SPECTRUM ANALYSIS OF AN ELECTROMAGNETIC FIELD GENERATED BY LINAC ELECTRON BEAMS

Hiroyasu Itoh , Shusuke Nisiyama , Satoshi Tomioka , Takeaki Enoto
Department of Nuclear Engineering, Hokkaido University
North 13,West 8,Sapporo,Hokkaido 060,Japan

Abstruct

Obtaining the spectrum of the electromagnetic field from electron beam is important to analyze the structure of micro-pulses. However it is difficult to measure the shape of the transient electric field radiated from micro-pulses of electron linear accelerator, because of the limitations of frequency bandwidth of detector systems. The spectrum can be also obtained by measuring the spatial field distributions of a standing wave, instead of the time variation which requires using an expensive oscilloscope. In this paper, we show the validity of the measurement method by both the temporal and spatial distributions. To analyze the electromagnetic field generated by an electron beam in a symmetric cylindrical waveguide, using the finite-difference time-domain (FD-TD) method. Making a comparison of between temporal and spatial distributions calculated, peak frequency values of these spectra showed good agreement, and thus it was confirmed that the frequency values of the electron beam can be determined from the spatial distribution. The actual standing wave distribution was also measured for the same configuration, using a quarter-wavelength monopole. The experimental result is compared with calculated results in the frequency domain.

Introduction

In contemporary research using linear accelerators, it has become important to measure not only an electron-beam's position, but also its emittance and energy distribution, as well as information on the temporal and spatial distribution of micro-pulses. To determine the bunched composition of an electron beam it is necessary to analyze the time dependent wave profile. However, high frequency responses require expensive measurement equipment. In contrast, the measurement technique proposed in this paper does not.

In our method, the standing wave distribution of the beam's electric field is measured. The beam's bunch composition and profile can thus be reconstructed through the frequency spectrum. The standing wave pattern is created by setting up two conductive plates inside the waveguide, perpendicular to the electron beam's path. Since the beam is pulsed, a transient standing wave is thought to be created between the plates. In this way it is feasible to determine the electron beam profile indirectly by analyzing the spatial distribution of the standing wave.

In order to investigate how closely the standing wave distribution corresponds with the time-dependent beam profile, the finite-difference time-domain (FD-TD) method is used to analyze the electromagnetic field excited within a cylindrical waveguide cavity as described above. The FD-TD method is often used to study various antennas [1] and microstrip discontinuities [2].

First the change of electric field with time is "observed" (simulated observation) at various points within the cavity. Frequency characteristics are directly determined from the Fourier transform of this data. In comparison, at each point on the FD-TD lattice, the maximum electric field amplitude is calculated. The spatial distribution of the standing wave is plotted from these maxima. Information on the wave number domain can be obtained from the Fourier transform of this measured data. By considering dispersion of the assumed mode, this wave number spectrum can be used to determine the frequency distribution.

Frequency characteristics calculated by these two methods are compared. The actual spatial distribution of the standing wave were also obtained by using a quarter-wavelength monopole, and the experimental results were compared with calculated frequency spectra. By these comparisons, this paper examines the feasibility of obtaining the electron beam profile from measurements of the standing wave fluctuations.

FD-TD Method

The electromagnetic field generated by electron beam bursts in a cylindrical waveguide is analyzed in the time domain, using the finite-difference time-domain (FD-TD) method. FD-TD method was introduced by Yee [3], and is a means of directly solving Maxwell's time-dependent curl equations using a central difference for the space and time derivatives. Here the electromagnetic field is excited by an axially symmetric current density J_z, substituted for the electron beam, which is independent of the cylindrical coordinate ϕ. Assuming that only the axially symmetric TM modes are excited, the Maxwell's equations in the cylindrical system are then

$$\mu_0 \frac{\partial H_\phi}{\partial t} = \frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z}, \quad (1)$$

$$\epsilon_0 \frac{\partial E_r}{\partial t} = -\frac{\partial H_\phi}{\partial z}, \quad (2)$$

$$\epsilon_0 \frac{\partial E_z}{\partial t} = \frac{1}{r}\frac{\partial r H_\phi}{\partial r} - J_z. \quad (3)$$

In the FD-TD formulation we denote any function of space and time as

$$E_r(r,z,t) = E_r(i\delta r, j\delta z, n\delta t) = E_z^n(i,j) \quad (4)$$

where δr and δz are the spatial increments, and δt is the time increment.

