

CALCULATIONS OF EXTERNAL COUPLING TO A SINGLE CELL RF CAVITY*

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Abstract

A method is proposed to calculate the intrinsic and external Q values of a cavity coupled to a waveguide using MAFIA code in time domain. Typical methods used for the Q calculation in time domain are also reviewed. The method works efficiently for high Q cavities with coupling strengths close to critical coupling case. Applications of the method to a single cell RF cavity coupled to a waveguide are presented. The comparisons between simulation and experimental results are given for a set of different coupling iris apertures.

1 INTRODUCTION

Calculations of external coupling to a resonant RF structure is important in many accelerator applications. There have been a few methods available for specific systems. The methods range from analytical approximations based on the equivalent circuit theory or analytical field solutions to numerical simulations. With the development of powerful computer codes such as MAFIA, SUPERFISH etc., numerical methods are favored for their versatility. Among the numerical methods, two types of the simulation techniques are often used: frequency domain and time domain. In the frequency domain, The Kroll-Yu method [1] has been used widely and proven to be efficient in the external Q calculations for systems having low Q values and strong coupling. The Kroll-Yu method uses the resonance frequencies computed from the MAFIA for different lengths of the waveguides and fits the induced phase variations with a theoretical formula. Both the loaded resonant frequency and the external Q are then obtained from the fitting. When the coupling becomes weak and Q gets higher, to fit the phase variation curve requires very high accuracy of the resonant frequency. This is difficult for many simulation codes. An improved version of this method is Kroll-Lin method, but we do not have enough data and experience to comment about it. We developed a method using the MAFIA time domain simulation results. Unlike the conventional decay method in the time domain, which calculates the natural decay time constant, we calculate the Q_0 , Q_{ext} using an energy method [2]. The method involves the use of waveguide boundary condition which is available in the MAFIA time domain module. Application of this method to a single cell RF cavity coupled to a rectangular waveguide, shown in Figure 1 will be presented. Comparisons between simulations and experimental measurement will be given for a

set of different coupling iris apertures.

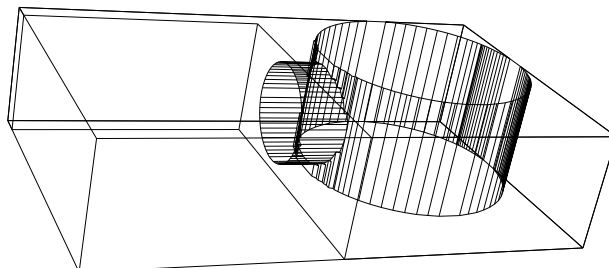


Figure 1: A 3D MAFIA model of a single cell pill-box cavity coupled to a rectangular waveguide through an open iris. The design resonant frequency for the cavity is 1.5 GHz.

2 THE METHOD

We define a cavity (or a chain of coupled cavities) coupled to an external RF component such as a waveguide or a coaxial line, through an open iris or a probe as a cavity-coupler system. The system, in a narrow band around its natural resonant frequency, can be very well presented by an equivalent RCL circuit coupled to a transmission line through a transformer. Due to the presence of the coupling between the cavity and the coupler, the electro-magnetic fields become quite complicated in the iris region. Numerical simulation codes are often used to model this region. The resonant frequency of the system will depend on how the coupler is terminated as well. In analogy to the definition of the unloaded quality factor Q_0 , an external quality factor Q_{ext} is defined in such a way that makes the coupling and the resonant frequency of the system unique by terminating the coupler with a matched load. It is difficult to numerically simulate a matched load in the frequency domain. We instead take the advantage of the built-in waveguide boundary condition in the MAFIA time domain, calculate the instantaneous stored energy and power loss over one RF period based on the time domain simulation results. We assume the following relations hold at any instant,

$$Q_0 = \omega_0 \frac{U(t)}{P_w(t)}, \quad Q_{\text{ext}} = \omega_0 \frac{U(t)}{P_{\text{out}}(t)} \quad (1)$$

where ω_0 is the natural angular resonant frequency; $U(t)$ and $P_w(t)$ are respectively the stored energy and power loss on the cavity averaged over one RF period at time t ; $P_{\text{out}}(t)$ is the RF power flowing out from the waveguide port over one RF cycle at time t . Two time domain runs are needed to determine both the resonant frequency and the Q_{ext} of the system: one for the resonant frequency and the other

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for the Q_L which is given by,

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}}. \quad (2)$$

As the MAFIA time domain module can handle lossy dielectric materials, system involving coupling with ceramic windows and lossy materials can also be simulated correctly.

