LOCALIZED SPACE-CHARGE WAVES FOR BEAM DIAGNOSTICS*

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Abstract

The propagation of localized space-charge waves can be measured easily and accurately in experiments. The results carry valuable information about many beam parameters such as the wave speed, the geometry factor g, the beam radius, the beam impedance, etc. The principle of this diagnostics technique and an experimental example are given in this paper.

Introduction

Space-charge waves play fundamental role in microwave generators [1] and instabilities in particle accelerators [2]. They also have applications in the diagnostics of plasmas [3] and charged particle beams. The conventional approach to these applications is to use sinusoidal signals in generating space-charge waves. We have found that space-charge waves in the form of localized perturbations have many advantages to diagnose some important beam parameters.

The generation of localized space-charge waves is described in very detail elsewhere [4]. With a localized perturbation typical beam current signals downstream at two different locations are depicted in Fig. 1, showing two separated space-charge waves, namely the slow wave and fast wave, and their propagation away from each other. Figure 2 shows an energy spectrum of a beam with such a localized perturbation, also revealing two separated space-charge waves.

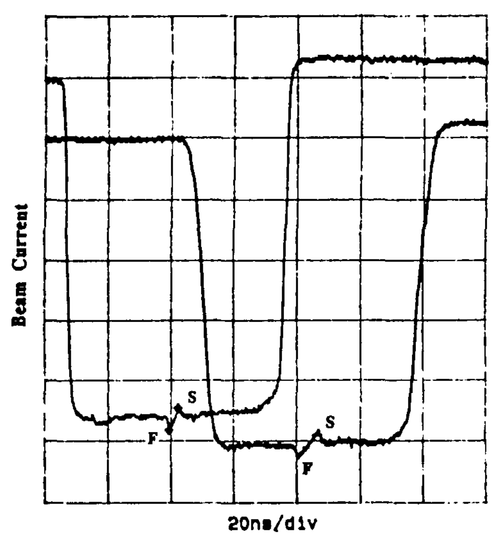

Fig. 1. Beam current signals modulated with localized perturbations showing the amplitude, polarity, and separation time of two space-charge waves.

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When the two space-charge waves are generated, they propagate away from each other in the beam frame. The separation time of the two space-charge waves at a given channel location can be easily and accurately measured from the beam current signal or the beam energy spectrum. It is this time dependence on the channel distance, that contains valuable information about the beam parameters.

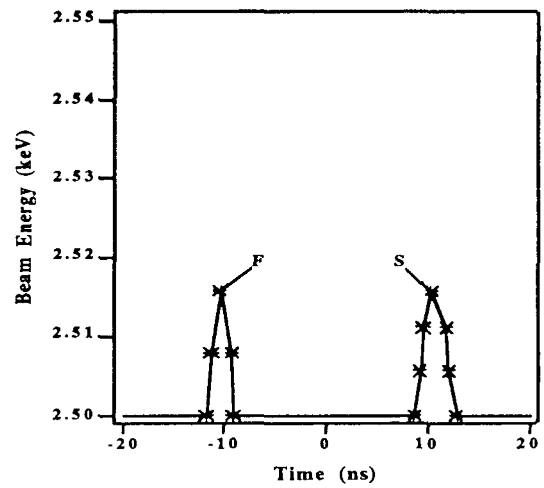

Fig. 2. Energy spectrum of a beam with localized space-charge waves.

Beam parameters diagnosed by localized space-charge waves

1. Propagation speed c_s of perturbations

In the linear theory, the two space-charge waves move in the lab frame at a constant speed of (v_0+c_s) for the fast wave and (v_0-c_s) for the slow wave, where v_0 is the beam velocity and c_s is the propagation speed of the perturbation in the beam frame. The separation time Δt of the two localized space-charge waves at a given channel distance s is a direct measurement of the speed c_s according to

$$\Delta t = \frac{2c_s}{v_0^2 - c_s^2} \cdot s \quad . \tag{1}$$

This speed c_s is also the speed of rarefaction waves at the edges of bunched beams with an initially rectangular profile [5].

2. Geometry factor g

The geometry factor g relates the longitudinal electric field associated with a perturbation in a beam with the line charge density variation. Under the long-wavelength limit this relationship can be expressed in the form [6]

$$E_z(z,t) \cong -\frac{g}{4\pi\varepsilon_0\gamma^2}\frac{\partial\Lambda_1(z,t)}{\partial z} \quad , \qquad (2)$$

where $\Lambda_1(z,t)$ is the perturbed line charge density, ε_0 is the permittivity of free space, γ is the Lorentz factor, and z is the axial coordinate in the beam frame . The relation between the geometry factor g and the perturbation speed c_s is given by [5]

$$c_s = \sqrt{\frac{qgI}{4\pi m\varepsilon_0\gamma^5 v_0}} \quad , \qquad (3)$$

where I is the beam current, and q/m denotes the ratio of charge to mass of the particles. The geometry factor g can be calculated according to Eq. (3) after c_s is determined by the technique with localized space-charge waves shown in Eq. (1).

