# THE MEASUREMENT OF ACCELERATING CELL EIGEN FREQUENCY

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#### Abstract

The accelerating tanks of modern high energy linear accelerators consist of many accelerating cells. The eigen frequency of each cell may have deviation from desired value because of inevitable production tolerances, so usually the tuning of each cell is needed. The tuning procedures require the number of the eigen frequency measurements for each cell and an accuracy and a simplicity of the measurements are important.

The measurements of eigen frequency with the help of two passive cells of simple geometry have been considered. It has been found that the certain dimensions of the passive cells provide a high accuracy of measurements. The method may be useful for tuning and investigation of accelerating structures of high energy linear accelerators.

#### 1. Introduction

The disk loaded waveguide (DLW) operating at  $\pi/2$  or  $2\pi/3$  modes is widely used for high energy accelerators. The errors of manufactoring lead to the deviations of eigen frequencies of the cells and the coupling coefficients from designed values. The deviations of frequency are eliminated by a tuning of the cells. For accelerating structures with a low coupling coefficient a tuning of each cell is needed at least at a stage of development, so the measurements of the eigen frequency of the cells, usually defined as a frequency of  $\pi/2$ -mode of an infinite uniform periodical structure, are needed.

The exact frequency of a  $\pi/2$  -mode is a frequency of a system consisting of an accelerating cell and two half-cells which have magnetic boundaries (Fig.1). The most part



Fig. 1. Disribution of field of  $\pi/2$ -mode in the system with magnetic boundaries.

of a stored energy in this unreal case is concentrated in a cell under interest. The close result may be achieved if one detunes the some end-cells with electric boundaries (metal) by a value much greater than a passband of accelerating structure. It should be remarked that the method is applicable for the cells of different eigen frequencies and coupling coefficients as well as without plane of symmetry. In this study we consider two methods of end-cell detuning, their systematic errors and give the method of estimation of these errors using experimental data.

### 2. Detuning of the end-cells

Let us consider a system consisting of a cell under investigation and two identical end-cells (the identity of end-cells is not necessary in general). A frequency of the cell is equal to  $f_0$  and a coupling coefficient in a uniform chain of such cells is equal to  $k_0$ . The frequencies of the end-cells are equal to  $f_e$ and a coupling coefficient between the cell and the end-cells is equal to  $k_e$ .

Let the frequency of the end-cells differ from the frequency of the accelerating cell:  $f_e = \alpha f_0$ ,  $1 - \alpha >> k_0$ .

The dispersion equation of the system derived as described in [1,2] is a cubic equation, so the system has three frequencies: two of them are close to  $f_e$  and the third  $f_{e0}$  is close to  $f_0$ . The frequency  $f_{e0}$  is an estimation of the eigen frequency of the accelerating cell and an error of the estimation is  $\Delta = f_{e0} - f_0$ . It may be shown by analysis of the dispersion equation that  $\Delta = F(\alpha) k_e^2 + O(k_e^3)$ . The numerical solution of the dispersion equation gives us the value  $\Delta / k_e^2$  as a function of  $\alpha$  (Fig. 2). So, it is clear that to decrease  $\Delta$  we have to decrease both  $\alpha$  and  $k_e$ .



Fig.2. The value of  $\Delta / k_e^2$  as a function of  $\alpha$ .

The methods of the end-cell detuning considered below were investigated by electrodynamic calculations using code MULTIMODE and the results are given in detail in [2].

Let the end-cells have an initial frequency close to  $f_0$  and are detuned by the plungers as shown in Fig. 3 (the parameters of accelerating cells are taken from [3]).



Fig.3. Detuning of the end-cells by the plungers.

A typical behavior of  $f_{e0}$  in dependence on a length 1 of plungers is shown in Fig. 4 .



Fig.4. Estimation  $f_{e0}$  as a function of plunger length l. 1 - end-cell length is equal to the length of investigated cell, 2 - it is reduced by two times.

The length 1 at which  $df_{e0}/dl$  is minimal may be choosed as an optimal one because the value of  $f_{e0}$  is close to  $f_0$  and a sensitivity of  $f_{e0}$  to the small errors of 1 is minimal. The systematic error at this point  $\Delta \cong f_0 k_e^2/2$  for  $\alpha < 0.75$ . The length of the end-cells must be equal to or greater than the length of the investigated cell to reduce  $k_e$ . Usually this is fulfilled because the method is always used when the accelerating structure is already brazed and the adjustent cells are used as the end-cells.

The systematic error may be reduced by several times if the end-cells are detuned by an increasing of their radius as it is shown in Fig. 5. The frequencies of the end-cells are lowered and simultaneously the coupling coefficient  $k_e$  is decreased.



Fig.5. Detuning of end-cells by increasing of their radius R.

