numerical simulations ($b = B/\sqrt{2}B_c$, with $B_c = 180\,\mathrm{mT}$ [16]).

The dotted lines show B_{sh} and B_{c1} of a bulk superconductor with constant κ like N-doped cavities. The blue area and the sum of pink and green areas show respectively the extension of the Meissner state and of the meta-stable Meissner state for constant κ cavities.

When the dirty layer is introduced the value of B_{c1} is constant and dependent only on κ_b (in this case assumed to be 1.04). Therefore, the Meissner state is now represented by the blue and the green areas, while the meta-stable Meissner state by the pink area alone.

N-infused cavities are assumed to have κ_s comparable to the κ of N-doped but confined only on a dirty layer at the surface where nitrogen is diffused [14, 15]. Their quench field is indeed higher than that of standard N-doped cavities, and well above $B_{c1}(\kappa_s)$, as expected when SS structures are present.

CONCLUSIONS

In this paper we addressed the theoretical description of the SS structure. The presence of the dirty layer is beneficial in order to push the Meissner state up to the bulk lower critical field, independently on the superficial κ.

The model presented is able to give insights for future developments of high quality factors at high gradients. Indeed, a superficial dirty layer may improve the gradient and the Q-factor at the same time—small mean free paths (~ 20 nm) minimize the Mattis-Bardeen surface resistance. Therefore, by a smart surface engineering it may be possible to achieve high Q-factors up to high gradients, allowing affordable future high gradient accelerators.

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