# BEAM POSITION CORRECTION IN THE FERMILAB LINAC 

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## Introduction

Orbit correction has long been an essential feature of circular accelerators, storage rings, multipass linacs, and lincar colliders [1-3]. In a drift tube linear accelerator (DTL) such as the H-Linac at Fermilab, beam position monitors (BPMs) and dipole corrector magnets can only be located in between accelerating tanks. Within a tank many drift tubes (from 20 to 60) each house a quadrupole magnet to provide strong transverse focusing of the beam. With good alignment of the drift tubes and quadrupoles and a sufficiently large diameter for the drift lubes, beam position is not typically a major concern. In the Fermilab DTL, $95 \%$ of the beam occupies only $35 \%$ of the available physical aperture $(4.4 \mathrm{~cm})$.

The recent upgrade of the Fermilab Linac [4] from a final energy of 200 MeV to 400 MeV has been achieved by replacing four 201.25 MHz drift tube linac tanks with seven 805 MHz side-coupled cavity modules (the high energy portion of the linac or HEL). In order to achieve this increase in energy within the existing enclosure, an accelerating gradient is required that is a factor of 3 larger than that found in the DTL. This in turn required that the physical aperture through which the beam must pass be significantly reduced. In addition, the lattice of the side-coupled structure provides significantly less transverse focusing than the DTL. Therefore in the early portion of the HEL the heam occupies over $95 \%$ of the available physical aperture $(3.0 \mathrm{~cm})$. In order to prevent beam loss and the creation of excess radiation, the ability to correct beam position throughout the HEL is of importance.

An orbit smoothing algorithm $[5,6]$ commonly used in the correction of closed orbits of circular machines has been implemented to achieve a global least-squares minimization of beam position errors. In order to accommodate several features of this accelerator a refinement in the algorithm has been made to increase its robustness and utilize correctors of varying strengths. Although a least-squares minimization technique is not the only possibility, it has been operationally shown to be an effective method of orbit correction.

## Theory

The orbit smoothing algorithm is briefly summarized here. In general we have N BPMs and M dipole correcting magnets with $\mathrm{N}>\mathrm{M}$. The initial distorted path down the linac as measured by these N BPMs is given by the N vector $\underline{X}_{\underline{Q}}$. The goal is to find an array of dimension $M$, denoted by $\underline{\theta}$, which consists of the angular kicks to be given by the dipole corrector magnets. The final particle position is then given by:

$$
\begin{equation*}
\mathrm{X}_{\mathrm{f}}=\underline{\mathrm{X}_{\mathrm{o}}}+\underline{T} \cdot \underline{\theta} \tag{1}
\end{equation*}
$$

where the NxM matrix T is:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{ij}}=\frac{\partial \mathrm{x}_{\mathrm{i}}}{\partial \theta_{\mathrm{j}}} \tag{2}
\end{equation*}
$$

[^0]namely, the change in beam position at BPM i caused by changing the strength of the angular kick at dipole $j$. The quantity $\chi^{2}$ is defined by:
\[

$$
\begin{align*}
\chi^{2} & =\left|\underline{X_{f}}\right|^{2}=\left(\underline{X_{\mathrm{o}}}+\underline{\underline{T}} \cdot \underline{\theta}\right) \cdot\left(\underline{\mathrm{X}_{\mathrm{o}}}+\underline{\underline{T}} \cdot \underline{\theta}\right) \\
& =\sum_{i=1}^{N}\left(X_{\mathrm{O}_{\mathrm{i}}}+\sum_{j=1}^{M} \mathrm{~T}_{\mathrm{ij}} \theta_{j}\right)^{2} \tag{3}
\end{align*}
$$
\]

To minimize $\chi^{2}$, differentiate Equation \#3:
For $k=1$.. $M$

$$
\begin{align*}
& \frac{\partial \chi^{2}}{\partial \theta_{k}}=\sum_{i=1}^{N} 2\left(X_{o_{i}}+\sum_{j=1}^{M} T_{i j} \theta_{j}\right) T_{i k} \\
&=\sum_{i=1}^{N} 2 T_{i k}\left(\underline{X_{o}}+\underset{=}{T} \cdot \underline{\theta}\right) \\
& i \\
&=2 \sum_{i=1}^{N} \tilde{T}_{k i}\left(\underline{X_{o}}+\underline{T} \cdot \underline{\theta}\right)=0  \tag{4}\\
& i \\
& \text { or } \tilde{=} \cdot\left(X_{o}+\underline{T} \cdot \underline{\theta}\right)=0
\end{align*}
$$

where the MxN matrix $\tilde{\mathrm{T}}$ is the transpose of matrix T . Solving for the desired array of dipole corrector strengths $\underline{\theta}$ yields:

$$
\begin{equation*}
\underline{\theta}=-(\underline{\tilde{\mathrm{T}} \mathrm{~T}})^{-1} \tilde{\underline{\mathrm{~T}}} \underline{\underline{\mathrm{O}}} \tag{5}
\end{equation*}
$$

Thus from measuring the matrix $T$ and the beam position displacements $\underline{X}_{\underline{Q}}$ the proper deflections at the dipole corrector magnets $\underline{\theta}$ can be found.