An example of a calculation of E_r is the following formula:

$$E_r^{n+1}(i,j) = E_r^n(i,j) - \frac{\delta t}{\epsilon_0 \delta z} \left[H_\phi^{n+\frac{1}{2}}(i,j+\tfrac{1}{2}) - H_\phi^{n+\frac{1}{2}}(i,j-\tfrac{1}{2})\right]. \quad (5)$$

The increments are chosen to satisfy the stability criterion:

$$\delta t \leq \frac{1}{c\sqrt{\frac{1}{\delta r^2} + \frac{1}{\delta z^2}}}. \quad (6)$$

In this study the spatial increments are $\delta r = \delta z = \delta = 1mm$, and the time increment is chosen as $\delta t = (2c)^{-1}\delta$.

The analytical model is shown in Fig.1. To generate the standing wave, two conductive plates being a millimeter thick are placed, spaced a set distance apart, perpendicular to the axis in the cylindrical waveguide. The plates have holes of 18mm in diameter in their centers, through which the electron beam passes.

For the boundary conditions, we assumed that both the waveguide wall and the conductive plates are perfect conductors, and we placed the hard truncation condition on the wall sufficiently far from the conductive plate in the +z direction.

The electron beam has a uniform diameter of 6mm, and pulses are a Gaussian shaped with the full-width at half-maximum (FWHM) equal to 20psec in the +z direction. Pulses are injected periodically (every 350psec) according to the acceleration period of the clystron.

Analytical Results

The temporal and spatial distributions of the excited electric field $E_r^n(r,z)$ are analyzed by using the FD-TD method. As one example, calculated results from the following specifications are compared. The distance between reflective plates is 354mm, and the time variations of field component $E_r^n(27,z)$ are calculated at various points with 27mm as off-axis distance in the symmetric cylindrical waveguide.

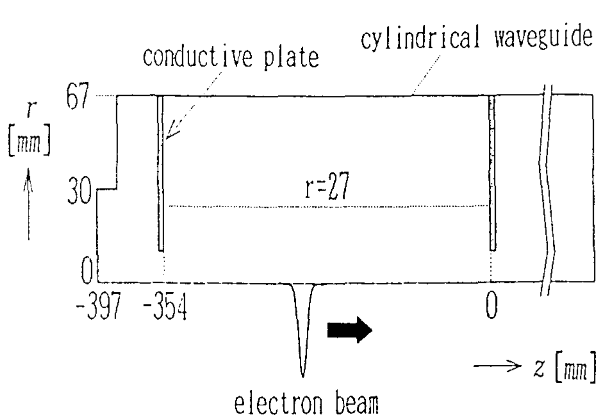

Fig. 1 Analytical model of a symmetric cylindrical waveguide.

In addition, the temporal distribution differs according to distance from the conductive plates. Therefore we examined the frequency spectra from electric field component E_r at two representative positions. Fig.2 and Fig.3 show the frequency spectra obtained directly from the Fourier transform of their temporal distributions.

On the other hand, the maximum amplitude of $E_r^n(27,z)$ at each point between reflective plates can be calculated. The spatial distribution of the standing wave is plotted from these maxima. The standing wave distribution of the pulsed electron beam is determined by a number of the incident micro-pulses within the computation time. In this paper, the computation time is set 11000δt ($\simeq$ 18$nsec$), the time by which fifty pulses inject within the waveguide.

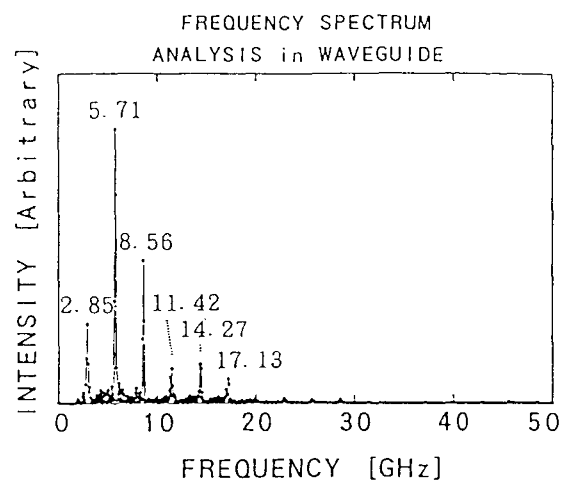

Fig. 2 Frequency spectrum of the temporal distribution for the field component $E_r(27,-53)$.

