BEAM SELF-EXCITED RF CAVITY DRIVER FOR A DEFLECTOR OR FOCUSING SYSTEM

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Abstract

A bunched beam from an accelerator can excite and power an rf cavity which then drives either a deflecting or focusing (including nonlinear focusing) rf cavity with an amplitude related to beam current. Rf power, generated when a bunched beam loses energy to an rf field when traversing an electric field that opposes the particle's motion, is used to drive a separate (or the same) cavity to either focus or deflect the beam. The deflected beam can be stopped by an aperture or directed to a different area of a target depending on beam current. The beam-generated rf power can drive a radio-frequency quadrupole that can change the focusing properties of a beam channel as a function of beam current (space-charge-force compensation or modifying the beam distribution on a target). An rf deflector can offset a beam to a downstream sextupole, effectively producing a position-dependent quadrupole field. The combination of rf deflector plus sextupole will produce a beam current dependent quadrupole-focusing force. A static quadrupole magnet plus another rf deflector can place the beam back on the optic axis. This paper describes the concept, derives the appropriate equations for system analysis, and gives examples. A variation on this theme is to use the wake field generated in an rf cavity to cause growth in the beam emittance. The beam current would then be apertured by emittance defining slits.

Deflector System

Figure 1 shows the concept in a system designed to aperture a high current beam. The RF generator and deflector are conceptually shown as two units. The beam deflection angle is proportional to the beam current. This deflection becomes a displacement at the beam collimator. Permanent magnet non-linear focusing magnets can enhance the operation of the RF deflector.

Beam Interaction with an Rf Field

In this section, a differential equation describing the rf field generated in a cavity excited by a bunched beam is derived and solved. This differential equation depends on the energy deposited in the cavity by the beam and the energy lost in the cavity due to resistive wall losses. We consider a TM_{010} mode single-cell cavity (DTL type) where the electric field is along the beam direction and is concentrated on the axis of the cavity between the drift tube noses.

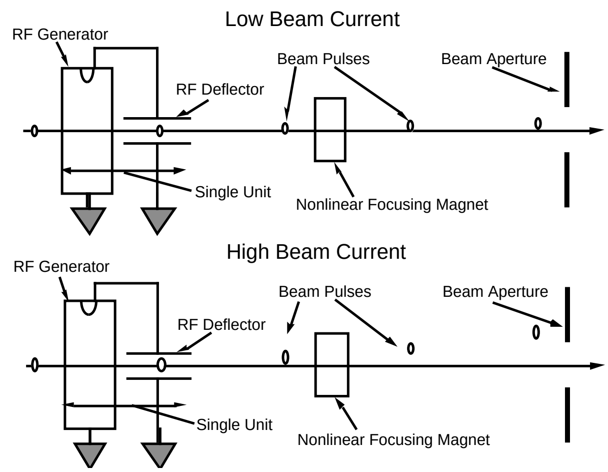

Fig. 1. RF Deflector Concept.

The bunched beam from the accelerator drives a cavity that produces rf power which then drives a beam deflecting cavity. The deflecting cavity could be followed by nonlinear magnets and then phase-space defining apertures to remove the deflected beam.

Work done against the rf electric field by particles traversing the gap adds energy to the rf field. This gain in the field energy, ΔU_P, due to one particle is

$$\Delta U_P = \int_{-Z_g/2}^{Z_g/2} eE_g \cos(\omega t + \varphi)\, dz \tag{1}$$

where

$$z = vt, \quad \frac{\omega}{v} = \frac{2\pi}{\beta\lambda}, \tag{2}$$

e is the charge on an electron, Z_g is the gap length, E_g is the gap voltage, ω is 2π times the rf frequency, β is the velocity of the particle with respect to the velocity of light, λ is the free space rf wavelength, v is the particle's velocity, t is time, and φ is the rf phase when a particle enters the gap.

Assuming that the change in particle energy in crossing the gap is small compared to the particle's kinetic energy and treating v as a constant, Eq. (1) is integrated to obtain

$$\Delta U_P = eE_0 T\beta\lambda \cos\varphi \tag{3}$$

where, E_o is an average field strength defined by

$$E_g Z_g = E_0\beta\lambda \;, \tag{4}$$

and T is a transit time factor. The transit time factor is defined as

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$$T = \sin\left(\frac{\pi Z_g}{\beta\lambda}\right) \Big/ \left(\frac{\pi Z_g}{\beta\lambda}\right) . \tag{5}$$

Equation (3) gives the energy gain in the rf electric field due to one particle crossing the rf-gap.

