A MODIFIED SPACE CHARGE ROUTINE FOR LINAC BEAM DYNAMICS CODES

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Abstract

In 1991 a space charge calculation for bunched beams with three-dimensional ellipsoidal symmetry was proposed for the PARMILA code, replacing the usual SCHEFF routines: it removes the cylindrical symmetry needed for the Fast Fourier Transform method and avoids the point to point interaction computation, where the number of simulation points is limited.

This routine has now been improved with the introduction of two (or more) ellipsoids, giving a good representation of actual, pear-shaped bunches (unlike the 3-D ellipsoidal assumption). The ellipsoidal density distributions are computed with a new method, avoiding the difficulty caused by statistical effects, encountered near the centre (the axis in 2-D problems) by the previous method. It also provides a check of the ellipsoidal symmetry for each part of the distribution. Finally, the Fourier analysis reported in 1991 has been replaced by a very convenient Hermite expansion, which gives a simple but accurate representation of practical distributions. Introduced in the new, versatile beam dynamics code, DYNAC, it should provide a good tool for the study of the effects of the various parameters responsible for the halo formation in high intensity linacs.

Introduction

Present linac beam dynamics codes often include a space charge routine, describing the evolution of intense beams [1]. They find their application in accelerators used as injectors for ultra high energy machines used in particle physics research. A new generation of linacs is now under consideration for various industrial applications like fission products waste incineration, generation of electrical power, pulsed neutron research facilities and other developments on advanced materials. What is needed in such machines is maintaining the loss level at less than 1 W/m, rather than conserving the beam quality as defined by its r.m.s. emittances. It is well known that when the intensity is increased in a linac, the beam never keeps a well defined shape ; instead a halo develops around it, due to the non linear character of the space charge forces. Theories are now being developed to understand the mechanisms of the formation of this halo and to quantify it. Simulation codes will however need to be improved for a more detailed study of the exact beam evolution. The present work, with its flexibility, is a step in this direction.

Existing Routines

Space charge routines can be classified in three categories.

The first type are the Particle In Cell (PIC) codes. They often use a Fast Fourier Transform (FFT). The space charge potential is computed on the nodes of a mesh, with an adequate smoothing interpolation from node to node. The SCHEFF routine in the PARMILA code [2] is of this type. These routines are quite fast and accurate, in spite of a slow apparent increasing emittance [3], which is a purely numerical effect[4]. Their main drawback is that, in order to avoid the long computation time required for 3-D FFT, they use rotational symmetry of the particle distribution around the axis for the computation.

The second type are Particle to Particle Interaction codes, used in MAPRO1 [5] and MOTION [6]. Here the space charge field is made linear inside spherical clouds of an optimized size around each macro-particle. It is quite accurate and avoids the increasing emittance given by the PIC codes. Its main drawback, however, is that when increasing the number of macro-particles N, the computing time increases as N^2.

In the last type, used in MAPRO2 [5] and SC3DELP [7], it is assumed that in transverse and longitudinal direction, the bunches keep an elliptical distribution. Such an assumption, reasonable in the transverse direction, is not justified in the longitudinal direction, where the motion is governed by a different, non-linear equation. In addition, the computation of the particle density is subject to difficulty near the centre (or the axis; it gives a zero divide). It can also be subject to statistical noise (like the PIC codes). Our new routine is of this last type, but it avoids these difficulties. Considering bunches to be constituted of several ellipsoids, it can treat a non-symmetrical longitudinal distribution. Using a different approach in the computation of the particle density, it avoids the problem around the centre (or axis). Finally, the noise resulting from statistical data is reduced by the use of Hermite polynomials.

Ellipsoidal Density Distribution Calculation

Scaling the r.m.s. dimensions a, b, c to be unity along the principal axes of an ellipsoidal distribution, the distribution function $n(x/a,\ y/b,\ z/c)$ becomes of spherical type.

