

# EFFECTS OF ALTERNATING CELL MISALIGNMENTS ON THE DDS

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## *Abstract*

We study some effects of cell misalignments in accelerator structures in which each cell is displaced the same amount and direction but with sign which alternates from cell to cell. In particular we study the manifold radiation in a damped detuned structure (DDS1) and wakefield effects in a uniform constant impedance structure. A synchronous wave aliasing phenomenon is observed. In previous treatments the effect of cell misalignment has been modeled by replacing it by beam displacement. Here we compare finite difference simulation of cell misalignment with the beam displacement model. We conclude with discussions about the relevance of our results for general misalignment profiles.

## 1 INTRODUCTION

In a damped detuned accelerator structure the dipole modes are detuned such that their synchronous frequencies vary with error function profile along the length of the structure, so that the frequency distribution function is Gaussian. In addition four damping manifolds, connected to each cell, extend along the length of the structure. The detuning leads to a rapid initial fall off of the transverse wakefield and the damping prevents the recoherence reappearance which would otherwise occur [1]. The utility of the manifold radiation as a source of information about beam position within the structure and about structure misalignment is discussed in [2] and [3]. The spectral distribution of manifold radiation, computed by means of the equivalent circuit theory, was found to be in good agreement with observations [2, Fig. 1]. Furthermore it was shown, by computing the effect of beam displacements localized to a few cells within the structure, that there was a strong correlation between the localization position and the frequencies emitted into the manifold [2, Fig. 2] allowing manifold radiation frequency to be mapped onto cell position. Reference [3] reports measurements of beam position which minimizes manifold radiation as a function of frequency. By assuming that this minimum position is centered with respect to the cells which provide the frequency being observed and applying the mapping function defined above, one obtains a measure of cell position as a function of cell number and good agreement with mechanical misalignment measurements.

Implicit in the above is the assumption that localized cell displacements can be taken into account by replacing them with localized beam displacements in the equivalent circuit theory. In reference [4], which discusses the computation of wake functions associated with actual beam misalignments as well as with structure misalignments, the same assumption is made. Also the computational and experimental evidence does not really support the above simple picture for sharp discontinuities (such as actually occurred in DDS1).

The remainder of this paper is devoted to a partial study of the effect of a special misalignment. We assume the cells to be displaced alternately upward and downward by equal amounts as one proceeds along the structure. Because this is the opposite of a smooth misalignment it has some bearing on the issues raised above. It is also simple enough to allow a direct test of the equivalence of localized beam displacement and cell displacement. Such a structure can be analysed by treating each cell pair as a single cell and carrying out an analysis completely analogous to the standard DDS theory. While we have carried out only a small part of such a program, a number of interesting phenomena have emerged. The concluding section includes some discussions on the applicability of what we have learned to the more general misalignment problem.

## 2 RELATIVE DISPLACEMENT OF 2 IDENTICAL STRUCTURES

The study of both the detuned structure without damping and the DDS begins with the study of individual cells as part of a uniform structure. Proceeding analogously here we consider a cell pair as a unit in a structure of period  $2p$ , where  $p$  is the period of the structure when the cell to cell displacement  $2d$  vanishes. Mode patterns and frequencies as a function of phase advance over the  $2p$  period were obtained from the finite difference program GdfidL [5] for a typical cell, similar to cell 103 of a detuned structure. The mesh for these calculations is shown in Fig. 1. Considerable care in mesh construction is required to preserve the symmetry of the problem, obtain sufficient accuracy, and avoid the introduction of mesh artifacts as  $d$  is varied. The Brillouin (dispersion) diagram for the accelerating mode and first two dipole modes is shown in Fig. 2.

As is conventional the diagram is folded so that the phase advance range is limited to 0 to 180 degrees. Also

shown on Fig. 2 is the (folded) light line. Computations were performed for  $d = 0, 0.1825,$  and  $0.365$  mm. For comparison we note that the diameter of the iris aperture is  $9.5926$  mm.

