BBU CODE DEVELOPMENT FOR HIGH-POWER MICROWAVE GENERATORS*

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Abstract

We are developing a two-dimensional, time-dependent computer code for the simulation of transverse instabilities in support of relativistic klystron-two beam accelerator research at LLNL. The code addresses transient effects as well as both cumulative and regenerative beam breakup modes. Although designed specifically for the transport of high current (kA) beams through traveling-wave structures, it is applicable to devices consisting of multiple combinations of standing-wave, traveling-wave, and induction accelerator structures. In this paper we compare code simulations to analytical solutions for the case where there is no rf coupling between cavities, to theoretical scaling parameters for coupled cavity structures, and to experimental data involving beam breakup in the two traveling-wave output structure of our microwave generator.

Introduction

The beam breakup (BBU) instability, a form of pulse shortening, has been observed in particle accelerators and high-power klystrons. This instability arises from the excitation of higher order resonant modes within various structures along the beamline. The high-power microwave program at LLNL has observed BBU in the SL4/TW relativistic klystron[1] and in the Choppertron microwave generator.[2] Transient phenomena during the short beam pulses associated with these devices complicates the analysis of the BBU mechanism. Significant theoretical work has been accomplished on BBU since it was first observed in 1957/58 including analytical solutions for cumulative BBU[3] and scaling parameters[4] for regenerative BBU. However, to account for realistic beam conditions and transient effects on BBU growth in complex high-power microwave generator structures, it is necessary to use numerical methods.

Description of BBU Code

The BBU Code advances slices of a beam through a series of resonant cavities which comprise the microwave generator structure. The code has the following features:

1. dipole fields for azimuthally symmetric cavities
2. self-consistent beam-cavity interaction
3. treatment of cumulative and regenerative BBU
4. time-dependent cavity excitation

The basic equations used are the single particle equations of motion in the x and y direction, and the coupled circuit equations governing cavity excitation. Assuming a single cavity mode is dominant, let the x-polarization of the electric field in the n^{th} cavity be given by

$$\vec{E}_n(\vec{r},t) = f_n(t)\vec{\xi}_n(\vec{r})\,e^{i\omega t}, \tag{1}$$

where ξ_n denotes an eigenmode with eigenfrequency ω_n. Here ω denotes a characteristic frequency of the device assumed near the transverse instability resonance. It is possible to show that the excitation amplitudes f_n are governed by the following circuit equations

$$\ddot{f}_n + \left(\frac{\omega_n}{Q_n} - 2i\omega\right)\dot{f}_n + \left(\omega_n^2 - \omega^2 - \frac{i\omega\omega_n}{Q_n}\right) f_n = K_n^{n-1} f_{n-1} + K_n^{n+1} f_{n+1} + \frac{\omega_n^3}{\varepsilon c^2}\left(\frac{Z_\perp}{Q}\right)_n \frac{\partial Ix}{\partial t}\, e^{-i(\omega t + \phi_n)}, \tag{2}$$

where Q_n denotes the quality factor of the n^{th} cavity, $K_n^{n\pm1}$ denotes coupling of the n and n±1 cavities, I is the current, x is the transverse displacement of the beam's centroid in the x direction from the center line, ϕ is a phase advance, and $Z_\perp$ denotes the transverse impedance. A second circuit equation is used for the y-polarization. The last term in equation (2) assumes the beam is near the axis.

Comparison With Theory

Numerous checks were made to verify the results of the code. The checks can be divided into the code response for cumulative and for regenerative BBU.

Cumulative BBU

For the case of cumulative BBU, comparison can be made with analytical solutions for specific boundary conditions. To simulate the effect of cumulative BBU, seven standing-wave cavities with narrow gaps and separated by drift spaces were used as the beam line. The parameters needed for the code were chosen to match the non-dimensional parameters used in Ref. 3 so that the results could be compared directly. Initial excitation was similar to a delta function accomplished by imposing the boundary condition that at Z = 0, X(0,T) = 1 for $0 \le T \le 0.2$ and zero otherwise. Z,

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X, and T are dimensionless parameters representing axial distance, transverse beam displacement, and time, respectively. Shown in Fig. 1 is the spatial distribution of X(Z,T) and the asymptotic envelope from an analytic solution.[3] There was excellent agreement with all four cases presented in Ref. 3.