## Implementation

The HEL consists of seven 805 MHz side-coupled cavity modules. Each module consists of four accelerating sections. In each of the drift regions between these four sections are located a quadrupole magnet and beam position monitor as well as various diagnostics. The BPMs are of a stripline design with 4 plates that can give beam position readings in both the horizontal and vertical planes simultaneously. Several BPMs at the entrance and exit of the HEL are read in both planes, however the majority of BPMs are only monitored in one plane. Each BPM is mounted in the bore of a quadrupole. In order to maintain transverse focusing of the beam these quadrupoles produce a nominal gradient of $21.4 \mathrm{Tesla} / \mathrm{m}$ over a length of 8.5 cm . At this gradient, a position error of 0.5 mm in the beam entering a quadrupole will produce an angular kick of 0.6 milliradians in the 116 McV beam entering the HEL and an angular kick of 0.3 milliradians at the final beam energy of 400 MeV .

In the first two drift regions of a module dipole corrector magnets are placed. The dipole corrector magnets consist of copper wire wound around a square iron magnet frame. These
dipoles are powered by 2 separate power supplies and can independently steer the beam in both the horizontal and vertical planes. The length of the drift region between accelerating sections is proportional to the beam velocity. Due to space limitations, dipole magnets early in the HEL are more compact in design. However this is offset by the fact that the beam energy is still relatively low. In later regions of the HEL adequate space is available to accommodate larger dipole magnets but the beam energy is higher and the resultant amount of heam deflection is smaller. The maximum amount of beam deflection provided by the dipole magnets ranges from 3.6 milliradians at the entrance of the HEL to 2.0 milliradians at the end of the HEL. For the region of the HEL in which the stecring algorithm is implemented the total number of beam position monitors is 30 ( 16 horizontal and 14 vertical) and the number of dipole corrector magnets is 11.

Due to the nature of the FODO lattice, a horizontal corrector placed near a vertically focusing quadrupole will have a much smaller effect than a horizontal corrector placed near a horizontally focusing quadrupole. Therefore of the two dipole corrector magnets located in each HEL module only one dipole will provide a significant steering capability in a given plane. With this variation in the effective strength of correctors and the desire to reduce beam position errors at all quadrupoles (and thus reduce radiation losses from beam scraping) the idea of a global optimization algorithm as described in the previous section is appealing. Solving Equation \#5 could lead to a minimum leastsquares condition that is not physically achievable due to limits on the maximum magnitude of dipole correction available. To make the algorithm as robust as possible, the solution must be confined to the physical range available.

The minimum point of $\chi^{2}$ in an $M$ dimensional space is found by locating the point of zero derivative (Equation \#4). However if constraints are placed upon the region of space under consideration then the endpoints must be examined for a minimum also. This can easily be accommodated in the algorithm by fixing one dipole to its maximum value and solving for the remaining $\mathrm{M}-1$ dipole values in the same manner. For example if the first dipole magnet is set to its maximum value:
For $k=2$.. M:

$$
\begin{aligned}
& \frac{\partial \chi^{2}}{\partial \theta_{k}}=\sum_{i=1}^{N} 2\left(X_{o_{i}}+\sum_{j=1}^{M} T_{i j} \theta_{j}\right) T_{i k} \\
&=\sum_{i=1}^{N} 2\left(X_{O_{i}}+T_{i t} \theta_{I}^{m a x}+\sum_{j=2}^{M} T_{i j} \theta_{j}\right) T_{i k} \\
&=\sum_{i=1}^{N} 2 T_{i k}\left(\underline{X_{o}^{*}}+\stackrel{T}{=} \cdot \underline{\theta}\right)=0 \\
& i
\end{aligned}
$$

$$
\begin{equation*}
\text { or } \underline{\tilde{T}} \cdot\left(\underline{X_{\mathrm{o}}^{*}}+\underline{\underline{T}} \cdot \underline{\theta}\right)=0 \tag{6}
\end{equation*}
$$

where the vector $\mathrm{X}_{0}{ }^{*}$ now contains the beam displacements as measured by the BPMs plus the position change incurred by fixing one dipole corrector. The matrix T now has a dimension of $N x(M-1)$ and the (M-1) dimension vector $\underline{\theta}$ can once again be found by Equation \#5.