2.1 Resonant Frequency of the System

We first drive the cavity-coupler system with a broad band dipole pulse with its center frequency approximately around the natural resonant frequency of the system. A Gaussian pulse built-in in MAFIA is often used with its bandwidth wide enough to cover the natural resonant frequency, but not to excite high order modes. The location and the polarization direction of the dipole should be at the place where the mode of interest has a strong electric field component in the same direction. Waveguide boundary condition at the waveguide port must be used. The bandwidth of the waveguide should be wide enough so that there are no reflected fields coming back from the port. The induced time varying electro-magnetic fields in the cavity are also monitored at a place where the mode of interest has a strong component, but not too close to the driving point. The electro-magnetic field whose frequency is equal to the natural resonant frequency of the system will be coherently excited, and be built up. If the driving source is removed, the field decays with its natural time constant. For high Q cavities, the decay time, therefore the CPU time is so long that makes it in-efficient to calculate Q in this way. Nevertheless this field does provide enough information to determine the resonant frequency. Having carefully chosen the driving bandwidth, the only field left after the driving source will be the field with the natural resonant frequency. A Fast Fourier Transform (FFT) of the field then can be used to find the frequency. However better accuracy of the frequency can be determined by measuring the oscillation period of the time varying field signal using the last 5 to 10 RF cycles. Based on our experiences, typically less than 30 RF cycles are long enough to get a good measurement. Figure 2 shows an example of the time varying electric field signal excited by a pulse. The field is calculated by the MAFIA time domain module, T3.

2.2 The Energy Method

The same procedure used to measure the resonant frequency is repeated except the dipole excitation will be at a single frequency equal to the resonant frequency obtained from the 1st time domain simulation. In addition to monitoring the time varying electro-magnetic fields at the same location, 3D electro-magnetic fields in the system are recorded as well at time $t_0 \pm \Delta t$. The t_0 is chosen at a time when the relative energy contribution from the excitation source to the total stored energy in the cavity is negligible. In order to calculate the average stored energy and the

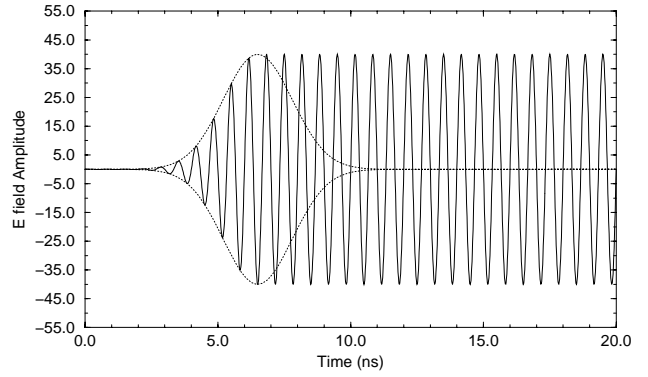


Figure 2: An electric field component is plotted as a function of time (solid line). The envelop of the driving pulse is plotted at the same time with dotted line. Only the last 5 to 10 RF cycles are used to calculate the resonant frequency of the system.

power loss at t_0 , we take,

$$\Delta t = \frac{1}{4f_{\text{drv}}}, \quad (3)$$

with $f_{\text{drv}} = \omega_0/2\pi$ as the driving frequency and ω_0 as the angular resonant frequency. With this driving source, the electro-magnetic fields will be building up coherently with time, e.g.,

$$\Phi(t) = \Phi_{\infty} \left(1 - e^{-\frac{t}{\tau}}\right) \sin(\omega_0 t + \theta_0), \quad (4)$$

where $\Phi(t)$ may denote any field component of $\vec{E}(\vec{r}, t)$ or $\vec{B}(\vec{r}, t)$, and Φ_{∞} is the field amplitude at steady state. $\tau = \frac{2Q_L}{\omega_0}$ is the time constant of the system. θ_0 is an initial phase. For a high Q_L system it is impractical to run MAFIA in time domain to obtain either an accurate value for the decay time or until the steady-state field amplitude Φ_{∞} is reached. We find that it is convenient to record the 3D fields at t_0 and $t_0 \pm \Delta t$ during the time while the fields build-up and use the fields to re-construct a quasi-time harmonic field (mode). Taking the advantage of many built-in features in the MAFIA post-processor (P code), the quasi-time harmonic field can be used readily for the calculation of Q_0 and Q_{ext} .

As indicated in Equation (4), the field build-up is approximately linear when $\frac{t}{\tau} \ll 1$. Figure 3 shows an electric field component as a function of time calculated by MAFIA during the field build-up. A linear build-up pattern of the field is exhibited. The same cavity-coupler system shown in Figure 1 is used for the simulation. The system is driven at its resonant frequency, obtained from the first time domain simulation discussed in Section 2.1.

