

**BEAM DYNAMICS AND LONGITUDINAL INSTABILITIES
IN HEAVY-ION-FUSION INDUCTION LINACS***

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Abstract

An induction linac accelerating a high-current pulse of heavy ions at subrelativistic velocities is predicted to exhibit unstable growth of current fluctuations. An overview is given of the mode character, estimates of growth rates, and their application to an IFE driver. The present and projected effort to understand and ameliorate the instability is described. This includes particle-in-cell simulations, calculation and measurements of impedance, and design of feedback controls.

Introduction

A heavy-ion, linear induction accelerator suitable as a driver for inertial confinement fusion is subject to an intrinsic, but slow growing instability. The unstable mode is a longitudinal bunching of the beam, which acts back on itself through the emf it induces via the acceleration modules. A growing wave moves backwards in the beam pulse and forwards in the accelerator. The essential beam dynamics are thought to be fairly simple at low frequencies ($v < 30$ MHz) and are often described using a 1-d cold fluid model. The interaction of the beam with the induction modules may be described by a complex coupling impedance $Z = -\delta E/\delta I$, which contains the complications of cavity design, drive circuitry, core materials, and multiple beam configuration. Since acceleration must be reasonably efficient for commercial generation of electricity from inertial fusion, the coupling impedance at very low frequencies is that of the external drive source (R) that must be approximately equal to the matched value $R_0 = G_0/I_0$, where G_0 and I_0 are the mean accelerating gradient and beam current. It is this unavoidable resistive component of impedance which causes wave growth at low frequencies, and it is generally in the range 100-1000 Ω/m over the entire high-current portion of the accelerator. There are, however, other impedance features which influence mode character and growth. Module capacitance C acts in parallel with R and causes the net coupling impedance to fall rapidly with increasing frequency. Similarly, the direct interaction of the beams' space charge with itself

and Landau damping by momentum spread become important as frequency rises. For $v > 50$ MHz cavity resonances are expected. The resistive character of a resonance near its peak and its inductive character for frequencies below the peak are highly destabilizing in principle. Work done to date in characterizing the coupling impedance of driver modules indicates that the resonance Q values will be very low (~ 10) due to dissipation in the tape core area and the gap transit-time effect. Momentum spreads on the order of $\Delta P/P \approx 10^{-3}$ - 10^{-4} could then effectively damp the resonant instability. The emerging picture for the longitudinal instability is therefore that of predictable danger at very low frequencies and probably benign resonances at high frequency. The low frequency disturbance is expected to have growth lengths on the order of 100 m or greater; it is reasonable to consider feed-forward control of accelerating wave forms such that the coupling impedance for these disturbances is effectively zero. Such a control system would probably be an addition to the simple waveform control required in the absence of instability and would increase the system cost.

Instability Model

A simple model of the longitudinal instability at low frequency⁽¹⁾ is reproduced here. We treat a cluster of beams drifting at velocity v , with line charge density λ and current $I = \lambda v$. It is assumed here that all of the beams (≈ 4 -32) effectively act in concert so that λ and I are total values, and v is their common velocity. The continuity equation, written in laboratory frame variables (z,t) is:

$$\frac{\partial \lambda}{\partial t} + \frac{\partial I}{\partial z} = 0 \quad (1)$$

A smoothed longitudinal field E , induced by interaction of I with the induction modules, acts on v :

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} = \frac{q e}{m} E \quad (2)$$

In general, perturbed components of E and I are related by an impedance at angular frequency ω :

$$\delta E(\omega) = -Z(\omega) \delta I(\omega) \quad (3)$$

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In the present study the low frequency interaction is modeled as that of a distributed resistance R (Ω/m) and capacity C (F-m) in parallel. We may use the circuit representation:

$$\frac{E}{RC} + \frac{\partial E}{\partial t} = \frac{I_0 I}{C} \quad (4)$$

Some previous work² neglected the capacity but included a direct space-charge force proportional to $\partial\lambda/\partial z$. The present model appears to be representative at low frequencies for the Heavy Ion Fusion application although a parallel inductance should be included to properly treat frequencies much lower than an inverse pulse length. In general the capacity reduces growth rates compared with the case of pure resistance by lowering the impedance as frequency increases.

A perturbation analysis is carried out for small variations from constant (drifting beam) values. For $v = v_0 + \delta v$, $\lambda = \lambda_0 + \delta\lambda$, $I = I_0 + \delta I$, $E = \delta E$, the perturbed equations are:

$$\delta I = \lambda_0 \delta v + v_0 \delta \lambda, \quad (5)$$

$$\frac{\partial \delta \lambda}{\partial t} + \frac{\partial \delta I}{\partial z} = 0, \quad (6)$$

$$\frac{\partial \delta v}{\partial t} + v \frac{\partial \delta v}{\partial z} = \frac{q e}{m} \delta E, \quad (7)$$

$$\frac{\delta E}{RC} + \frac{\partial \delta E}{\partial t} = - \frac{\delta I}{C} \quad (8)$$

The values of R and C are related to beam parameters by considerations of system efficiency.³ For a good match of source to beam load, R must not be too different from the matched value $R_0 = G_0/I_0$, where G_0 is the average accelerating gradient. For the typical parameters $G_0 = 10^6$ volts/m and $I_0 = 1000$ A, we have $R_0 = 1000 \Omega/m$. In this case R could be reduced to 300 Ω/m without a drastic loss of efficiency (~25% of source power is reflected). The characteristic time $RC \equiv \alpha^{-1}$ should be a small fraction of the pulse length to avoid excessive energy flow in charging the module gaps. For the typical value $C = 3 \times 10^{-10}$ F-m, we then have $RC = 90$ ns, which is short compared with a typical 500 ns pulse length, but is somewhat longer than the ~25 ns desired for source and core economy.

