A POSSIBLE PARTICLE-CORE APPROACH TO MISMATCHED BEAMS IN A PERIODIC FOCUSING CHANNEL

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Abstract

A method is derived for applying the particle-core analysis to mismatched beams in a periodic focusing channel. By carefully choosing the parameters to yield a favorable core frequency, Poincaré surface of section plots are obtained. The plots for a periodic solenoid channel exhibit a striking resemblance with the continuous focusing case, while those for an FODO channel exhibit strong chaosity which is not seen in the corresponding continuous situation. Some typical numerical results and a way to find adequate parameters are presented.

1 INTRODUCTION

In recent years, halo formation in intense ion beams has been extensively studied in both theoretical and numerical ways. In these studies, the so-called *particle-core model* [1] has been frequently used. In this model, we usually consider a beam propagating in a continuous focusing channel and assume that its core has the Kapchinskij-Vladimirskij (KV) distribution. In the macro-particle simulation studies of the continuously focused beams, features such as the separatrix and fixed point locations are found to be in good agreement with those obtained with the particle-core model [2].

The halo properties in periodically focused cases have also been studied self-consistently, and a close resemblance to the continuous focusing cases is found for a periodic solenoid channel unless instabilities due to structure-driven resonances occur [3]. Though the role of the particle-core resonance in periodic focusing situations can be directly investigated by applying the particle-core model, it has never been done in mismatched cases mainly due to the difficulty in finding the fundamental frequency of the system. In the particle-core model, Poincaré mapping technique is an essential tool to examine the stability properties of test particles, but we need to know the fundamental frequency of the system to use this technique. It is generally difficult to know the fundamental frequency in periodic focusing cases because there are two sources of periodicity, namely; the external focusing field periodicity and that due to initial beam-size mismatch. As the envelope is known to be stable with a reasonable choice of parameters, we try to obtain the fundamental core frequency restricting our interest to the cases where core oscillation is stable.

2 PERIODIC SOLENOID CHANNEL

Assuming the axial symmetry of the focusing channel, the time evolution of the beam envelope is governed by the envelope equation,

$$\frac{d^2 R_b}{ds^2} + \kappa(s) R_b - \frac{K}{R_b} - \frac{\varepsilon^2}{{R_b}^3} = 0, \qquad (1)$$

where R_b is the beam radius, κ(s) is the periodic function representing the external focusing field strength, K is the generalized perveance, ε is the rms emittance of the beam, and independent variable s is the distance measured along the beam line. Then, in terms of dimensionless variables, Eq. (1) becomes

$$\frac{d^2 R}{d\tau^2} + \vartheta(\tau) R - \frac{\Gamma}{R} - \frac{1}{R^3} = 0, \qquad (2)$$

where $\tau = s/S$ is taken as the independ variable with S being the focusing period. The function $\vartheta(\tau)$ is related to the zero-current phase advance σ_0, and Γ is related to the tune depression η, namely, the ratio of the space-charge depressed phase advance to the zero-current phase advance. The matched solution R_0 of Eq. (2) can be obtained with the help of an optimization code. For later reference, we here introduce a mismatch factor defined as

$$M = \left[R(0) - R_0(0)\right] / R_0(0). \qquad (3)$$

It should be noted that $R_0(0)$ corresponds to the maximum of the matched beam radius since the origin of the coordinate τ is located at the center of a focusing solenoid. With use of the smooth-approximation, Eq. (2) can be written as

$$\frac{d^2 R_s}{d\tau^2} + {\sigma_0}^2 R_s - \frac{\Gamma}{R_s} - \frac{1}{{R_s}^3} = 0, \qquad (4)$$

where R_s is the scaled beam radius in the approximation. In weakly mismatched cases, the phase advance of the breathing mode oscillation of the envelope can be approximated by

$$\sigma_m = \sqrt{2\left(1+\eta^2\right) + \frac{1}{2}\left(1+9\eta^2\right) M^2}\,\sigma_0, \qquad (5)$$

where we use a combination of a simple perturbation method and an averaging method.

We here assume that the oscillation of the core can be approximated by a simple composition of two oscillation modes, namely; one is excited by the initial beam-size mismatch (*mismatch mode*) and the other is excited by the periodic nature of a focusing structure (*structure mode*). Based on the smooth-approximation analysis above, the phase advance of the mismatch mode is expected to be σ_m. On the other hand, the fundamental period of the structure mode is apparently synchronized with the focusing structure. Thus, it is obvious that if $\sigma_m/2\pi$ is a rational number *n/m*, the mismatched envelope is exactly periodic in τ with the period of *m* times a focusing period. In such cases, we can easily obtain a Poincaré surface of section plot by plotting test particle location every *m* focusing periods. That is our strategy to apply the particle-core method to mismatched beams in a periodic channel.