Beam Bunch Rf Energy Gain

The individual charges in a beam bunch enter the rf-cavity at different phases φ. The phase distribution of the particles is described by the function ρ(φ). The total charge per beam bunch is

$$q = \int_{-\pi}^{\pi} \rho(\varphi) d\varphi . \tag{6}$$

Let,

$$\Delta U_B = \frac{\text{average rf field power gain}}{\text{beam bunch}}$$

be the energy deposited in the cavity by a complete beam bunch, and use Eq. (3) to obtain

$$\Delta U_B = E_0 T \beta\lambda \int_{-\pi}^{\pi} \rho(\varphi)\cos(\varphi) d\varphi . \tag{7}$$

The integral in Eq. (7) can be defined in terms of a dimensionless charge-distribution form factor as

$$F = \frac{1}{q}\int_{-\pi}^{\pi} \rho(\varphi)\cos(\varphi) d\varphi . \tag{8}$$

The value of F is less than 1 (it is equal to 1 for a δ function distribution). This form factor is rather insensitive to beam bunch length. For example, assume that ρ(φ) is described by the rectangular distribution

$$\rho(\varphi) = \begin{cases} q/2\varphi_0 & -\varphi_0 < \varphi < \varphi_0 \\ 0 & \text{otherwise} \end{cases} , \tag{9}$$

where φ_o is the phase extent of the distribution. Then,

$$F = \sin(\varphi_0)/\varphi_0 . \tag{10}$$

When $\varphi_o = 0$, $F = 1$, and when $\varphi_o = \pi/2$ (severe debunching), $F = 0.64$. Equations (24) and (25) (derived below) show that the maximum-generated electric field scales as F. We see that the rf-electric field is somewhat insensitive to significant beam debunching.

Let U_T equal the total rf field energy in the cavity. The rf electric field will scale as the square root of U_T. Combining Eqs. (7) and (8) and defining the constant k_1 as

$$k_1 = E_0 / U_T^{1/2} \tag{11}$$

gives

$$\Delta U_B = q k_1 T \beta\lambda F U_T^{1/2} . \tag{12}$$

The constant k_1 depends on the electric field distribution in the cavity and is a function of the cavity geometry. We will later assume a model for the electric field distribution that will permit a rough calculation of k_1.

Resistive Wall Losses and *Q*

Equation (12) gives the rf field energy gain due to one beam bunch crossing the rf-cavity gap against the rf electric field. There are power losses in the cavity due to the finite resistance of the cavity walls. This power loss can be determined from the Q of the cavity defined as

$$Q = \omega U_T / W_L \tag{13}$$

where W_L is the average rf power loss per unit time. The rf energy loss in one rf cycle (time $\tau = 2\pi/\omega$) is then

$$W_L \frac{2\pi}{\omega} = \frac{\omega U_T}{Q}\frac{2\pi}{\omega} . \tag{14}$$

RF Time Dependent Field Equation

The change in total rf power per time is

$$\frac{\Delta U_T}{\Delta t} = \frac{\Delta U_T}{2\pi/\omega} = \frac{\Delta U_B}{2\pi/\omega} - \frac{W_L\, 2\pi/\omega}{2\pi/\omega} .$$

Using Eqs. (12), and (13) gives

$$\frac{dU_T}{dt} \approx \frac{\omega q k_1 T \beta\lambda F}{2\pi} U_T^{1/2} - \frac{\omega}{Q} U_T . \tag{15}$$

Equation (15) is easier to solve if E_o [from Eq. (11)] is substituted for U_T. Equation (15) becomes

$$2\frac{dE_0}{dt} = \frac{\omega q k_1^2 T \beta\lambda F}{2\pi} - \frac{\omega}{Q} E_0 . \tag{16}$$

Assuming that the rf power is zero when $t = 0$, Eq. (16) can be integrated to give

$$E_0 = \frac{Q q k_1^2 T \beta\lambda F}{2\pi}\left(1 - e^{-\omega t/2Q}\right) . \tag{17}$$