3. Beam radius a

It has been shown that the geometry factor g for a space-charge dominated coasting beam of radius a is related to the channel radius b by the simple formula [7]

$$g = 2\ln\frac{b}{a} \quad . \qquad (4)$$

After the geometry factor g is determined by Eq. (3), the average beam radius can be calculated by Eq. (4).

4. Longitudinal impedance X_s^*

The space-charge complex wave impedance per unit length is defined as

$$Z_s^*(k,\omega) = -\frac{E_z(k,\omega)}{i_1(k,\omega)} \quad , \qquad (5)$$

where E_z and i_1 are the perturbed longitudinal electrical field and the perturbed beam current in the complex frequency domain (k,ω), respectively. At a given perturbation frequency ω_0, this impedance can be approximated under the assumption $\omega_0 \approx kv_0$ as [8]

$$Z_s^*(\omega_0) = -iX_s^*(\omega_0) \approx -i\frac{g\omega_0}{4\pi\beta^2\gamma^2 c}Z_0 \quad , \qquad (6)$$

and can be normalized to

$$X_s' = \frac{X_s^*(\omega_0)\lambda_0}{Z_0} = \frac{g}{2\beta\gamma^2} \quad . \qquad (7)$$

Here c is the speed of light, $Z_0=1/(\varepsilon_0 c)=377\ \Omega$ is the characteristic impedance of free space, $\beta=v_0/c$, and λ_0 is the wavelength of perturbation.

According to Eq. (7), the normalized space-charge impedance can be readily calculated if the beam energy (β, γ) is known and the geometry factor g is measured from this technique. Under a given perturbation wavelength λ_0, the space-charge wave impedance X_s^* per unit length can be found according to Eq. (6).

5. Other measurements

The localized space-charge waves can also be used to measure the longitudinal instability, i.e. the growth of slow waves and decay of fast waves in a resistive channel. In a pure resistive channel, the spatial growth rate k_i of the slow wave can be found as [9]

$$k_i = \frac{\pi R^*}{Z_0}\left(\frac{K\beta}{X_s'}\right)^{\frac{1}{2}} \quad , \qquad (8)$$

where R^* is the channel resistance per unit length and $K=(I/I_0)(2/(\beta\gamma)^3)$ is the generalized perveance with I_0 being the characteristic current of the charged particles. With X_s' measured from this technique, the spatial growth rate can be calculated by Eq. (8). This value can be compared with the amplitude of the localized slow wave in experiments.

Another example of beam diagnostics with localized space-charge waves is to measure the reflection and transmission of space-charge waves on bunched beam ends [10]. The speed of the reflected and transmitted waves can also be determined with this technique in experiments.

Application of the technique

An experiment has been performed to illustrate how to employ localized space-charge waves to diagnose beam parameters. The facility consists of an electron beam injector and a 5-meter long periodic solenoidal focusing channel. The key device in the injector is a gridded electron gun which is able to produce the desired beam parameters with localized perturbations [11]. The transport channel consists of 36 short focusing solenoids. The diagnostic tools along the channel include five fast wall-current monitors and three beam energy analyzers. At the end of the channel a diagnostic chamber houses a beam transverse image identifier. Typical beam parameters in the experiment are: beam energy of 2.5 keV to 5 keV, beam current of 30 mA to 70 mA, and pulse length of 30 to 70 ns.

When the beam is modulated with localized perturbations in the gun, the typical beam current signal and a beam energy spectrum are shown in Figs. 1 and 2, respectively. The time interval between the two space-charge waves can be measured at each location of the current monitors and energy analyzers. Figure 3 plots the data points of such a measurement from the five wall-current monitors, where the beam energy is 5 keV and the beam current is 56.2 mA. A least square fitting yields the solid line with a slope of $\Delta t/s=2.04$ ns/m. Applying this value to Eq. (1) leads to the space-charge wave propagation speed of $c_s = 1.76\text{x}10^6$ m/s while the beam velocity is $v_0=4.16\text{x}10^7$ m/s. Using this value of c_s in Eq. (3), one gets the geometry factor g=1.52. The transport channel has a radius of b=1.91 cm. Equation (4) thus gives the average beam radius of a=0.89 cm. The normalized space-charge wave impedance X_s' is obtained from Eq. (7) to be 5.37. If a perturbation frequency of 100 MHz is considered, the space-charge wave impedance per unit length would be X_s^*=4.87 kΩ/m.

