# HIGH PRECISION BEAM POSITION MONITOR USING A RE-ENTRANT COAXIAL CAVITY

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# Abstract

The analysis and performance of a beam position monitor using a re-entrant coaxial cavity at 200 MHz are presented. The monitor measures both the horizontal and vertical beam positions independently of each other. The beam bunches excite an evanescent dipole mode  $H_{11}$  in the cavity proportionally to the transverse beam displacement from the centre axis. Microwave cavities are often used at the resonant frequency of the cavity with a small bandwidth. For beam observation over a broad bandwidth, the evanescent field of the fundamental cavity mode is used with advantage at frequencies which are about half of the cavity resonant frequency. Due to the larger bandwidth, a beam monitor below resonance provides more signal power from single beam bunches than a narrow band monitor at resonance. The broad band monitor has a lower beam coupling impedance than the same monitor at resonance, and causes less beam break-up forces to a long bunch train. The beam position measurement at frequencies below the fundamental H<sub>11</sub> mode is linear, since the higher order modes are damped with resistive ferrite material to reduce the beam impedance for short bunches. The theory of the evanescent fields in a detuned microwave cavity is illustrated with the example of a re-entrant coaxial cavity constructed for the SPS, for which measurements and theory are compared.

#### Introduction

The design and manufacture of microwave beam position monitors with an accuracy of < 0.01 mm is a major challenge for the construction of future linear accelerators. In order to obtain the best possible mechanical precision for the construction and alignment of the beam monitors, cavities with circular geometry, which can be milled on CNC working stations, are preferred.

The electrical precision of the beam position monitor is limited by the quality of the signal feed-throughs from the microwave cavity to the electronic equipment for the measurement of the cavity signals. If the centre position of the beam must be measured with an absolute accuracy of  $< 10^{-3}$  of the radius of the vacuum chamber, the cavity must be excited at the lowest dipole resonance frequency, and the position signal extracted from the cavity by a single feed-through<sup>1</sup>). If several feed-throughs are connected to the cavity, unavoidable electrical asymmetries between the feed-throughs cause additional errors to the measurement of the centre position. With four orthogonal feed-throughs, however, these errors can be calibrated accurately.

In linear accelerators, the beam intensity  $I(\omega)$  covers a rather large frequency range  $\omega$ , as the length of the bunches and the bunch train are relatively short. Therefore, a resonant beam monitor with a high shunt impedance covers only a small fraction of the frequency spectrum of the beam.

Away from resonance, the shunt impedance of the cavity is nuch smaller, but the broader bandwidth of the evanescent

mode provides more signal power. The larger bandwidth provides also a better resolution of the beam signals in the time domain, and allows the monitoring of intensity and position variations inside a bunch train.

A cavity design suitable for broadband operation below the fundamental resonance mode is the re-entrant coaxial cavity. The Q-factor of the cavity excited at frequency  $f_o$ decreases by the factor  $\sqrt{f_o} / f_r$  with respect to the resonant frequency  $f_r$ . The standing waves induced by the beam in the cavity below the resonant frequency are strongly attenuated in space and in time. As for the evanescent fields in waveguides below the cut-off frequency, the frequency bandwidth of a cavity below resonance is relatively large. The attenuation constant of the evanescent fields below half of the cut-off frequency is practically constant.

At the SPS accelerator at CERN, eight beam position monitors, BPA, using a re-entrant coaxial cavity at 200 MHz work since  $1975^{2}$ ). The beam aperture of the BPA monitor has a diameter of 270 mm. It is possible to adapt the re-entrant cavity to higher frequencies by reducing the beam aperture and the cavity dimensions accordingly.

## Oscillation Modes of Re-entrant Coaxial Cavity

The re-entrant coaxial cavity is arranged around the beam tube and forms a coaxial line which is short-circuited at the downstream end. The re-entrant coaxial cavity of a beam monitor consists of three distinct regions (Fig. 1): I. beam tube, II. gap, III. coaxial cylinder. By appropriate shaping of the gap, a large shunt impedance for the fundamental monopole and dipole modes can be obtained for the re-entrant coaxial cavity. In the transition region of the gap (II), the radial electric field lines of the coaxial cylinder (III) are bent by 90° into the direction of the beam axis, so that the H-modes of the coaxial cavity interact with the beam.



