LINAC DESIGN ALGORITHM WITH SYMMETRIC SEGMENTS

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Abstract

The cell lengths in linacs of conventional design are typically graded as a function of particle velocity. Use of symmetric cells in short segments of both the coupled-cavity drift-tube linac (CCDTL) and coupled-cavity linac (CCL) simplifies the cavity design. Mechanical design and fabrication are also simpler without compromising the performance. We have implemented a design algorithm in the PARMILA code for symmetric cells and symmetric multiple-cavity segments. This feature significantly reduces the number of unique components. We have compared the performance of a symmetric-segment linac with a more conventional graded-cell-length linac.

Elements in a Symmetric Unit

Figure 1 defines a cell as an acceleration unit that includes a single rf accelerating gap. A cavity contains one or more accelerating cells and may be resonantly coupled to other cavities.

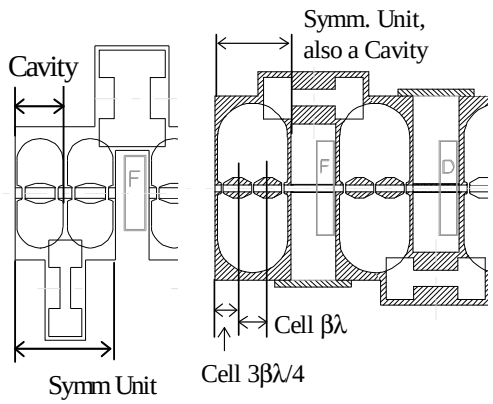

Figure 1. Cell, cavity, and symmetric unit in a 1- and 2-drift-tube CCDTL.

CCDTL [1] cavities may contain two types of cells. One type extends from the center of one drift tube to the center of the next. The other type starts from the up-stream face of a cavity and extends to the center of the first drift tube. The reverse of this later type occurs in the last cell of a CCDTL cavity. In a graded-β design, each cell in a CCDTL would have a unique length and the resulting cavities would be asymmetric. In a CCL, cells and cavities are synonymous and are typically symmetric.

We define a symmetric unit or segment as a series of coupled cavities all of which by themselves are symmetric and all of which have identical geometry. In symmetric units the rf fields in each cavity are equal. In the CCDTL, the field amplitude in each cell may be different but is fixed relative to the other cells within the cavity. In a symmetric unit, symmetrically placed cells have equal fields.

Symmetric units may include space between cavities for focusing elements. Symmetry does not constrain the placement of the focusing elements. Ignoring the quadrupole lenses, a symmetric unit is non-directional. It will perform correctly with either end located up-stream. The quadrupoles position is independent of the cavity position so long as it fits within the space between the cavities.

Algorithm of Linac Design

In a graded-β linac design (β = relativistic particle velocity) a synchronous particle determines the cell length by requiring it to arrive at the center of the accelerating gap when the rf fields are at the "synchronous" phase. Any phase programming along the linac is folded into the increasing cell lengths which are otherwise proportional to βλ. (λ = the rf wavelength.) In PARMILA [2] the thin-lens approximation determines the acceleration across a gap [3]. Longitudinally, the Prome term [4] corrects the phase advance across the gap ensuring that thin-lens approximation conserves emittance.

PARMILA divides the gap at its electrical center z_c. In a multiple-drift-tube CCDTL cavity, the fields in the end cells are asymmetric and the geometrical and electrical centers do not coincide. Therefore, we use the code CDTFISH [5] to design the CCDTL cavities so that the electrical center of the gap coincides with the center of the gap as defined by PARMILA. The value of z_c, can be found by satisfying the equation:

$$\int_{z_c}^{\infty} E_z(z)\cdot\sin(kz)dz = -\int_{-\infty}^{z_c} E_z(z)\cdot\sin(kz)dz \,. \quad (1)$$

CDTFISH adjusts the length of the cavity noses until the electric field integrals in equation (1) are equal for a given value of z_c. It does this while maintaining its resonant frequency. For each geometrical velocity β_g SUPERFISH [6] then calculates the transit time factors and other relevant cavity parameters. This procedure is now used by PARMILA to determine the cell lengths of a graded-β linac. In principle, a linac designed by a graded-β method accelerates slightly more efficiently than one designed by the symmetric method.