For such a sphere, one can introduce its radial density distribution $n(R)$ and also consider the density distribution for each individual coordinate by integration of the two others. Refering to Fig. 1, one has :

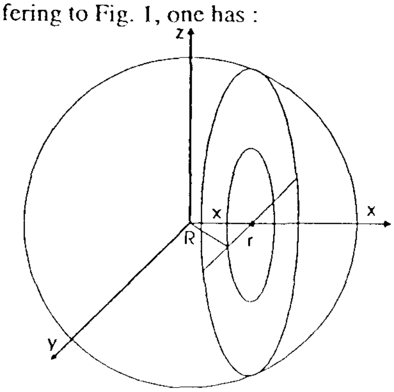

Figure 1. Density distribution coordinates

$$m\left(\frac{x}{a}\right) = m\left(\frac{y}{b}\right) = m\left(\frac{z}{c}\right) = \int n(R) 2\pi r dr \qquad (1)$$

Setting

$$R^2 = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = t \qquad (2)$$

and

$$(x/a)^2 = u \qquad (3)$$

$$r^2 = t - u \qquad (4)$$

gives, with constant x:

$$m\left(\frac{x}{a}\right) = \pi \int_u^\infty n(R)\, dt \qquad (5)$$

With (4) and t to represent the radial variable R:

$$m(u) = \pi \int_u^\infty n(t)\, dt \qquad (6)$$

from which one obtains [8] :

$$n(t) = -\frac{1}{\pi} \frac{dm(u)}{du} \qquad (7)$$

For the application of the above, instead of directly computing the particle density $n(t)$ as in [5] or [7], one determines $m(x/a)$, $m(y/b)$, and $m(z/c)$, as explained in the next paragraphs. One takes the average value :

$$m(u) = \frac{1}{3}\left[m\left(\frac{x}{a}\right) + m\left(\frac{y}{b}\right) + m\left(\frac{z}{c}\right) \right] \qquad (8)$$

and subsequently applies the relation (7). Moreover one can check that the terms in (8) are similar. Around the coordinate planes the densities are high and there is no singularity of $m(x/a)$ around $x = 0$ (similarly for y and z).

Hermite Expansion

The distributions in x, y and z, for a beam with a halo (i.e. without well defined limits), resemble Gaussians. It is then appropriate to express them in the form of a Hermite expansion :

$$m\left(\frac{x}{a}\right) = \sum_i A_i \exp\left(-\frac{x^2}{2a^2}\right) H_i\left(\frac{x}{a}\right) \qquad (9)$$

with

$$A_i \approx \frac{1}{i!\sqrt{2\pi}} \sum_{n=0}^{N} H_i\left(\frac{x_n}{a}\right) \qquad (10)$$

where the H_i are Hermite polynomials, and N represents the total number of macroparticles per bunch. As $m(x)$ is symmetrical around $x=0$, only even order polynomials are to be used (in practice up to $i=8$), with even parity terms in x (i.e. in u). This simplifies the computation of $n(t)$ (through equation (7), as the differential is an analytical expression) as well as the space charge field integration for each ellipsoid representing the bunch :

$$E_x = \frac{q\, abc\, x}{2\varepsilon_0} \int_0^\infty \frac{n(t_1)}{\left(a^2+s\right)^{3/2}\left(b^2+s\right)^{1/2}\left(c^2+s\right)^{1/2}} ds \qquad (11)$$

where q is the macroparticle charge and

$t_1 = x^2/\left(a^2+s\right) + y^2/\left(b^2+s\right) + z^2/\left(c^2+s\right)$;

computed as explained in [7]. Analogous expressions are valid for E_y and E_z.

Longitudinal Charge Density

The density distribution in a bunch along the longitudinal axis is usually pear shaped (Fig. 2). The new routine introduces an ellipsoid computed for the denser part of the longitudinal distribution; this part is very similar to the transverse distribution. Symmetrizing it, the remaining part (usually less than 30 % and sometimes very small) can be satisfactorily represented by a second ellipsoid.

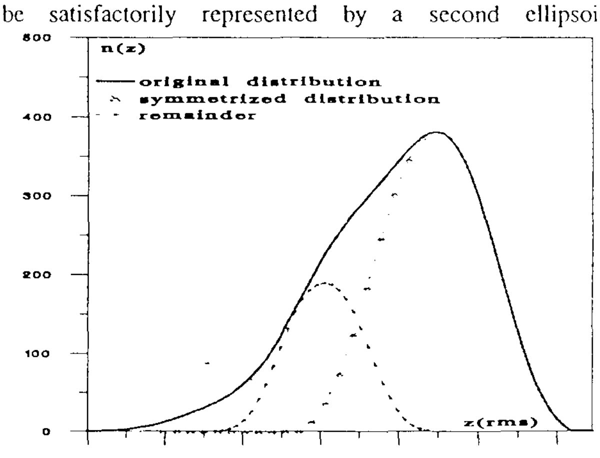

Figure 2. Original longitudinal beam density distribution (continuous line), and its computed decompostion into two ellipsoids (dashed lines)

Results

The new space charge routine has been installed in the very accurate and versatile code DYNAC [9]. This code can treat transport lines and many types of accelerating structures, even long elements like independent complex cavities or multi-cell structures, Quasi-Alvarez and also IH cavities. Space charge effects can be introduced not only once per accelerating element but at several places in case it is felt useful with regard to the change in shape of the beam along its length. Several cases have been studied, in particular the IH structure of the new lead ion linac at CERN [10] for 10 mA of Pb^{27+} and the CERN proton linac [11] for 213 mA of protons. The latter case is discussed below.