Fig. 1 Re-entrant Coaxial Cavity of Beam Monitor

The fundamental resonance mode of the re-entrant coaxial cavity is the  $\lambda/4$ -resonance of the coaxial line. The resonance wavelength  $\lambda_1$  of the transverse electromagnetic monopole mode depends only on the effective length  $a_1$  of the coaxial cavity:  $\lambda_1/4 = a_1$ . The monopole TEM field is proportional to beam intensity and does not depend on beam position.

The lowest dipole resonance frequency is found for the coaxial waveguide mode  $H_{11}$ . If the difference  $r_3 - r_2$  between the outer and inner radius of the coaxial line is sufficiently small,  $r_3 - r_2 < 0.2 r_2$ , the cut-off wavelength  $\lambda_c$  of the  $H_{11}$ -mode is  $\lambda_c = 2 \pi r_m$ , where  $r_m$  is the mean radius of the coaxial cavity,  $r_m \equiv (r_3 + r_2)/2$ . The resonance wavelength  $\lambda_2$  of the mode  $H_{111}$  of the re-entrant coaxial cavity with an effective length  $a_2$  is given by:

$$\lambda_2 = 2 \pi r_m / \sqrt{1 + (\pi r_m / 2a_2)^2}$$
 (1)

The dipole field  $H_{111}$  is excited by the beam proportionally to the beam intensity and proportionally to the distance of the beam from the centre axis of the monitor. The polarization plane of the dipole mode in the beam tube and in the coaxial cavity is defined by the vector  $r_0$  of the beam displacement (Fig. 1). The voltage of the dipole field adds to the monopole voltage in the direction of the beam displacement and subtracts from the monopole voltage on the opposite side. Therefore, the beam position P(x,y) can be measured from the output voltage of a pair of feed-throughs mounted on opposite sides of the cavity on the x- and y-axis.

$$x = L(V_1 - V_3) / (V_1 + V_3), y = L(V_2 - V_4) / (V_2 + V_4)$$
(2)

The factor L is the dipole constant of the beam monitor. The difference voltages  $V_1 - V_3$  and  $V_2 - V_4$  correspond to the voltage of the dipole field along the x- and y-axis. The sum voltage  $V_1 + V_3 = V_2 + V_4 = I \cdot Z_1$  is the sum of the TEM-monopole signals which are proportional to the beam intensity I and the transfer impedance  $Z_1$  of the monitor.

## Evanescent Dipole Mode of Coaxial Cavity

When the beam excites the re-entrant cavity below the cut-off frequency of the dipole mode  $H_{11}$ , the electromagnetic dipole field of the beam does not penetrate far into the coaxial cavity (III). The field energy is concentrated in the gap (II), and the dipole field in the coaxial cavity is attenuated exponentially away from the gap. Because of the exponential attenuation of waves below the cut-off frequency, these fields are called evanescent. The attenuation constant  $\alpha$  of evanescent  $H_{m1}$  fields along the coaxial line is given for m = integer by

$$\alpha = \frac{2\pi}{\lambda_c} \sqrt{1 - \frac{\lambda_c^2}{\lambda_o^2}}, \quad \lambda_c \cong 2\pi r_m / m$$
(3)