If a global solution is found within the acceptable physical range of dipole corrector strengths then the problem is solved. However if the global solution is not acceptable, an iterative
procedure is used setting each dipole corrector in turn to its maximum value and searching for the minimum $\chi^{2}$ with the remaining M-1 correctors. Informational output to the user is in the form of a root mean square (RMS) position error:

$$
\begin{equation*}
\mathrm{RMS}=\sqrt{\frac{\sum \chi_{\mathrm{i}}^{2}}{\mathrm{~N}}} \tag{7}
\end{equation*}
$$

where N is the number of BPM measurements.

## Results

The goal of the steering algorithm is to steer the beam through the center of each quadrupole which is also where the BPMs are located. Since the alignment of the quadrupole center to the BPM center is typically good to only 0.3 mm , the desired positions of the beam have been determined experimentally. Beam traveling through the magnetic center of the quadrupole will not be steered by the quadrupole. Therefore upstream dipole correctors are used to position the beam at a quadrupole so that no change in the beam position is recorded by downstream BPMs when the gradient of the quadrupole is varied. This point is then defined to be the zero position of the BPM.

The elements of the T matrix given in Equation \#2 are experimentally determined by measuring the beam position at all BPMs while varying the current in a dipole corrector. To accommodate pulse to pulse variations in the linac, an average of five readings from each BPM are used for the position measurement and this average typically has a standard deviation of $\pm 0.2 \mathrm{~mm}$. This is the dominating source of uncertainty in measurements since the BPMs provide a 0.002 mm resolution over their $\pm 3 \mathrm{~mm}$ lincar operating region. A least squares linear fit is then made to the data points. The statistical uncertainty in the slope of the linear fit is typically 0.1 $\mathrm{mm} / \mathrm{Amp}$ which represents a $5-10 \%$ uncertainty in the value of many of the T matrix elements. The beam-dynamics program Trace 3-D [7] has also been used to predict the T matrix values by calculating the transfer matrix between a dipole and BPM. Agreement between the predicted and measured values is typically within $\pm 10 \%$ for the first two BPMs following a dipole, however as the distance between dipole and BPM increases the agreement between predicted and measured values becomes very poor, even exceeding $100 \%$. The reason for this discrepancy is unknown, but may be the cumulative effect of survey uncertainties in the longitudinal position of lattice elements.

A measurement of the T matrix elements has the added benefit of providing information on coupling between the horizontal and vertical planes. To within the statistical accuracy of the measurement, the response of a BPM to changes in a dipole in the opposite plane is found to be zero.

The steering algorithm provides the best solution and is not an iterative procedure. However due to the measurement uncertainties of the T matrix elements if significant changes in dipole settings are called for, the procedure may require one iteration before converging. In the typical course of day-to-day tuning where changes are small no iteration is needed to achieve a final solution.

Figure 1 shows the typical improvement in beam position at each BPM achieved by the algorithm. In the horizontal plane the RMS error is reduced from 0.41 mm to 0.06 mm while in the vertical plane the RMS error is reduced from 0.52 mm to 0.15 mm . Figure 2 shows the improvement in RMS beam
position over a wide range of operational conditions. Clearly the correction achievable in the vertical plane is poorer than that of the horizontal plane. From survey data it has been determined that a 3.1 mm vertical offset exists between the DTL and the HEL and that the RMS deviation in quadrupole position from the linac axis is $10 \%$ higher in the vertical plane than the horizontal plane. These misalignments make the task of orbit correction more difficult in the vertical plane.

The addition of proper boundary examination to the algorithm has allowed correctors of marginal strength to be utilized in beam steering while maintaining stability in the algorithm's solution. Having dipole correctors at their maximum value is clearly not a desirable operational situation; it is anticipated that further examination and correction of quadrupole misalignments will improve this situation.


Figure 1. Typical algorithm results. Initial (X) and Final ( $\bullet$ ) beam position offsets for (a) the horizontal and (b) the vertical planc.


Figure 2. Final beam position error versus Initial beam position error for (a) horizontal and (b) vertical planc for one iteration of the beam steering aggorithon.

## References

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[^0]:    * Operated by Universities Research Association Inc. under contract to the U.S. Department of Energy.