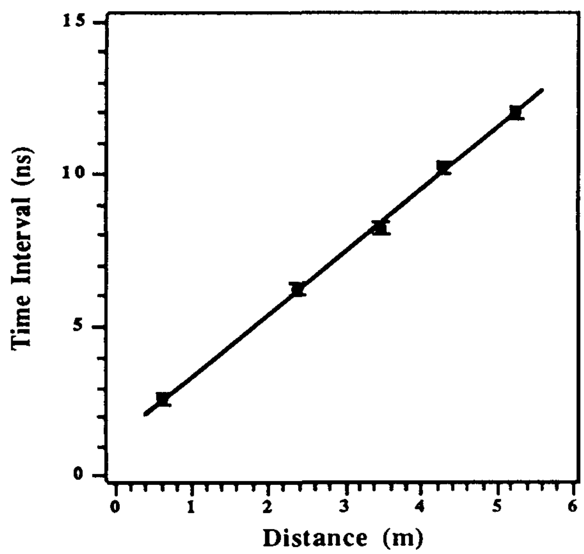

Fig. 3. Time interval between two space-charge waves vs. drifting distance, as measured by the five current monitors. The solid line is a least-square fitting of the experimental data.

The average beam radius measured by this technique is compared in Fig. 4 with an independent measurement by the beam image identifier in the end chamber. The beam image identifier consists of an axially movable phosphor screen and a CCD camera [12]. The transverse dimension can be obtained at any location along the channel. Good agreement is found.

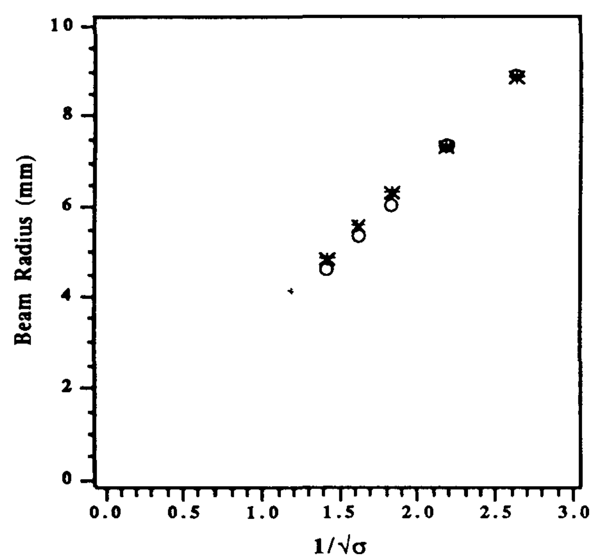

Fig. 4. Comparison of the measured beam radii from two different approaches: the stars are from the space-charge wave method, while the circles are from the beam image identifier, where σ is the phase advance of betatron oscillation with space charge.

The bunch end effect on space-charge waves has also been studied with this facility. The localized perturbations are initially launched very close to one end of initially rectangular bunches. Both reflection and transmission of localized space-charge waves at bunch ends are observed. The speed of the reflected wave and the transmitted wave has been measured with this technique. It is very hard, if it is not impossible, to diagnose the bunch end effect with sinusoidal space-charge waves. The details of this study are reported elsewhere [10].

Summary

Localized perturbations can be introduced to charged particle beams through modulation on passive gaps or active gridded guns. The space-charge waves thus generated can be diagnosed downstream from the beam current signals or beam energy spectrums. The information about the separation time of the two space-charge waves as a function of the propagation distance along the channel can be used to deduce a number of important beam parameters such as the wave speed, the geometry factor g, the beam radius (a non-destructive method of beam size diagnostics), the space-charge wave impedance, and reflection of waves at bunch ends, etc. An example is given to show a practical diagnostics.

References

[1] A. H. W. Beck, Space-Charge Waves and Slow Electromagnetic Waves, (Pergamon Press, New York, 1958).

[2] J. D. Lawson, The Physics of Charged-Particle Beams, (Oxford University Press, New York, 1988), second edition.

[3] N. A. Krall and A. W. Trivelpiece, Principles of Plasma Physics, (San Francisco Press, Inc., San Francisco, 1986).

[4] J. G. Wang, D. X. Wang, and M. Reiser, Phys. Rev. Lett., **71**(12), pp. 1836-1839, Sept. 20, 1993.

[5] A. Faltens, E. P. Lee, and S. S. Rosenblum, J. Appl. Phys., **61**(12), 15 June, 1987.

[6] V. K. Neil and A. M. Sessler, Rev. Sci. Instr. **36**(4), 429 (1965).

[7] J. G. Wang, H. Suk, D. X. Wang, and M. Reiser, Phys. Rev. Lett., 72(13), pp. 2029-2032, March 28, 1994.

[8] J. G. Wang and M. Reiser, Physics Fluids B, **5**(7), July 1993.

[9] J. G. Wang, M. Reiser, W. M. Guo, and D. X. Wang, Particle Accelerators, Vol. **37-38** , 181 (1992).

[10] J. G. Wang, D. X. Wang, H. Suk, and M. Reiser, submitted for publication to Phys. Rev. Lett.

[11] J. G. Wang, D. X. Wang, and M. Reiser, Nucl. Instr. & Meth. in Phys. Res. **A316**, 112 (1992).

[12] D. Kehne, K. Low, M. Reiser, T. Shea, C. R. Chang, and Y. Chen, Nucl. Instr. & Meth. in Phys. Res. A278, pp. 194-197, 1989.