DO IMAGE FIELDS AFFECT RFQ EFFICIENCY?

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Abstract

Mere knowledge of tune depression and illumination of the acceptance is sufficient to estimate limitations. Evidence comes out of a plain smooth approximation theory.

Foundations

High current RFQ linacs necessarily begin with a shaper, an accelerator section almost without modulation. Therefore the corresponding theory may answer the purpose within a two-dimensional scope, where transversally hyperbolic quadrupole boundaries generate the image fields. In [1] these image forces have been explained in detail, some new results will be discussed in this report. In smooth approximation equations of motion may be expressed in a rather general form

$$\frac{d^2x}{ds^2} + \frac{1}{4}x - \frac{0.103}{1 - I/I_{max}}\left(+ x^3 + 0.29\,x^7 + 0.075\,x^{11} + 0.019\,x^{15} + \ldots\right) = 0 \qquad (1)$$

Here x stands for the normalized radial coordinate with quadrupole aperture R_1, $x = r/R_1$ and s for normalized time $s = \frac{\sigma}{\pi}\omega t$ with radio frequency $f = \omega/2\pi$. The depressed tune shift σ is in smooth approximation given by

$$\frac{\sigma^2}{\pi^2} = \frac{\sigma_o^2}{\pi^2} - \frac{2eI}{\pi\varepsilon_o m v \omega^2 R_1^2}$$

Maximum beam current is related to zero current phase advance σ_o according to

$$I_{max} = \frac{\sigma_o^2}{\pi}\,\frac{\varepsilon_o m v R_1^2\,\omega^2}{2e}$$

Further meanings are I beam current, e/m specific ion charge, ε_o dielectric constant and v ion velocity. Numbers appearing in equ. (1) result from a field harmonic analysis of the image potential Φ_{norm} within hyperbolic quadrupole boundaries [1, 2]

$$\Phi_{norm} = \sum A_N r^N \cos N\varphi$$

(polar coordinates r, φ). Table I informs on the A_N. Then realistic field harmonics a_n corresponding to any given aperture, beam current and particle velocity follow from

$$a_n = \frac{A_N I}{4\pi\varepsilon_o v R_1^N} \qquad (2)$$

Data of table II were determined by numerical integration of equ. (1). Here the ratio I/I_{max} was raised toward such values, which caused loss of even one particle out of the matched beam at given illumination of the acceptance by the beam emittance. Fig. 1 illustrates typical emittance behaviour after 4 betatron oscillations (16π) in case of $I/I_{max} = 0.44$. Filamentation is evident and in particular symbols * correspond to particles, which initially started at points ◊ in phase space and should at 16 π reproduce in the absence of image forces. Then fig. 2 demonstrates the sensitiveness with respect to beam current.

More detailed studies of beam behaviour have just started in our institute, where for numerical computer simulations PARMTEQ [3] had to be supplemented by proper image field terms. Table III indicates tendencies. Computations are based on an RFQ set-up [4] for a 10 keV He_4^{1+} beam. Electrode voltage 2 kV, aperture R_1 = 5 mm and frequency f = 16.9 MHz determine a maximum beam current I_{max} = 6.8 mA and tune σ_o = 44.23° is depressed to σ/σ_o = 0.75 by 3 mA. This RFQ is equipped with a 2 βλ radial matcher (without image forces) and illumination is supposed to be 100 %. For this first approach only the octupole term a_4 according to equ. (2) together with table I was involved and all terms associated with the elliptical beam shape as well as a_8, a_{12} . . . were neglected. Table III lists particle losses and separates them into such that are due to the image forces and such, which are caused by space charge effects anyhow. Wall effects are evident. Methodical investigations are in progress and will be reported elsewhere.

Summary

Limitations of minimum tune depression and maximum illumination due to wall effects must surely be observed, when a high current RFQ is taken into consideration. Table II demonstrates restrictions more or less stringent.