Finally, we write down the equation of motion for a test particle. Assuming that the core has a KV distribution and test particles have no augular momentum, the equation of motion in terms of the dimensionless variables is given by

$$\frac{d^2x}{d\tau^2} + \vartheta(\tau)x - \frac{\Gamma}{R^2}x = 0 \quad (|x| \le R), \tag{6a}$$

and

$$\frac{d^2x}{d\tau^2} + \vartheta(\tau)x - \frac{\Gamma}{x} = 0 \quad (|x| > R). \tag{6b}$$

Figure 1 shows an example in which we consider a periodic solenoid channel having σ_0=45° and a 50% filling factor, and the beam parameters are set to be η=0.5 and M=0.3. These parameters are determined to yield σ_m=360/5=72° by Eq. (5) with the help of an optimization code. As shown in Fig. 1, the fundamental period of the core oscillation coincide with five focusing periods with a very good accuracy. We can also see in Fig. 1 that the core oscillation is almost dominated by the mismatch mode, and the contribution from the structure mode is fairly small. Plotting the single particle position every five focusing periods, we successfully obtain a Poincaré surface of section shown in Fig. 2, which exhibits a striking resemblance with continuous focusing cases.

3 FODO CHANNEL

The same method is also applicable to the beams in channels without an axial symmetry such as FODO channels. Assuming that the zero-current phase advance and emittance are the same in the horizontal and vertical directions, the envelope equations are given in terms of dimensionless variables as

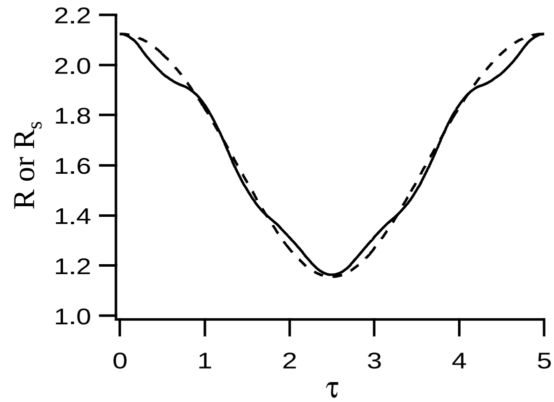

Figure 1: Time-evolution of the beam envelope in a periodic solenoid channel. Solid line: periodic solenoid channel. Broken line: smooth-approximation.

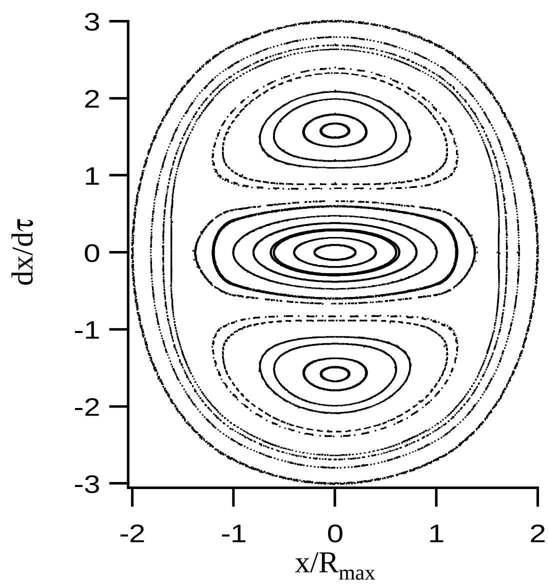

Figure 2: Poincaré surface of section plot for a periodic solenoid channel. The same parameters with Fig. 1 are employed.

$$\frac{d^2X}{d\tau^2} + \vartheta(\tau)X - \frac{2\Gamma}{X+Y} - \frac{1}{X^3} = 0, \tag{7a}$$

and

$$\frac{d^2Y}{d\tau^2} - \vartheta(\tau)Y - \frac{2\Gamma}{X+Y} - \frac{1}{Y^3} = 0, \tag{7b}$$

where X and Y are, respectively, the scaled beam half-width for the horizontal and vertical directions.