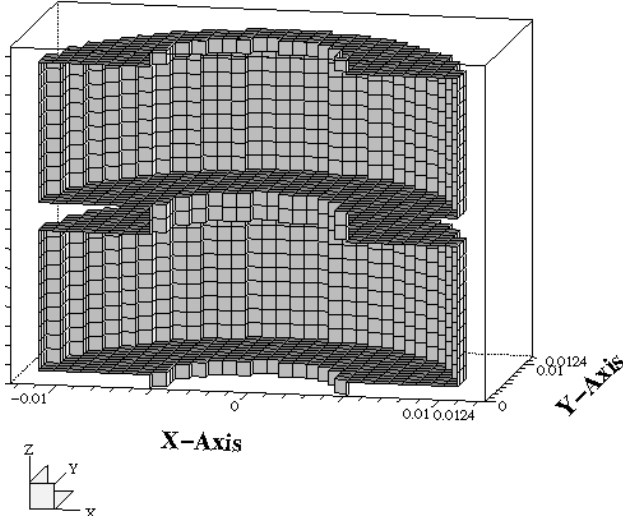


Figure 1: GdfidL mesh used for double period structure, viewed from the  $y$  symmetry plane. A coarse mesh is used to illustrate the geometrical features.

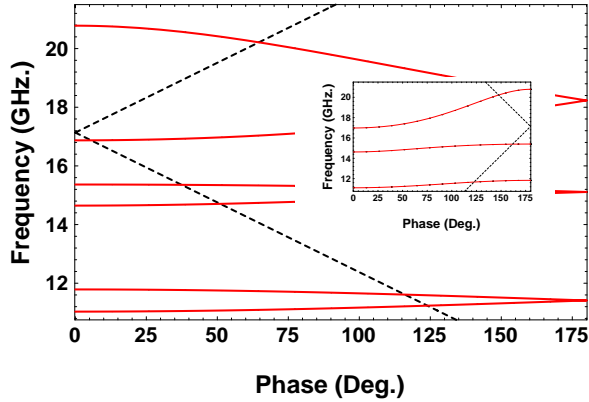


Figure 2: Brillouin diagram for double period structure with  $0$ mm displacement of structure.

We first discuss Fig. 2 for the case  $d = 0$ . In that case the structure is periodic over the distance  $p$  (as well as  $2p$ ), and the conventional Brillouin diagram, folded with respect to  $p$  rather than  $2p$ , is shown in the Fig. 2 inset. One sees that the single mode curves of the conventional form become doubled by folding back the  $90$  to  $180$  degree sections, leading to the paired curves of the  $2p$  representation connected at  $180$  degrees. One also sees that the three synchronous frequency points (light line intersections) of the conventional representation become supplemented by three more which we refer to as alias synchronous frequencies. The longitudinal voltages for velocity of light particles vanish identically at these frequencies.

We next discuss what happens when one cell of the pair is offset to the left a distance  $d$  and the other offset to the right by  $d$ . As far as the Brillouin diagram is concerned there are small frequency shifts, varying as  $d^2$ , at each phase advance. The shifts of the six synchronous frequencies in KHz for  $d = 10$  micrometers were inferred from the simulations to be  $-7.58, -13.2, 15.3, 27.5, 17.8,$  and  $-6.95$  respectively, starting from the lowest frequency to the highest. Numbering them one to six, the odd numbers correspond to the alias cases. The fact that the coincidence of the curve pairs remains at  $180$  degrees is evidence that the mesh variation as  $d$  is varied is under control. The curves separate if the cells differ from one another.

The interaction of a  $v = c$  beam with a mode of the structure is controlled by the longitudinal voltage integral:

$$V_p(x, y) = \int_0^p E_z(x, y, z) \exp(j\omega z / c) dz$$

where  $P$  is the structure period ( $p$  or  $2p$  in our case), and  $E_z$  and  $\omega$  refer to the mode under study. We base our comparison of beam displacement with cell displacement on the behaviour of  $V_p$  and  $V_{2p}$ .

We consider the beam displacement model first. The beam displacement approximation is based upon the  $d = 0$  structure, a structure with period  $p$ . At a synchronous point we have besides the expression given above:

$$V_p(x, y) = \int_p^{2p} E_z(x, y, z) \exp(j\omega z / c) dz$$

while for the alias synchronous modes this relation is

$$V_p(x, y) = -\int_p^{2p} E_z(x, y, z) \exp(j\omega z / c) dz$$

Thus at a synchronous point the beam displacement model yields  $V_{2p}(x, y) = V_p(x-d) + V_p(x+d)$ , while at an alias synchronous point it yields  $V_{2p}(x, y) = V_p(x-d, y) - V_p(x+d, y)$ . We recall that at a synchronous point of a periodic structure,  $V_p$  is a solution of the 2D laplace equation at all points within a region  $A$  defined by the area common to the beam apertures. Since the  $d = 0$  structure is also rotationally symmetric about the beam axis, within  $A$   $V_p$  is constant for the monopole case and a constant times  $x$  or  $y$  for the two dipole mode polarizations. Hence for all synchronous cases of the modes we study here,  $V_{2p} = 2 V_p$ . Away from a synchronous point  $V_p$  no longer has this simple behaviour. However the departure from it is found to be very small, and the symmetry properties under reflection are preserved. It follows that