To maintain an average phase synchronism over a symmetric unit, PARMILA sets the entry phase to the symmetric unit so that average phase equals the design phase. The average phase is the phase, averaged over all gaps in the symmetric unit, of the reference particle when it arrives at the gap centers. In conventional graded-β designs, the reference particle sees cells of increasing length. However, imposing

equal cell lengths, optimized near the mid section of the symmetric unit, inevitably results in cells that are the incorrect length at both ends of the symmetric unit. A reference particle sees a longer cell length than preferred in the earlier cells, and a shorter cell length in the later cells. This results in a phase slip, shown in Fig. 2 through the symmetric unit. We determine β_g so that the reference particle phase increases at successive gaps until the mid section, then, decreases from above the design phase until the end of a symmetric unit.

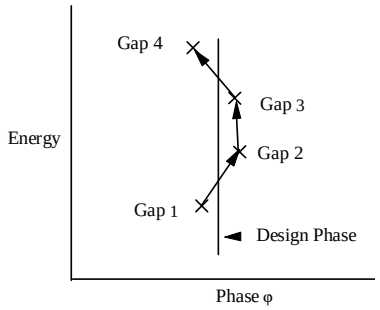

Figure 2. PARMILA adjusts the symmetric unit length and the entry phase until the design phase equals the average reference particle phase.

Determining the Geometrical β_g

In designing a symmetric linac, we define a "design particle" and a "reference particle." We no longer use a "synchronous particle" that arrives at the center of cells at the synchronous phase. The purpose of the loosely-defined design particle is to keep track of the phase programming. The design phase may not correspond to phase at any particular gap for multiple-gap symmetric unit.

The reference particle is a sample particle that obeys the particle dynamics. PARMILA chooses the length of the symmetric unit such that all the phases seen by the reference particle at the gap centers are close to the design phase. We determine the geometrical velocity β_g from the requirement that the time required for the reference particle to traverse a symmetric unit is equal to a time that should lapse to maintain synchronism at both ends of the unit. We impose no restrictions on the phase or energy of the particle at the gaps within the unit. The total length of a symmetric unit must be $n_s\beta_g\lambda/2$ (n_s is an integer). The geometrically determined velocity β_g is constant over the symmetric unit. We have investigated alternate approaches for determining β_g such as choosing its value corresponding to half the energy gain in the segment, or to half the velocity gain in the segment. These schemes were unsatisfactory, particularly when the number of cells in a symmetric unit was small.

In our calculation of β_g we include the Prome-phase correction [4] as an extra time contribution in a gap transformation. Previously PARMILA used this only in the particle-dynamics portion of the calculation. The effect of a positive Prome value is a reduction in the cell lengths. For example, for the 2-drift-tube CCDTL in Fig. 1, The following expression determines β_g.

$$\frac{\frac{1}{4}\beta_g\lambda}{\beta_{1i}c} + \frac{\frac{1}{2}\beta_g\lambda}{\beta_{1o}c} + P_1 + \frac{\frac{1}{2}\beta_g\lambda}{\beta_{2i}c} + \frac{\frac{1}{2}\beta_g\lambda}{\beta_{2o}c} + P_2 + \frac{\frac{1}{2}\beta_g\lambda}{\beta_{3i}c} + \frac{\frac{1}{4}\beta_g\lambda}{\beta_{3o}c} + P_3 = (\frac{1}{4}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{4})T \quad (2)$$

where T is the rf period (λ/c). β_{ji} and β_{jo} are the relativistic velocities of entry to and exit from the gap j. Terms P_j (j=1,2,3) are the time delays derived from the Prome phase corrections for each gap ($P_j = \lambda/(2\pi c) * (\text{phase correction})_j$.) With this correction, the cell lengths and the particle dynamics through the symmetric unit become consistent. An error the order of 0.1 degrees per cell accumulates during the cell generation process when the Prome correction is neglected.

After determining the symmetric-unit length, PARMILA adjusts its longitudinal position so that the reference particle arrives at the entrance to the symmetric unit at the correct phase. The exit phase of the reference particle traversing a symmetric unit differs from its entrance phase to the next symmetric unit. Also the exit phase of a reference particle through a symmetric unit does not necessarily equal its entrance phase. PARMILA adjusts the entry phase to minimize this phase difference using single particle dynamics through each symmetric unit. After determining the entry phase, the code calculates the length of the drift space between the units to provide the exit to entry phase difference. If the design requires a ramp in the phase, adjusting the drift-space length provides the ramp. If external-quadrupole magnets require additional space, it is added in units of $\beta\lambda/2$ to maintain the correct reference particle phasing.