The charge per beam bunch, q, can be calculated from the instantaneous average beam current, I, and is

$$q = 2\pi I/\omega . \tag{18}$$

Substituting Eq. (18) into (17) gives

$$E_0 = E_{0_{max}}\left(1 - e^{-\omega t/2Q}\right) . \tag{19}$$

where

$$E_{0_{max}} = \frac{Q I k_1^2 T \beta\lambda F}{\omega} . \tag{20}$$

Relationship Between RF Electric Field and RF Power

A crude estimate of k_1 can be obtained by assuming that most of the rf electric field is concentrated between the drift-tube noses and is a constant. The maximum stored energy in the electric field can be calculated and related to U_T to give k_1. The value of U_T calculated from the electric field is

$$U_T = \frac{\varepsilon_0}{2}\int E^2 dV \approx \frac{\varepsilon_0}{2} E_g^2 \pi R_g^2 Z_g \ . \qquad (21)$$

Solving this equation for E_g and using Eq. (4) gives

$$E_0 = \left(\frac{2Z_g}{\varepsilon_0 \pi R_g^2 \beta^2 \lambda^2}\right)^{1/2} U_T^{1/2} \ . \qquad (22)$$

Comparing Eqs. (11) and (22) gives

$$k_1 = \left(\frac{2Z_g}{\varepsilon_0 \pi R_g^2 \beta^2 \lambda^2}\right)^{1/2} \ . \qquad (23)$$

Equation Summary

Combining Eqs. (4), (11), (13), (20) and (23) give

$$E_{0_{\max}} = \frac{2QITFZ_g}{\omega \varepsilon_0 \pi R_g^2 \beta\lambda} = \text{max. average electric field,} \qquad (24)$$

$$E_{g_{\max}} = \frac{2QITF}{\omega \varepsilon_0 \pi R_g^2} = \text{max. gap electric field,} \qquad (25)$$

$$U_{T_{\max}} = \frac{2Q^2 I^2 T^2 F^2 Z_g}{\omega^2 \varepsilon_0 \pi R_g^2} = \text{max. total rf energy,} \qquad (26)$$

and

$$W_{L_{\max}} = \frac{2QI^2 T^2 F^2 Z_g}{\omega \varepsilon_0 \pi R_g^2} = \text{max. rf power loss/ time.} \qquad (27)$$

Examples

We calculate $E_{g_{\max}}$ and $W_{L_{\max}}$ using Eqs. (25) and (27) for a 100 μA beam at 800 MeV. Assume a 10% beam duty factor, then I = 1.0 mA. We let $\omega/2\pi$ = 200 MHz, Q=1000, φ = 1 deg, Z_g = 1.0 cm, and R_g = 1.0 cm ($\varepsilon_0 = 10^{-9}/36\pi$). Equations (25) and (27) give $E_{g_{\max}}$ = 5.7x10^5 V/m (rf-gap voltage) and $W_{L_{\max}}$ = 5.7 watts (maximum power extracted from the beam).

We calculate the beam deflection due to a transverse rf-electric field. From,

$$\frac{dP}{dt} = eE\cos(\omega t), \qquad (28)$$

we obtain

$$\Delta P = \int_{-\varphi_0}^{\varphi_0} eE\cos(\omega t)dt = \frac{2eE\sin\varphi_0}{\omega} \ . \qquad (29)$$

The deflection angle

$$X' = \Delta P / P_{//} \ . \qquad (30)$$

For E = 0.57 MV/m, $\phi_0 = \pi/2$ (complete rf half cycle), 5 rf deflection cavities (each of length $\beta\lambda/2$ = 0.68 m), and an 800 MeV beam, we find that $X' = 10^{-3}$ radians. This will produce a deflection of 1 cm in 10 meters. Given this same geometry, a 10 mA beam will have a deflection angle of 10^{-2} radians and will be deflected 10 cm.

There are issues to be addressed if this system is to be used for limiting beam current for personnel safety. These include: sensitivity of the rf cavities to detuning, possible long term degradation in cavity Q due to oxidation of cavity surfaces, determining the envelope of off-nominal linac operational parameters that will cause the beam to sufficiently debunch so that the rf deflection system will no longer work, and rf cavity conditioning.

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