# BEAM DYNAMICS DESIGN OF THE INS SPLIT COAXIAL RFQ FOR RADIOACTIVE NUCLEI 

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#### Abstract

A $25.5-\mathrm{MHz}, 8.6-\mathrm{m}$ long split coaxial RFQ has been constructed and is now undergoing rf tests. The electrodes are modulated vanes, and the vane-tip profile is such that variable $\rho_{\mathrm{T}}$ vanes $u_{\mathrm{p}}$ to the distance 1.1 m down from the RFQ entrance and $\rho_{\mathrm{T}}=r_{0}$ vanes in the remaining. Computer simulations taking account of higher-order multipole fields show that this geometry yields beam performance better than those other geometries do.


## Introduction

The split coaxial RFQ presented here is a linac that operates at 25.5 MHz and accelerates ions with a charge-to-mass ratio $(q / A)$ greater than $1 / 30$ from 2 to $172 \mathrm{keV} / \mathrm{u}$. The cavity comprises 12 module cavities; the whole length is 8.6 m , and the inner diameter is 0.9 m . The electrodes are modulated vanes same as those in the four-vane RFQ. The cavity was set up in INS last spring, and low-power tests have been conducted [1]. This RFQ and an interdigital-H linac following it will accelerate radioactive nuclei in the E-Arena Test Facility, now under construction in INS.

The $R F Q$ is an extended version of a prototype ( 25.5 MHz, 3 module cavities, 2.1 m long, $q / A \geq 1 / 30,1 \rightarrow 45.4$ $\mathrm{keV} / \mathrm{u})$ [2]. At the prototype the vanc-tip geometry was the $\rho_{\mathrm{r}}=r_{0}$ one: the vane tips are machined to a circular arc with a transverse radius of curvature ( $\rho_{\mathrm{T}}$ ) equal to the mean aperture radius ( $r_{0}$ ). The $A_{10}$ correction (Sect. Design Procedure) was not made on the vanes. Because of this, experimentally obtained transmission efficiencies were lower than those predicted by a ParmTEQ simulation that uses an electric fiold derived from the Kapchinskii-Teplyakov's two-term potential function [3]. From this experience we made the $A_{10}$ correction on the vanes for the present. RFQ. After comparing good and bad points of different vane-tip geometries, we chose the following geometry: variable $\rho_{\mathrm{T}}$ for the low-energy part from the entrance to the distance of 1.1 m , and $\rho_{\mathrm{T}}=r_{0}$ for the remaining part.

This paper presents the considerations for the choice of the vane-tip geometry, procedure of the beam dynamics design, and discussion on the performance of the beam in vanes with different geometries.

## Choice of the Vane-Tip Geometry

Among the three vane-tip geometries of variable $\rho_{\mathrm{T}}, \rho_{\mathrm{T}}=$ $r_{0}$, and $\rho_{1}=0.75 r_{0}$, we abandoned first the last geometry. Though it has a small field enhancement factor ( $\kappa$ ),${ }^{1}$ the intervane capacitance is lower, and the cavity diameter goes

[^0]larger accordingly. The tank cylinders of the prototype $R F Q$ were to be utilized in the present $R F Q$; hence, the diameter must be maintained at 0.9 m .

We expected that a good choice would be a combination of variable $\rho_{\mathrm{T}}$ vanes and $\rho_{\mathrm{T}}=r_{0}$ ones. The former are to be used in a lower-energy part, and the latter in the remaining part. The choice was due to the following considerations: 1) $\rho_{\mathrm{T}}=r_{0}$ vancs are cheaper in cutting and surface finish; 2) variable $\rho_{\mathrm{T}}$ vanes would be better than $\rho_{\mathrm{T}}=r_{0}$ ones in the radial matching section; and 3) the pseudo-octapole field (most harmful higer-order multipole) would be minimized by using variable $\rho_{\mathrm{T}}$ vanes in a lower-energy part and $\rho_{\mathrm{T}}$ $=r_{0}$ ones in a higher-energy part.

## Design Procedure

The optimization of the cell parameters was first carried out by using two computer codes: GENRFQ and PARMTEQ2. The former is a cell gencrator for an RFQ accelerating a low-current beam [4], and the latter is a ray tracer that uses an electric field derived from the Kapchinskii-Teplyakov's two-term potential function. After genrfa/PARMTEQ-2 runs, we selected an RFQ that has the best beam performance, and then investigated its performance further for different vanc-tip geometries by using another code PARMTEQ-II, where higher-order multipole fields are included. The potential function is expressed as:

$$
\begin{align*}
U(r, \psi, z)=\frac{V}{2} & {\left[\sum_{i=1}^{3} A_{0 i}\left(\frac{r}{r_{0}}\right)^{2 i} \cos 2 i \psi\right.} \\
& \left.+\sum_{i=0}^{3} \sum_{j=1}^{3} A_{j i} I_{2 i}(j k r) \cos 2 i \psi \cos j k z\right] \tag{1}
\end{align*}
$$

From the fourth symmetry condition, $(-1)^{i}(-1)^{j}=-1$. In Eq. 1, $V$ is the intervane voltage, $I_{2 i}$ the modified Bessel function of the order $2 i$, and $r_{0}$ is given by

$$
\begin{equation*}
r_{0}=\frac{a}{\left[1-A I_{0}(k a)\right]^{1 / 2}}, A=\frac{m^{2}-1}{m^{2} I_{0}(k a)+I_{0}(m k a)} \tag{2}
\end{equation*}
$$

where $k=2 \pi / \beta \lambda, a$ the aperture radius, and $m$ the modulation index. The $A_{j i}$ values are obtained through an interpolation procedure on the Crandall's table values [5].