$$V_{2p} = -2d(d/dx)V_p$$

to an excellent approximation. This means that the alias synchronous modes have their  $x$  reflection behaviour reversed in parity. The monopole becomes an  $x$  dipole, the  $x$  dipoles become monopoles, and the  $y$  dipoles become quadrupoles. Note that to the extent that  $V_p$

satisfies pure 2D laplacian behaviour the x dipoles behave like a pure monopole and the  $V_{2p}$  vanish for the monopole and y dipoles. Hence the alias dipole and quadrupole effects are very weak.

We have compared these results with the behaviour of GdfidL simulations at  $y = 0$  and values of  $x$  over the interval  $[-4,4]$  mm. To compare different cases we require that all have the same stored energy over the  $[0,2p]$  interval. Also since the GdfidL simulations have an arbitrary overall phase, we compare absolute values of  $V_{2p}$ .

Comparisons were made at  $d = 0.365$  and  $0.1825$  mm. First a few general observations. Since all modes are synchronous with respect to the  $2p$  period, all of the  $V_{2p}$  satisfy the 2D laplace equation. The rotational invariance about the beam axis is, however, broken, and hence the simple  $x$  dependence of pure multipoles may be altered. On the other hand, one can show that the behaviour under  $x$  reflection should be preserved. Hence we expect the alias monopole to be principally dipole, and the alias x dipoles to be principally monopole, an expectation born out by the simulations.

We discuss the even numbered modes first. For the monopole case, there is no detectable difference between the displaced beam model and the simulation at  $0.1825$  and no detectable variation from pure monopole behaviour. For the first dipole there is a 0.1% deviation for  $d = 0.1825$  and 0.25% for  $d = 0.365$  and no evidence for departure from pure dipole behaviour. For the second dipole the discrepancy was 10% at  $d = 0.1825$  and 30% at  $d = 0.365$  and there was slight evidence for a deviation from pure dipole behaviour. We note that the voltage integral is abnormally small for this mode and therefore seems to be more sensitive to field changes associated with the displacement. The discrepancy does appear to be decreasing with  $d$  and may well become very small at  $d$  values likely occur in practice. These results provide no evidence against the use of the displaced beam approximation for calculating the transverse wakefunction as was done in [4].

For the odd numbered modes, which correspond to the aliased synchronous modes, the comparison is much poorer. The departure from pure multipole behavior is substantial but not significant at beam displacements which are likely to occur in practice. The result of most practical interest is the deflecting mode 1, which occurs at a frequency within the accelerating mode band and would thus be unaffected by manifold damping. The magnitude is 8 times as large as that given by the beam displacement model and appears to be proportional to  $d$ . The kick factor associated with it then varies as  $d^2$ , and assuming this to be the case is a factor  $10^3$  smaller than a typical first band dipole mode for  $d = 36 \mu\text{m}$ . For mode 3 the dominant monopole component is over estimated 15% for  $d = 0.1825$  and 20% at  $0.365$  by the beam displacement model. The figures for mode 5 are 65 and 75% respectively. Being non deflecting and subject to

manifold damping these modes are of little practical wakefield interest.

### 3 MANIFOLD RADIATION PHENOMENA

An equivalent circuit computation of the manifold radiation spectrum from DDS1 for beam displacements alternated from cell by equal amounts yielded a striking shift in the spectrum which we interpreted as implying that each cell was excited at its aliased synchronous frequency rather than its actual synchronous frequency. In the future we plan to investigate the extent to which this effect is localised. The other striking result was the fact that the radiation was predominantly upstream rather than downstream, an effect which was consistent with the explanation of directional asymmetry given in [1].

### 4 CONCLUDING COMMENTS

While the evidence is limited to the special situations that we have studied, we attempt here to draw some tentative general conclusions. First, frequency shifts due to misalignments are unimportant compared to those likely to arise from dimensional errors in the cells. Second, for the computation of transverse wakes due to dipole modes the displaced beam approximation is reliable. These misalignment may also cause a longitudinal wake from the dipole modes, but it is small and of little practical importance. Third, we have no evidence that misalignment produces any significant effect on the accelerating mode band. The transverse wakes from aliasing are very small, and we have no evidence of transverse wakes produced directly by the RF drive.

### 5 ACKNOWLEDGMENTS

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