Note here that not only the breathing but also quadrupole mode oscillation can be excited in an FODO channel. For the quadrupole mode oscillation, the frequency of the mismatch mode is given by

$$\sigma_m = \sqrt{1 + 3\eta^2 + 5\eta^2 M^2}\,\sigma_0 . \qquad (8)$$

The equations of motion for a test particle initially located on the horizontal plane can be written [4] as

$$\frac{d^2x}{d\tau^2} + \vartheta(\tau)x - \frac{2\Gamma}{X(X+Y)}x = 0, \qquad (|x| \le X) \text{ (9a)}$$

and

$$\frac{d^2x}{d\tau^2} + \vartheta(\tau)x - \frac{2\Gamma}{x^2 + |x|\sqrt{x^2 + Y^2 - X^2}}x = 0. \; (|x| > X) \text{ (9b)}$$

First, we will consider the case where the breathing mode oscillation of the core is excited. Figure 3 shows an example in which we consider an FODO channel having σ_0=45° and a 50% filling factor, and the beam parameters are set to be η=0.5, and M=0.3. We can see in Fig. 3 that the modulation of the core oscillation due to the periodic nature of the focusing field is much larger than in periodic solenoid cases. A Poincaré surface of section plot is obtained as shown in Fig. 4.

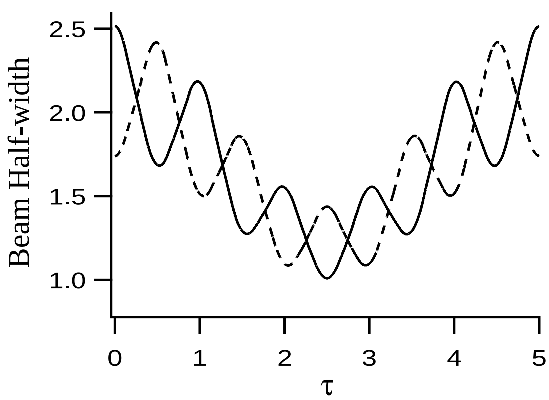

Figure 3: Time-evolution of the beam envelope in an FODO channel (breathing oscillation case). Solid line: horizonral half-width. Broken line: vertical half-width.

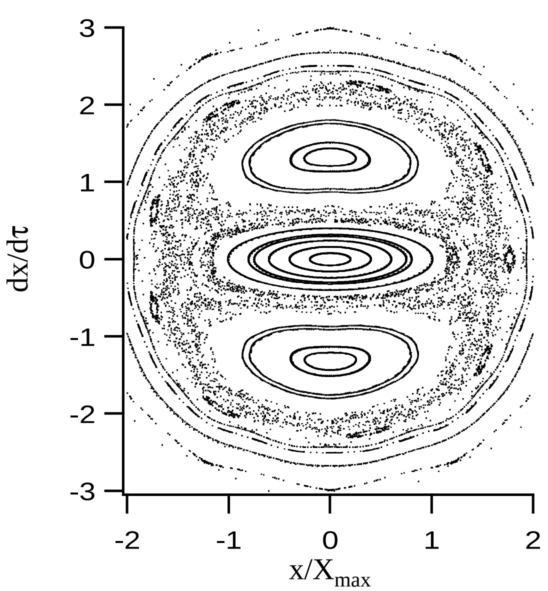

Figure 4: Poincaré surface of section plot for an FODO channel (breathing oscillation case). The same parameters with Fig. 3 are employed.

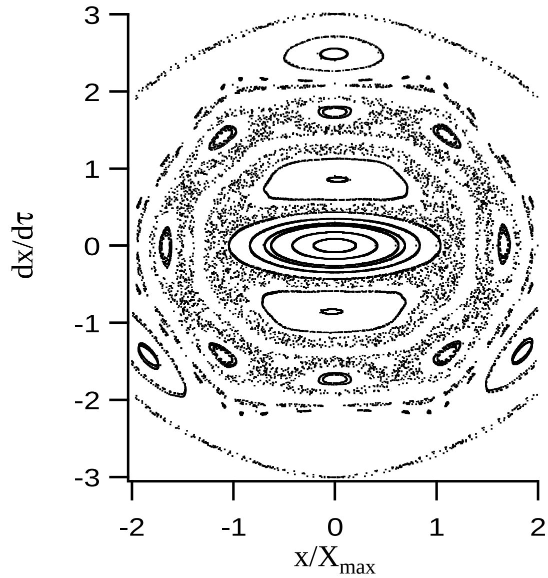

Figure 5: Poincaré surface of section plot for an FODO channel (quadrupole oscillation case).

Second, we consider the case where the quadrupole mode oscillation is excited. Figure 5 shows the Poincaré plot for a core executing the quadrupole mode oscillation. An FODO channel having σ_0=52° and a 50% filling factor is considered, and the beam parameters are set to be η=0.5, and M=0.3. These parameters are determined again to yield σ_m=72°.

Both pictures show strong chaosity which is not observed in solenoid cases. It suggests that the strong modulation of the core oscillation due to an alternating-gradient focusing field affects test particle stability, and causes an increase of the halo intensity.

4 ACKNOWLEDGEMENTS

We thank Dr. R. A. Jameson and Dr. S. Machida for helpful discussions on several points in this paper.

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