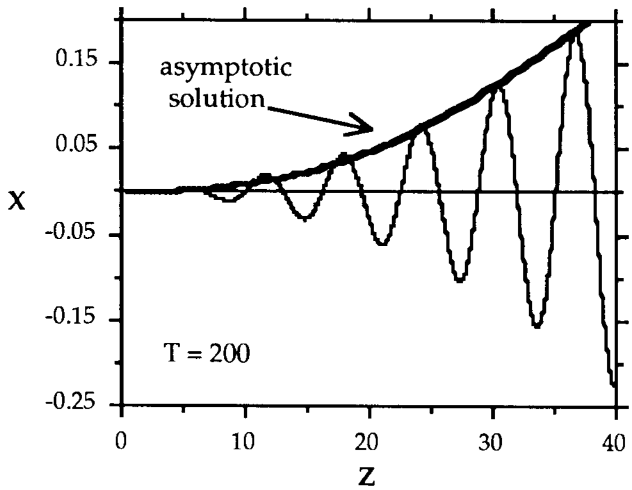

Figure 1. Comparison with analytical solution.

Regenerative BBU

Initial checks for the case where the cavities are electromagnetically coupled were done with no beam interaction. A large number of cavities were joined together to form a traveling wave structure. At T = 0 electromagnetic energy stored in the center cavity of the structure was allowed to propagate. By measuring the energy in the various cavities as a function of time it was possible to ascertain the group velocity of the structure and verify energy conservation ($Q = \infty$).

Final checks involved comparison with scaling parameters. For a uniform impedance structure, the threshold current (for infinite buildup time) has been shown[4] for regenerative conditions to vary as

$$I_{th} \sim v_g \left(\frac{r_\perp}{Q}\right)^{-1} L^{-3}, \tag{3}$$

where v_g is the group velocity, $r_\perp$ is the transverse shunt impedance, Q is the quality factor, and L is the length of the structure. The dependence of the threshold current on these parameters was checked using models of traveling-wave structures similar to the Choppertron output structure. The threshold current was defined as the maximum current for which the stored energy of the structure did not increase after an initial transient period. As the stored energy grew or decayed exponentially away from the threshold, only relatively short runs were needed to determine the threshold current. The results of these checks are shown in Fig. 2 and 3. Scaling with v_g is linear over the values shown in Fig. 2, but differs significantly from linearity at values above 0.3 c. This variation from linear behavior at high group velocities appears to be caused by the finite length of the modeled structure. If the length of the modeled structure is increased, the range and linearity of scaling with v_g is increased. The scaling with $r_\perp/Q$ was in agreement with theory as evidenced in Fig. 3.

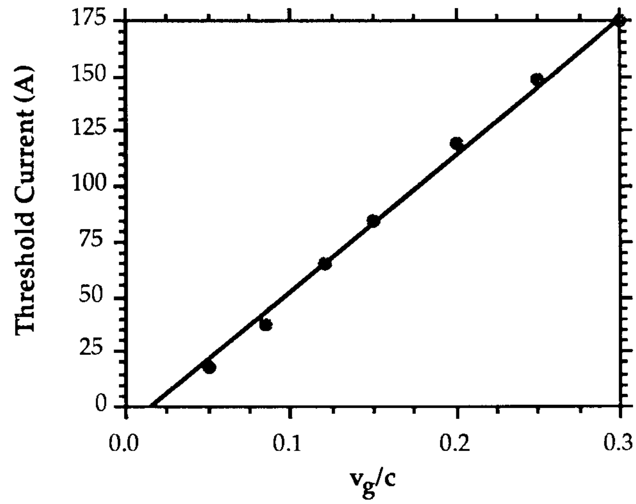

Figure 2. Threshold current scaling with v_g.

Due to the way that parameters are handled in the code, it was not possible to arbitrarily vary the length of the structure without changing other parameters. The most straight forward approach was to add or subtract cavities from the structure. Results of this check are shown in Fig. 3 and agree with theory.