The two-term potential function is the lowest-order version, having a form of Eq. 1 with $A_{01}=1, A_{10}=A$, and $A_{j i}$ $=0$ for $i, j \geq 2$. Generally $A_{10} \neq A$, as shown in Fig. 1, and consequently PARMTEQ-H simulations yield results different from those parmteq-2 do. In order to obtain a result close to PRANTEQ-2 one, we must make $A_{10}$ correction: $a$ and $m$ values are changed from $a_{\text {old }}$ and $m_{\text {old }}$ (before correction) to $a_{\text {new }}$ and $m_{\text {new }}$ (after correction) so that $r_{0}$ is preserved and $A_{\text {i0 }}\left(a_{\text {new }}, m_{\text {new }}\right)=A\left(a_{\text {old }}, m_{\text {old }}\right)$. We figured out $a_{\text {new }}$ and $m_{\text {new }}$ by using the MOD12 code [6]. In Fig. 1 the $A_{10} / A$
curves cross at the 76 th cell; therefore the $a_{\text {new }}$ 's and $m_{\text {new }}$ 's of the two geometrics are same, as shown in Fig. 2. At the cell center the variable $\rho_{\mathrm{T}}$ vanes have a $\rho_{\mathrm{T}}$ equal to $r_{0}$. We have thus connected the vanes smoothly.

Through the above procedure we fixed the design of the RFQ. The resulting parameters are listed in Table 1, and cell parameters are plotted in Fig. 3.


Fig. 1. $A_{10} / A$ ratios for the variable $\rho_{\mathrm{T}}$ and the $\rho_{\mathrm{T}}=r_{0}$ ones vs cell number. The synchronous phase $\phi_{s}$ is also plotted.


Fig. 2. Corrected $a$ and ma: $\Delta a=a_{\text {new }}-a_{\text {old }}$, and $\Delta(m a)=$ $m_{\text {new }} a_{\text {new }}-m_{\text {old }} a_{\text {old }}$.


Fig. 3. Cell parameters vs cell number. The parameters $a$ and $m$ are the ones before the $A_{10}$ correction.

TABLE 1
Design Parameters of the RFQ

| Frequency $(f)$ | 25.5 MHz |
| :--- | ---: |
| Charge-to-mass ratio $(q / A)$ | $1 / 30$ |
| Kinetic energy $\left(T_{\text {in }} \rightarrow T_{\text {out }}^{\prime}\right)$ | $2 \rightarrow 172 \mathrm{keV} / \mathrm{u}$ |
| Normalized emittance $\left(\varepsilon_{\mathrm{n}}\right)$ | $0.06 \pi \mathrm{~cm} \cdot \mathrm{mrad}$ |
| Input emittance $\left(\varepsilon_{\mathrm{in}}\right)$ | $29.1 \pi \mathrm{~cm} \cdot \mathrm{mrad}$ |
| Vane length $\left(L_{\mathrm{V}}\right)$ | 858.5 cm |
| Number of cells $\left(N_{\mathrm{c}}\right)$ | 172 |
| Intervane voltage $(V)$ | 108.6 kV |
| Maximum surface field $\left(E_{s, \max }\right)$ | $178.2 \mathrm{kV} / \mathrm{cm}$ |
|  | $(2.49 \mathrm{Kilpatrick})$ |
| Max. field enhancement factor $\left(\kappa_{\max }\right)$ | 1.615 |
| Mean aperture radius $\left(r_{0}\right)$ | 0.9846 cm |
| Minimum aperture radius $\left(a_{\text {min }}\right)$ | 0.5388 cm |
| Max. modulation index $\left(m_{\text {max }}\right)$ | 2.53 |
| Final synchronous phase $\left(\phi_{\mathrm{f}}\right)$ | $-30^{\circ}$ |
| Focusing strength $(B)$ | 5.5 |
| Maximum defocusing strength $\left(\Delta_{\mathrm{b}}\right)$ | -0.17 |

## Discussion on Beam Performance

We examined transmission efficiencies for different vanetip geometries. The results of PARMTEQ runs for input beams with $\varepsilon_{\mathrm{n}}=0.06 \pi \mathrm{~cm} \cdot \mathrm{mrad}$ and currents of 0,5 , and 10 mA are listed in Table 2. In Fig. 4, transmission efficiencies as functions of the input emittance are plotted for zero-current beams. The actual geometry 'variable $\rho_{\mathrm{T}}$ $\& \rho_{\mathrm{T}}=r_{0}$ ' is the best: the transmission efficiencies and acceptance are close to those of the ideal vanes.