The computation with two ellipsoids instead of one (Fig.3) shows a significant difference in the longitudinal emittance growth (i.e. current emittance divided by input emittance). The effect on the transverse behaviour is small.

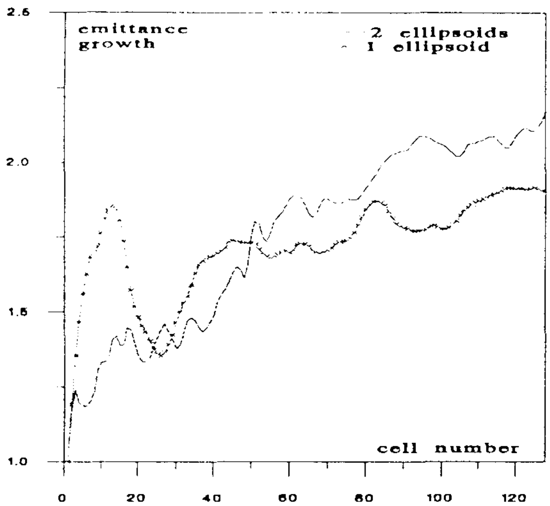

Figure 3 Longitudinal emittance growth for space charge calculations with 1 and 2 ellipsoids

Fig.4 shows a comparison of a space charge computation for two ellipsoids made in gaps only, quadrupoles only and made in both gaps and quadrupoles.

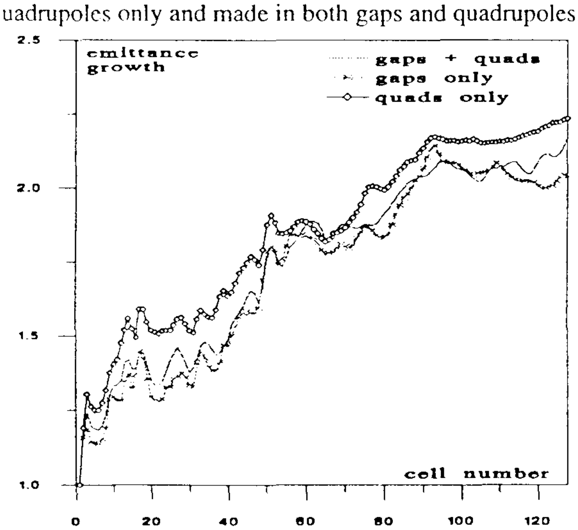

Figure 4 Longitudinal emittance growth for 2 ellipsoids computation for three cases (gaps only, quadrupoles only and gaps and quadrupoles)

In Fig.5 another case study shows that DYNAC with 250 particles gives similar results as DYNAC and PARMILA with 1000 particles, apart from a zone of losses between gaps 10 and 40 where the emittance values are perturbed.

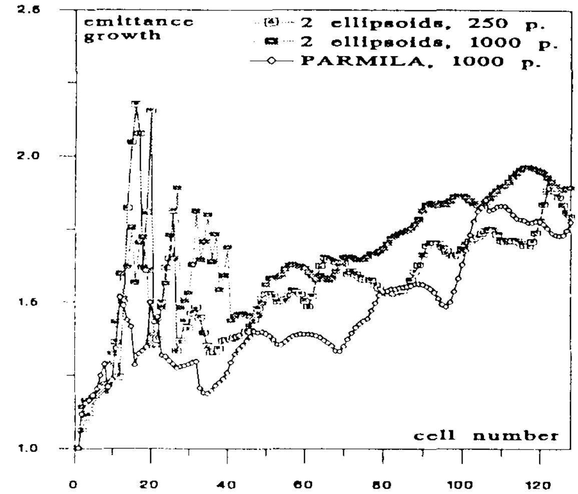

Figure 5 Longitudinal emittance growth as computed by PARMILA (1000 particles, SCHEFF routine) and DYNAC (250 and 1000 particles, new routine, 2 ellipsoids).

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