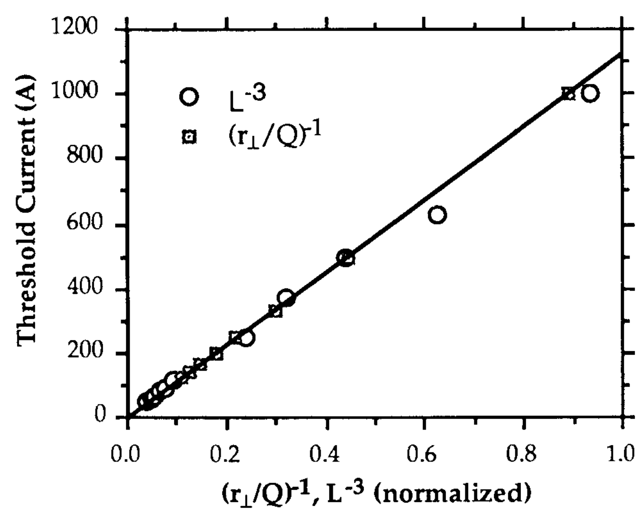

Figure 3. Threshold current scaling with L and $r_\perp/Q$.

Analysis of BBU in the Choppertron

The BBU code was used to simulate the growth in transverse displacement of the beam due to excitation of higher order modes in the output section of the Choppertron. The output section consists of two six-cell, traveling-wave structures with approximately 6 cm of pipe between the two structures. Parameters used in the code to model the Choppertron are given in

Table 1. The resonant mode and frequency were chosen such that the phase velocity is approximately the speed of light. Fig. 4 shows the beam's displacement as a function of axial position for two different currents; one below the threshold for beam loss and the other above. Fig. 4 is divided into three regions to represent the two traveling-wave structures and the connecting pipe. Little growth in displacement is seen in the first structure, but the small additional transverse momentum imparted for the higher current causes sufficient displacement for the beam to be rapidly driven into the wall of the second structure. These "snapshot" plots are taken 45 ns into the pulse. At 420 amperes, the displacement in the second structure begins to exceed 3 mm (threshold for current loss) after about 38 ns.

TABLE 1
Modeling Parameters for Choppertron Output

resonant mode	lower "HEM_{11} branch"
frequency	13.6 GHz
phase advance	142.5° per cell
# cells per structure	6
# structures	2
cell length	0.8754 cm
aperture	14 mm
group velocity	0.12 c
Q_{wall}	3000
$r_{\perp}/Q$	18.33 Ω/cell
current rise time	5 ns
initial beam offset	0.1 mm
beam radius	4 mm

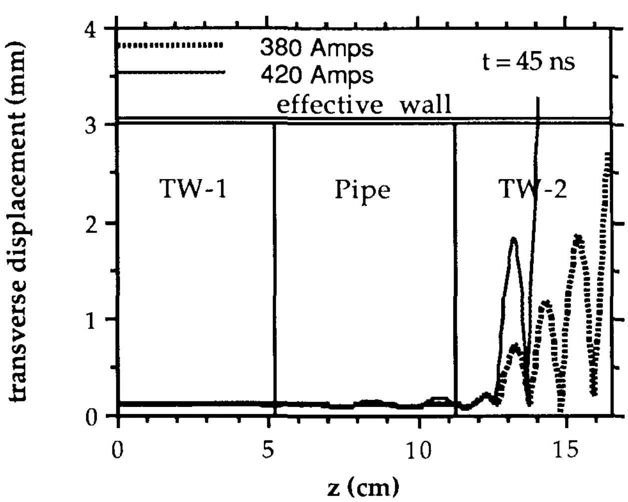

Figure 4. Simulation of Choppertron output section.

Although it is not possible to measure the threshold current for infinite buildup time in the experiment, the current threshold at which beam loss occurs and the power extracted from the output section as a function of current can be measured. By varying the current used in the code it was possible to determine a threshold of 400 to 420 amperes at which beam loss would appear. This value agrees with the experimentally measured value of approximately 400 amperes. The code also predicted that the power extracted at the BBU frequency would vary exponentially with the current, and agreed within a standard deviation of the experimentally measured growth rate. In contrast, the power extracted at the primary (TM_{010}) frequency varies as the square of the modulated current.

Future Work

A planned improvement for the BBU code is a more accurate determination of coupling coefficients between dissimilar structures. We also intend to add options to allow for acceleration/deacceleration of the beam and to allow more realistic initial beam conditions.

Summary

We have developed a code to study the effect of higher order resonances on transverse beam instabilities in high-power microwave generators. The code has been successfully checked against theoretical predictions as well as experimental results. The most important feature of this code is the ability to analyze effects of transient behavior associated with the interaction of the short beam pulse with resonant cavities. We are using the code to assist with the design of experiments to be performed at the LLNL Microwave Source Facility.[5]

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