TABLE 2
Transmission Efficiencies

| PARMTEQ | Vane-tip geometry | 0 mA | 5 mA | 10 mA |
| :---: | :--- | :--- | :--- | :--- |
| 2 | ideal | $91.4 \%$ | $87.6 \%$ | $68.4 \%$ |
| H | var. $\rho_{\mathrm{T}} \& \rho_{\mathrm{T}}=r_{0}$ | $91.4 \%$ | $86.0 \%$ | $65.2 \%$ |
| H | $\rho_{\mathrm{T}}=r_{0}$ | $90.8 \%$ | $81.8 \%$ | $60.2 \%$ |
| H | variable $\rho_{\mathrm{T}}$ | $90.0 \%$ | $80.0 \%$ | $55.0 \%$ |
| H | $\rho_{\mathrm{T}}=0.75 r_{0}$ | $88.0 \%$ | $67.0 \%$ | $46.4 \%$ |



Fig. 4. Transmissin efficiencies vs input emittance ( $0-\mathrm{mA}$ beam).

The observed difference in transmission efficiency comes from that in $A_{12}$ coefficient. The $A_{10}$ and $A_{12}$ multipoles generate radial field components, $E_{r, 10}$ and $E_{r, 12}$. Averaging them over the half rf period, we have

$$
\begin{align*}
& \bar{E}_{r, 10}=-\frac{1}{4} k V A_{10} I_{1}(k r) \sin \phi  \tag{3}\\
& \bar{E}_{r, 12}=-\frac{1}{4} k V A_{12} I_{4}^{\prime}(k r) \cos 4 \psi \sin \phi \tag{4}
\end{align*}
$$

where $\phi$ is the rf phase at the cell entrance ( $\phi<0$ when a particle is accelerated). Since $A_{10}>0$, then $\bar{E}_{r, 10}$ yiclds isotropic rf defocusing, whose strength is proportional to the parameter $\Delta_{\mathrm{rf}}$. On the other hand, $\bar{E}_{r, 12}$ is octapole. If $A_{12}>0$ (this is usual), the beam is pulled in the horizontal and vertical directions and pushed in the diagonal ones. As a result, the $x-y$ profile is distorted from a circle to a rhombus, as shown in Fig. 5. Particles near the verteces may hit a vane, consequently the transmission efficiency will be decreased.

We measure the strength of the octapole field in terms of the following ratio:

$$
\begin{equation*}
R(r)=\frac{A_{12} I_{4}^{\prime}(k r)}{A_{10} I_{1}(k r)} \tag{5}
\end{equation*}
$$

In Fig. 6, the $R$ ratios at $r=0.3 \mathrm{~cm}$ are plotted for vanetip geometries. The connection of the variable $\rho_{\mathrm{T}}$ and the $\rho_{\mathrm{T}}=r_{0}$ vanes looks to have advantages of the both geometries. The variable $\rho_{\mathrm{T}}$ vanes have the weakest octapole field at low energies, and at high energies $\rho_{\mathrm{T}}=r_{0}$ vancs have negative $A_{12}$ with smaller $\left|A_{12}\right|$. The negative $A_{12}$ seems preffrable, because the directions of the $\bar{E}_{\tau, 12}$ forces are reversed, and the forces will push/pull the rhombus profile back to a circular one.
The $\rho_{\mathrm{T}}=r_{0}$ vanes throughout the RFQ have an acceptance almost same as that of the ideal vanes for a zerocurrent beam. For $5-$ and $10-\mathrm{mA}$ beams, however, the transmission efficiencies are lower. This will be due to the large $A_{12}$ coefficients at low energies. The variable $\rho_{\mathrm{T}}$ vanes have disadvantages at high energies: $A_{12}$ coefficients are large, and the aperture is reduced ( $\sec \Delta a=a_{\text {new }}-a_{\text {old }}$ in Fig. 2). Since the $A_{10} / A$ ratio is nearly constant over the vane length (Fig. 1), we had better not make the $A_{10}$ correction but increase the intervane voltage by multiplying the design value by a factor of $\sim A / A_{10}$. The $\rho_{\mathrm{T}}=0.75 r_{0}$ vanes have the largest $A_{12}$ coefficients at low energies. The strongest octapole field seems to have lead to the lowest transmission efficiencies.


Fig. 6. Plot of $R(r=0.3 \mathrm{~cm})$ and the rf defocusing strength $\Delta_{\mathrm{rf}}$.

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## References

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Fig. 5. Profiles of zero-current beams in the $x-y$ plane at cell 70 (unit: cm ). The numbers in the graphs indicate the ones of particles: from top to bottom, surviving particles, those lost by hitting a vane, and those not captured by the rf bucket.


[^0]:    ${ }^{1}$ In the present RFQ case, the maximum $\kappa$ 's after the $A_{10}$ correction are calculated to be 1.615 (variable $\left.\rho_{\mathrm{T}}\right), 1.510\left(\rho_{\mathrm{T}}=r_{0}\right)$, and $1.423\left(\rho_{\mathrm{T}}=0.75 r_{0}\right)$.