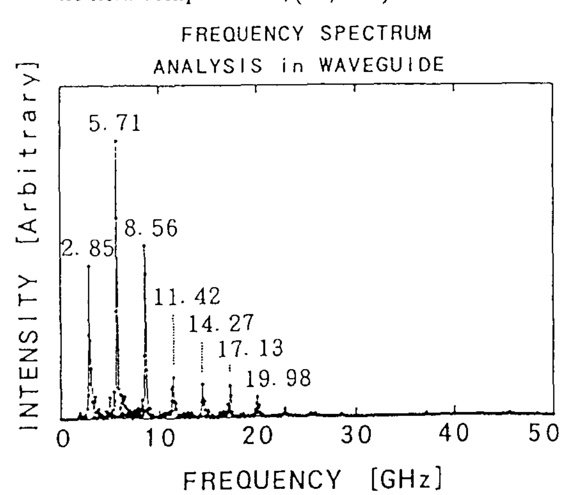

Fig. 3 Frequency spectrum of the temporal distribution for the field component $E_r(27,-79)$.

After the values of the spatial distribution were windowed by the Hanning window, we transformed them into the wave number spectrum by applying the Fourier transform. Fig.4 shows the frequency spectrum obtained from the wave number spectrum, taking into account the dispersion of TM_{01} mode in the cylindrical waveguide.

By comparing Fig.4 with Fig.2 and Fig.3, it can be seen that since the frequency changes with distance from the reflective plate, the corresponding envelopes vary. However the peak frequency of these spectra showed good agreement, and thus it was confirmed that the frequency values of electron beam pulses can be estimated by using the spatial distribution.

Measurement

Following the simulated apparatus set-up, we placed two conductive plates, spaced $354mm$ apart, perpendicular to the axis in the cylindrical waveguide, and measured the spatial distribution, by moving the stepping motor controlled stage in $1mm$ steps along the axis. Output from the antenna was passed through a crystal detector for square-law detection and input to an oscilloscope for observation.

In the experiment reported here, a quarter-wavelength monopole antenna is used, made of coaxial cable of Semi-Rigid type, and of length $3mm$.

The maximum output voltage (ground to peak) was plotted at each point. Fig.5 shows the frequency spectrum from the spatial distribution, obtained in the same way as the analysis.

In comparison with calculated frequency spectra, the experimentally observed peak frequency values were confirmed to match well.

Conclusion

The FD-TD method makes it possible to calculate temporal and spatial distributions of the electromagnetic field radiated from pulsed electron beam. For the field component E_r these distributions were calculated, and the frequency characteristics were obtained from the Fourier analysis. A good level of agreement on the peak frequency was found between the frequency spectrum obtained from the time distribution, and the one determined from the spatial distribution of the standing wave. For a comparison with experimental results, the actual spatial distribution of the standing wave was also measured, using a monopole antenna, and the results showed good agreement with those calculated above. Thus, it is feasible to obtain the electron beam profile from the measurement of the spatial distribution of the standing wave. Future numerical research will require inclusion of scattering effects and frequency characteristics of antennas.

References

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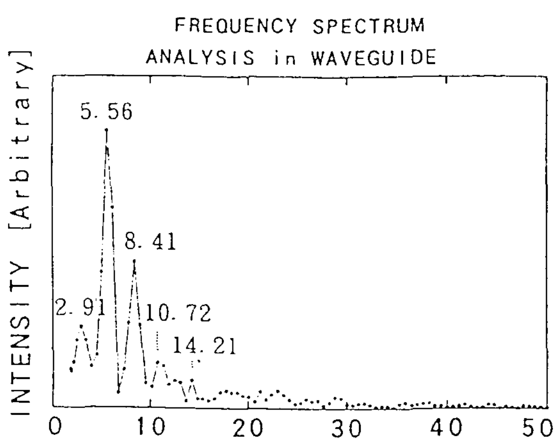

Fig. 4 Frequency spectrum of the spatial distribution on $r = 27mm$.

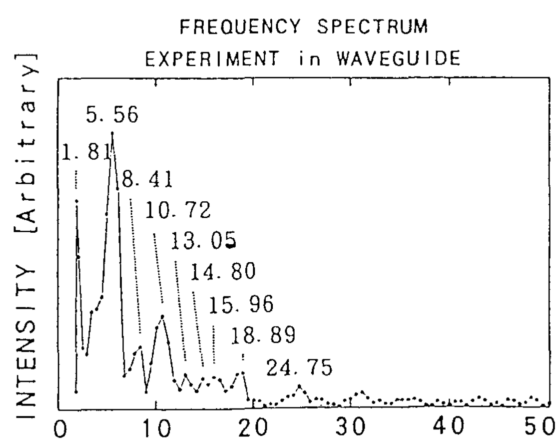

Fig. 5 Frequency spectrum of the spatial distribution measured experimentally.