As we pointed out earlier, the excitation dipole will inevitably contribute its energy to the total stored energy in the cavity, but its relative contribution decreases with time. Therefore it is preferable to record the fields at several times until we are sure that approximately the same Q values are obtained. Usually about 30 RF cycles are needed. To calculate $U(t)$, $P_w(t)$ and $P_{\text{out}}(t)$, the MAFIA P code

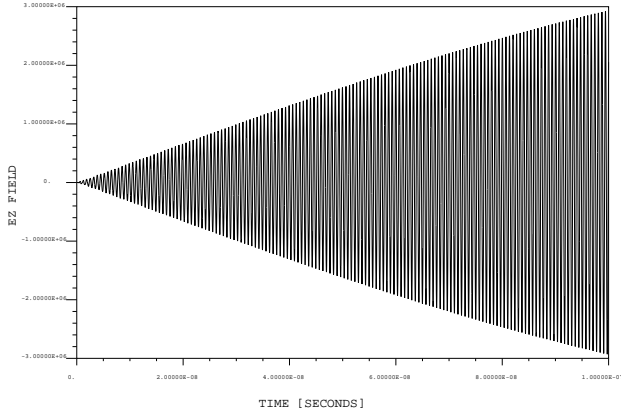


Figure 3: An electric field component is built up linearly as a function of time for a high Q cavity-coupler system driven at its resonant frequency.

is needed. In fact, the P code treats the electro-magnetic fields at time t_0 and $t_0 \pm \Delta t$ as time harmonic fields. Both the energy and power loss calculations from the P code have phase dependent terms. However, the phase dependent terms are cancelled out automatically by the careful choice of Δt in Equation (3). The stored energy at t_0 is then given by,

$$U_{\Phi}(t_0) \approx \frac{\alpha}{2} \oint_V \{ \Phi^2(t_0) + \Phi^2(t_0 \pm \Delta t) \} dv \quad (5)$$

with

$$\alpha = \begin{cases} \epsilon_0 & \text{electric energy, } \Phi = \vec{E} \\ \frac{1}{\mu_0} & \text{magnetic energy, } \Phi = \vec{B} \end{cases}$$

Similarly the power loss is calculated by,

$$P_w \approx \frac{R_s}{2} \oint_S \{ H(t_0)^2 + H(t_0 \pm \Delta t)^2 \} ds \quad (6)$$

The unloaded Q_0 is then computed by,

$$Q_0 \approx \omega_0 \frac{U_E(t_0) + U_B(t_0)}{P_w(t_0)}. \quad (7)$$

2.3 Poynting Vector Calculations

To calculate Q_{ext} , P_{out} has to be calculated using the Poynting vector $\vec{S} = \vec{E} \times \vec{H}$, which is constructed by the MAFIA P code using the recorded fields at t_0 and $t_0 \pm \Delta t$,

$$P_{\text{out}}(t_0) \approx \frac{1}{2} \int_s \{ S_{\perp}(t_0) + S_{\perp}(t_0 \pm \Delta t) \} ds \quad (8)$$

S_{\perp} is the transverse component of the Poynting vector in the waveguide. The integral in Equation (8) may be conducted at any cross section s in the waveguide away from the iris. It is also worthy to point out that E_{\perp} and H_{\perp} are in phase in the waveguide, therefore the phase dependent terms are cancelled out in Equation (8) for the same reason stated earlier.

Considering the simple geometry of the rectangular waveguide, $P_{\text{out}}(t_0)$ may be computed by an analytical formula combining with an energy density W_{wg} obtained from MAFIA by,

$$P_{\text{out}}(t_0) = c \sqrt{1 - \left(\frac{f_c}{f_{\text{drv}}} \right)^2} W_{\text{wg}}(t_0)$$

$$W_{\text{wg}}(t_0) = \frac{1}{d} [U_E(t_0) + U_B(t_0)]_d, \quad (9)$$

where $v_g = c \sqrt{1 - \left(\frac{f_c}{f_{\text{drv}}} \right)^2}$ is the group velocity; f_c is cut-off frequency determined by the geometry of the waveguide; $W_{\text{wg}}(t_0)$ is stored energy per unit length in the propagation direction with d as the length used for the stored energy calculation. The same P_{out} values are achieved from either Equation (8) or Equation (9). Q_{ext} is then directly calculated from Equation (1).

3 NUMERICAL RESULTS

Numerical simulations of the system have been performed for a set of different iris apertures, and compared with experimental measurements on a simple cold-test model. The results are shown in Figure 4.

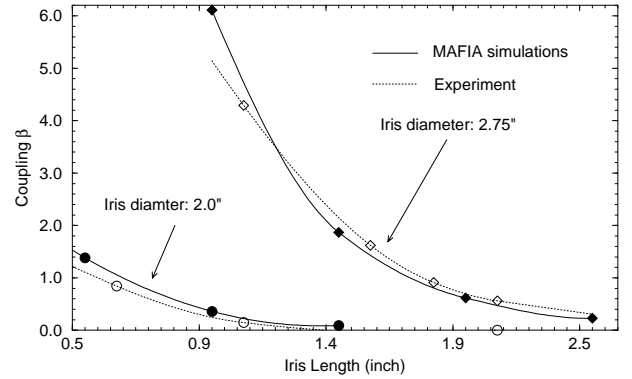


Figure 4: Coupling coefficient $\beta = \frac{Q_0}{Q_{\text{ext}}}$ is plotted as a function of iris length for two different iris diameters.

4 CONCLUSION

The time domain method works efficiently for the cavity-coupler system with high Q s and weak coupling. Good agreements are achieved between the MAFIA simulations and the experimental measurements. The method can be easily applied to other systems as well.

5 REFERENCES

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