Modes with higher cut-off frequencies, m > 2, need not be considered, since the fields of the higher modes are attenuated much more than the fundamental dipole mode. The strong damping of higher modes, the absence of mode coupling and mode degeneration are the major advantages of the broadband microwave beam monitor excited at about half of the fundamental resonant frequency. The field impedance  $Z_F$  of an oscillation mode is defined as the ratio between the radial electric field  $E_r$  and the azimuthal magnetic field  $H_{\phi}$  at the field point  $Q(\phi, r, z)$ . Similarly to the TEM mode, the field impedance of the  $H_{m1}$  field is constant at any point  $Q(\phi, r)$  over the cross-section of the coaxial line (III). The field impedance  $Z_F$  of the  $H_{m1}$  - modes at the open end of the coaxial line of length  $\ell$  at frequency  $\omega$  is expressed by <sup>3</sup>):

$$Z_{\rm F}(\omega) = E_{\rm r}/H_{\phi} = j(\omega\mu_{\rm o}/\alpha) \tanh(\alpha \ell) \tag{4}$$
$$\mu_{\rm O} = 4\pi \cdot 10^{-7} \ {\rm Vs/Am}$$

At the open end, the coaxial line is bent by 90° towards the beam tube. With some approximation, the gap (II) between the beam tube (I) and the coaxial cavity (III) can be considered as prolongation of the coaxial line, at least if  $g \ge b$ (Fig. 1). A better approximation of the impedance of the gap is obtained from an impedance model of third order, which can be derived also for narrow gaps, g < b.

## Transfer Impedance of Beam Intensity Monitor

The transfer impedance  $Z_t$  indicates the output voltage  $U_0$  of the monitor induced by the beam of intensity  $I_0$  located at the centre axis of the beam monitor:  $Z_t = U_0/I_0$ . The output voltage  $U_0$  is measured across the load resistance of 50  $\Omega$  at each of the coaxial feed-throughs of the beam monitor. For the beam in centre position, only the monopole mode needs to be considered. For the TEM-mode, the impedance  $Z_2$  of the coaxial line (III) at distance  $\ell$  from the short-circuit is given by  $3^{(1)}$ 

$$Z_2(\omega) = j \ 60 \ \Omega \ \ln(r_3/r_2) \ \tan(\omega \ell/c) \tag{5}$$

The impedance of the gap (II) with the fringe fields at the boundaries with region I and III of the BPA-monitor has been calculated by the computer programme URMEL<sup>4</sup>) for three different frequencies, 400, 600 and 800 MHz, corresponding to different cavity lengths. From the impedance at three frequencies, the impedance model of the gap represented by three circuit elements as shown in Fig. 2 has been constituted :  $L_2 = 6.58$  nH,  $C_2 = 7.85$  pF,  $C_4 = 4.72$  pF.



Fig. 2 Equivalent Impedance Model of Monopole Mode of Beam Monitor

The impedance model of the gap is a third order approximation valid below the eigen resonance of the gap, and has been checked by an URMEL simulation at 214 MHz, which agrees to  $6.10^{-3}$  with the model.

The connection between the cavity and the coaxial feedthrough constitutes a series inductance  $L_o$  in the signal path and has been measured at 200 MHz for the monopole mode of the BPA-monitor :  $L_o = 20 \pm 2$  nH.

Knowing the circuit elements of the equivalent circuit diagram for the monopole mode of the re-entrant coaxial cavity, the transfer impedance  $Z_t$  of the BPA monitor at 200 MHz is calculated from  $Z_t = U_0/I_0 = 5.7 \ \Omega/39^\circ$ . For comparison, the transfer impedance of the BPA monitors in the SPS measured with beam, amounts to  $Z_t = 5.9 \pm 0.5 \ \Omega^{3}$ . Not included in the equivalent circuit diagram in Fig. 2, which is valid in the frequency range  $0 \le f \le 800 \text{ MHz}$ , are the higher order resonances (f > 1 GHz), which can be damped by lossy ferrite rings at the short-circuit end of the coaxial cavity.

The longitudinal beam impedance  $Z_{|||}$  of the re-entrant coaxial cavity is measured at the boundary between region I and II and can be deduced directly from the equivalent circuit diagram of the monopole mode (Fig. 2). For the BPA monitor we get  $Z_{1||} = U_4/I_0 = 2.8 \ \Omega + j6.1 \ \Omega$  at  $f = 200 \ \text{MHz}$ , and  $n = f/f_{\text{rev}} = 4620 \ \text{for SPS}$ .