If the linac design requires a ramp in the accelerating field gradient, we step E_0, the average axial electric field, from unit to unit while maintaining E_0.constant within the unit. For a given ramp in E_0, the value of β_g is required to calculate the needed value of E_0. Therefore, we iterate about 5 cycles through each symmetric unit until the correct β_g is found for the E_0 calculation.

The Entry Phase into a Symmetric Unit

The average phase of reference particle must bear some relation with the design particle phase. Because the design particle is simply a programmed phase angle, there are a number of ways to approach this problem. Using a reference particle that follows the correct beam dynamics, one can constrain the entry and exit phases of the reference particle through the symmetric unit to be equal. However, if the number of cells in a symmetric unit is small (2 or 3), we often encounter a difficulty. The extreme phases (phase at gap 1 through 4 in Fig. 2) do not bracket the design phase. Another approach requires the equal phase angles at the center of first and the last gap in the unit as seen by the reference particle. This approach also suffers from the same problem.

We therefore require that the average of reference particle phases at gap centers in a symmetric unit equal the design phase. This approach ensures the extreme phases in the symmetric unit bracket the design phase. The entry and exit phases are not necessarily equal. In the PARMILA code, we employ Brent's zero-crossing technique [7] in determining the entry phase angle. For each iteration, a single particle is propagated through the symmetric unit. This method converges quickly and accurately.

When the number of gaps becomes large in a symmetric unit, β_g is appreciably different from the actual particle β near both ends of the segment. This situation requires a correction in the transit time factors for the reference particle. We use CDTFISH to design the cavity for specified β_g. Then SUPERFISH calculates the transit time factors (T, T′, S, S′, etc.) assuming that the reference particle velocity $\beta=\beta_g$. When the reference particle β is not equal to β_g, we expand the transit time factors around β_g with respect to the wave number k (k is $2\pi/\beta\lambda$). PARMILA uses this expansion in for both the single-particle-dynamics calculations in the linac design and in the multiple-particle simulation sections of the code.

After determining the entry phase, PARMILA stores the phases at the entrance, center and exit of each cell as well as the energy at the exit of the unit. Normally, the exit phase of one unit is not equal to the entry phase of next one. The source of this can be partly a ramp in the phase, and partly the phase slip through the multiple cells in the unit. Adding extra space between units corrects this small phase discontinuity. The PARMILA code generates one additional symmetric unit beyond the end of linac. It uses this additional unit to calculate the correct spacing between the end of this section of linac and the next section if more follows. The design of the next unit uses the exit energy of this unit as its starting energy. The linac is designed by repeating this process until the required energy is achieved. All of the pertinent design information is stored in memory for the multiple-particle beam-dynamics simulation.

In a more general CCDTL structure (see Fig. 1, CCDTL with 2 drift tubes), the cell lengths within a cavity may differ according to their position in the cavity. Multiple cavities that are completely interchangeable may comprise symmetric units. In a symmetric unit, if there are no external quadrupoles between cavities, either end can be placed to the beam up-stream. If there are multiple drifts within a symmetric unit for quadrupoles, the unit may not be reversible because the drift lengths may be different. Each symmetrically designed unit has a unique β_g and all of the cell lengths within the segment are proportional to $\beta_g\lambda$.

Example: Design of 2-Drift-Tube CCDTL

We designed a 2-drift-tube CCDTL that accelerates a 100-mA proton beam from 8 to 20 MeV in two ways; The conventional graded-β approach and the symmetric unit approach. Table 1 compares the two of designs at the end of 59 cavities.

Table 1.
CCDTL with 2 drift tubes: graded β and symmetric designs.

	Length (cm)	Energy (MeV)	# of cavities	#of cells
Graded-β	1639.9	20.274	59	177
Symmetric	1637.9	20.215	59	177

These designs use a ramp in both the cavity field amplitude and the synchronous phase: The cavity phase varies from -54° to -40° and the field E0 varies from 1.56 MV/m to 2.26 MV/m. The two designs differ by 0.06 MeV and about 2 cm in length.

Summary

The new version of PARMILA can design symmetric cavities and symmetric linac sections. It does this for both CCDTLs and CCLs. Then it calculates the beam dynamics through the linac. The symmetric design process simplifies the engineering and eases the fabrication. The difference in performance of the graded β design and the symmetric design of a CCDTL is small.

Acknowledgment

Work is supported by the US Department of Energy.

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