The estimated frequency  $f_{e0}$  is changed in dependence on L in similar way as in previous case and also has the optimal point . At this point the systematic error is several times less than for previous method of detuning and is negative. The calculations show that the radius of the end-cells should be choosed b + 0.04  $\lambda < R < b + 0.08 \lambda$  (b - radius of accelerating cell,  $\lambda$  - wavelength of operating mode ). If the dimensions of accelerating cells are changed from cell to cell (constant gradient structure) one can use one pair of the end-cells of the intermediate dimensions for all cells.

There is a possibility to detune the end-cell by decreasing of their diameter, but this variant is worse than the ones mentio - ned above in all aspects [2].

All results and estimations are valid also for measurements of the frequencies of  $\pi/2$  and  $2\pi/3$  modes in a system consisting of two accelerating cells and detuned end-cells (Fig.6). In this case systematic error  $\Delta$  will be less twice.

## 2. Systematic error of eigen frequency measurement.

The estimations of systematic error made above permit to compare the considered methods. In practice the exact value of  $\Delta$  is interesting and it can be estimated on a base of experimental data [3].

Let we have two accelerating cells with egien frequencies  $f_a$  and  $f_b$  and let  $|2(f_a - f_b)/(|f_a + f_b)| \le k_o$  ( that is usually true).





Fig. 6. System of cells for estimations of frequencies of  $\pi/3$  and  $2\pi/3$  modes.

The frequency measurements of each cell and the combination of both cells (Fig. 6) will give us:

$$f_{1} = f_{(\pi/2)a} = f_{a} \{1 + k_{o} (1 + \eta k_{e}^{2})\}^{1/2}$$

$$f_{2} = f_{(\pi/2)b} = f_{b} \{1 + k_{o} (1 + \eta k_{e}^{2})\}^{1/2}$$

$$f_{3} = f_{(\pi/3)} = (f_{a} + f_{b})\{1 + k_{o} (1 + \eta k_{e}^{2}/2)/2\}^{1/2}/2$$

$$f_{4} = f_{(2\pi/3)} = (f_{a} + f_{b})\{1 + 3k_{o} (1 + \eta k_{e}^{2}/2)/2\}^{1/2}/2$$
(1)

From these equation one can derive the estimation

$$\Delta = \eta k_e^2 = (\gamma - 1)/(1 - \gamma/2), \qquad (2)$$

where

$$\begin{aligned} k_0 &= 2(f_4^2 - f_3^2)/(3 f_3^2 - f_4^2) , \\ \gamma &= (f_1 - f_2) \{ (2 + k_0)/(1 + k_0)/2) \}^{1/2}/2f_3 . \end{aligned}$$

For all methods of end-cell detuning  $\Delta$  increases with increasing of the coupling coefficient  $k_e$ , which grows fast with increasing of the iris radius a. As the example we have considered  $\Delta$  calculated for the cells with parameters taken from [1] and for the second method of end-cell detuning. The relative systematic error  $\Delta/f_0$  as a function of iris diameter is given in Fig. 7.

The formulas (1,2) have been derived on a base of a dispersion equation taken from [1]. If the equation describes a dispersion curve of DLW even with a small deviation, the value of  $\Delta$  may be overestimated drastically. In this case the correction of the estimation is necessary. One has know the true values of the frequencies of modes  $\pi/2$ ,  $\pi/3$  and  $2\pi/3$  from exact electrodynamical calculations or experiments. The application of (1,2) to these exact frequencies gives us the value of the correction  $\Delta_{cor}$ , so the real systematic error will be  $\Delta_r = \Delta - \Delta_{cor}$  ( $\Delta_{cor} = 0$ , if the dispersion equation describes the accelerating structure with good accuracy). Corrected estimation for this method is also given in Fig. 7.



Fig.7. Relative systematic error as a function of iris diameter. 1 - without correction, 2 - corrected one.

From Fig. 7 one may conclude that the relative systematical error of the method less than  $3 \times 10^{-5}$  without correction is possible for  $a/\lambda < 0.096$ , with correction for  $a/\lambda < 0.14$ . Relative error less than  $10^{-4}$  is possible for  $a/\lambda < 0.11$  without and for  $a/\lambda < 0.168$  with correction. This range of the iris diameters overlaps practically all types of the DLW used in electron linaes for scientific and industrial purposes.

## Conclusion

We have considered the method of estimation of eigen frequency of accelerating cells with the use of detuned end-cells and derived the formulas for evaluation of systematical error using the experimental data. Several methods of the end-cell detuning have been considered. It was shown that the detuning of the end-cells by increasing of their radius has the minimal systematic error and provides needed accuracy of the measurements for all types of disk loaded waveguide used in electron linacs for scientific and industrial applications.

### References

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