## Linearity and Orthogonality of Position Monitor

For an ultra-relativistic beam located at point  $P(\phi_0, r_0)$  the azimuthal magnetic field induced at azimuth  $\phi$  and radius  $r_3$  on the inner surface of a circular vacuum tube is given by<sup>5</sup>):

$$H_{\varphi}(\varphi, r_{3}) = \frac{I_{0}}{2\pi r_{3}} \cdot \frac{r_{3}^{2} - r_{0}^{2}}{r_{3}^{2} + r_{0}^{2} - 2r_{3}r_{0}\cos(\varphi - \varphi_{0})} = \frac{I_{0}}{2\pi r_{3}} \cdot \left\{ 1 + 2\sum_{n=1}^{\infty} \left(\frac{r_{0}}{r_{3}}\right)^{n}\cos n(\varphi - \varphi_{0}) \right\}^{(6)}$$

The magnetic field of the beam excites the  $H_{11}$  mode at the open end of the re-entrant cavity by the dipolar component n = 1, at azimuth  $\phi = 0$ :

$$H_{\varphi 1} = \left(I_{0}r_{0} / \pi r_{3}^{2}\right)\cos(\varphi - \varphi_{0}) = xI_{0} / \pi r_{3}^{2}$$
(7)

At the horizontal feedthroughs,  $\varphi = 0$  or  $\pi$ , the dipole field is proportional to the horizontal beam position x, since  $r_0 \cos (\varphi - \varphi_0) = \pm x$ . At the vertical feedthroughs,  $\varphi = \pi/2$  or  $3\pi/2$ , the dipole signal is proportional to the vertical beam position y, because  $r_0 \cos (\varphi - \varphi_0) = \pm y$ .

The response of a re-entrant coaxial cavity is linear for ultra-relativistic beams, if there are no other modes than the fundamental monopole and dipole mode. With four orthogonal feed-throughs mounted on the circumference of the coaxial cavity, the dipole signals are orthogonal, i.e. independent of the coordinate of the beam position on the orthogonal axis. Linearity and orthogonality are complementary and indispensable for an accurate beam position measurement. The length of the coaxial cavity (III),  $\ell \simeq r_m$ , has been chosen such that the linearity and orthogonality are optimum. A nearly perfect linearity and orthogonality were measured with the first beam position monitor using a re-entrant coaxial cavity excited at 200 MHz<sup>2</sup>). The linear error of the position measurement x was 0.01 x. The zero error of the centre position was less than 0.004 r<sub>1</sub>, where r<sub>1</sub> is the radius of the beam tube.

## Calibration and Accuracy of Position Measurement

The sensitivity of a beam position monitor is characterized by the transfer impedance and by the dipole constant. The dipole constant L can be measured in the accelerator, when the beam in the position monitor is displaced by orbit bumps of calibrated amplitudes  $\pm x$ . For the BPA monitor with a beam tube radius of 134.5 mm (Fig. 1), the monitor constant measured with beam bumps amounts to L = 128  $\pm$  6 mm.

The beam position P(x,y) can be evaluated directly from the voltage measurements at the four feed-through ports by means of eq. 2. A 180° hybrid junction provides the difference and sum signals which are further processed by the electronic equipment. The zero error of the 180° hybrid junction used for the BPA monitors at the SPS is  $\Delta/\Sigma < -50$  dB for equal input voltages V<sub>1</sub> = V<sub>3</sub> or V<sub>2</sub> = V<sub>4</sub><sup>2</sup>). The zero error of the position measurement of the output of the hybrid junction can be calibrated to an accuracy of better than 5-10<sup>-4</sup> by injecting a 200 MHz signal at the orthogonal feed-throughs.

The subsequent electronic equipment can be calibrated very accurately for zero input voltage of the  $\Delta$ -channel corresponding to zero beam position<sup>6</sup>). For this reason, the homodyne and super-heterodyne RF-receivers provide an excellent precision for the beam position measurements in the SPS accelerator.

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