Acknowledgement

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References

[1] P. Junior, Part. Acc., Vol. 39 (1992) pp. 1 - 10
[2] P. Junior, A. Harth, Part. Acc., Vol. 37/38 (1991) pp. 47
[3] PARMTEQ computer code, LANL, Los Alamos
[4] N. Zoubek, thesis, Universität Frankfurt a. M. 1987

Table I
Normalized Field Harmonics in a Quadrupole

$A_4 = -0.206$ $\quad A_{12} = -0.512 \cdot 10^{-2}$

$A_8 = -0.296 \cdot 10^{-1}$ $\quad A_{16} = -0.997 \cdot 10^{-3}$

Table II
Current Limits, Minimum Tune Depression and Maximum Emnittance Illumination of Acceptance

Illum. %	81	64	49	36	25	16
I/I_{max}	0.44	0.61	0.68	0.76	0.82	0.88
σ/σ_o	0.75	0.62	0.57	0.49	0.42	0.35

Table III
Particle Losses with and without Image Forces

I/I_{max}	Input behind matcher	Output without image fields	Output with image fields	Relative loss due to wall
0.44	943	869	824	4.8
0.51	907	815	759	6.2
0.59	897	804	747	6.4
0.66	881	762	691	8.1
0.74	849	735	662	8.6

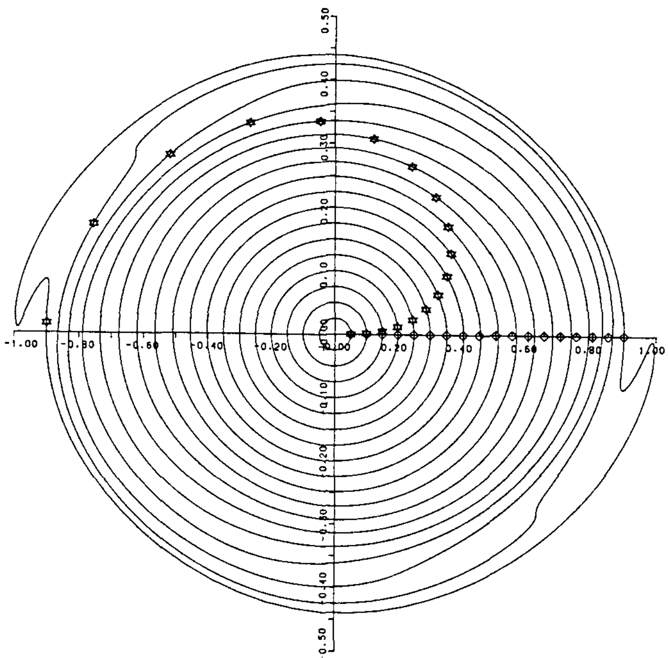

Fig. 1 Solutions of equ. (1) in phase space after 4 betatron oscillations for I/I_{max} = 0.44. Particle states * correspond to initial states ◊, resp.

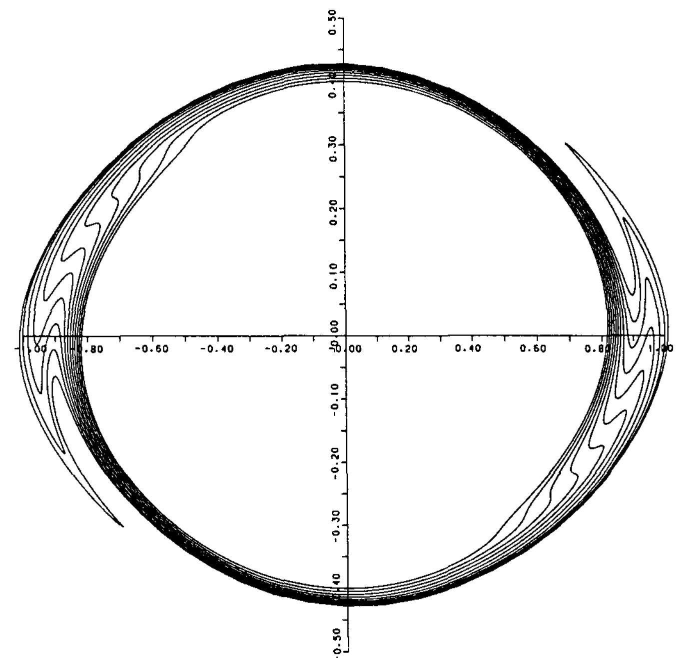

Fig. 2 Behaviour of outer particles for I/I_{max} = 0.47 in phase space after 4 betatron oscillations