In this simple model, time scales with $RC = \alpha^{-1}$, where the "retarded time" variable $\tau = t - z/v_0$ is used. A second scale quantity

$$K = \sqrt{\frac{q e I_0}{m v_0^2 C}}$$

appears in the theory and scales the variable z . That is, $\alpha\tau$ and Kz appear in a dimensionless formulation of Eqs. (5-8).

If we neglect the self-force from space charge, proportional to $\partial\lambda/\partial z$, the coupled equations for the perturbed field and current are conveniently written using z and the retarded time $\tau = t - z/v_0$ as independent variables. We have

$$\frac{\partial^2 \delta I}{\partial z^2} = K^2 C \frac{\partial \delta E}{\partial \tau}, \quad (9)$$

$$\left(\alpha + \frac{\partial}{\partial \tau} \right) \delta E = - \frac{\delta I}{C} \quad (10)$$

A dispersion relation and growth rate may be derived immediately from Eqns. (9) and (10). Taking δI and δE to vary as $\exp(-i(\omega t + \Omega z))$, we find for general impedance $Z(\omega) = -\delta E/\delta I$:

$$\Omega^2 = -i\omega K^2 C Z(\omega) \quad (11)$$

The maximum growth rate in z is found for given real $\omega > 0$ to be:

$$\Omega_i = \left[\frac{K^2 C \omega (|Z| - Z_i)}{2} \right]^{1/2}, \quad (12)$$

which for a parallel R-C impedance becomes (with $x = \omega RC$)

$$Z = R(1 - ix)^{-1}, \quad (13)$$

$$\Omega_i = \frac{K}{\sqrt{2}} \left[\frac{x}{\sqrt{1+x^2}} - \frac{x^2}{1+x^2} \right]^{1/2} \quad (14)$$

Note from Eq. (12) that the positive Z_i (from parallel C) is stabilizing, while negative Z_i (from parallel inductance) would be destabilizing. At very low frequency ($x \ll 1$), the growth rate with distance (z) is

$$\Omega_i \rightarrow K \sqrt{\frac{x}{2}} \quad (15)$$

However, for $x \gg 1$ the growth rate decreases with increasing ω as

$$\Omega_i \rightarrow \frac{K}{2x} \quad (16)$$

This is the consequence of the parallel capacity, which strongly reduces the impedance [Eq. (13)] that appears in the growth formula Eq. (12).

For calculations of transient growth a saddle point analysis of the dispersion relation [eq.(11)] may be made. Sparing the reader the details of this process, it is found that maximum growth occurs at $\alpha\tau = (3/\sqrt{32})Kz$ where the exponent is $Kz/(2\sqrt{2})$. To translate these results into meaningful numbers, we use the specific parameters given in Table 1 and discussed above. The perturbation then grows as $\exp(2.6z)$, where z is given in kilometers, out to about 1.4 kilometers where $\alpha\tau_p = (3/\sqrt{32})Kz$. The exponentiating rate, $Kz/(2\sqrt{2})$, is what one would get from the simple growth formula [Eq. (14)] for a sinusoidal velocity perturbation at a frequency of $\nu = \alpha/(2\pi\sqrt{3}) \sim 1$ MHz, which is too low to appear on a 500 ns pulse. For a more reasonable 10 MHz perturbation, where several cycles fit within the 500 ns pulse length, the exponentiation rate from Eq. (14) would be $.64 \text{ km}^{-1}$ instead of the maximum value of 2.6 km^{-1} . Both of these growth rates are significantly smaller than earlier estimates which neglected the role of C. Therefore one might contemplate the use of feed-forward techniques to prevent serious growth.

Current Efforts on Longitudinal Instability

In addition to simple calculations of growth rates and impedances, an effort is underway to gain a more detailed understanding of the longitudinal instability. The principal activities are as follows:

Detailed Code Calculations of Impedance

The 2-d electromagnetics AMOS code developed at LLNL^{4,5} for application to electron induction linacs is now being used for heavy ion module impedance calculations. Modifications needed for the HIF application have been made, which include non-relativistic beams ($\beta \neq 1$) and extension to 3-d. The difficult problem of incorporating tape core properties into the AMOS formulation is also receiving attention. The higher dimensional capability is necessary to treat accurately the effect of localized drive leads (there are 1 - 4 of them), as well as multiple beams. These latter features are expected to be important only at fairly high frequency.

Impedance Measurements

There are as yet no realistic prototype modules for a heavy ion driver at full scale. However, at LBL a scaled core model has been characterized⁶ for resonance frequencies and Q values. An essential feature of impedance tests with metglas is that they be made on an excited driver core, e.g., half-way to saturation, instead of in an unmagnetized state. The latter circumstance gives a near short circuit for small signals at low-to-medium frequencies.

1-d Simulation of Instability

An existing 1-d simulation code (SHIFTz) has been extended to study the use of feedforward control of both pulser errors and longitudinal instability. The code consists of a 1-d particle pusher coupled to a circuit equation for the module response and a long-wave-length space charge force. It has been used particularly for studying the role of momentum spread in eliminating resonant mode growth at high frequency.⁷ With recent structural changes made in the code, perturbed waveforms can be amplified (with added noise) and translated to downstream correction points. A similar

TABLE 1
Application to Heavy Ion Drivers

Kinetic Energy T	1000 MeV	Beam current I_0	1.0 kA
Ion Mass M	200 amu	Pulse Length τ_p	500 ns
Ion Charge State q	+1	$K = (qe\lambda_0/mv_0^2C)^{1/2}$	$7.33 \times 10^{-3} \text{ m}^{-1}$
Module Capacity C	$3 \times 10^{-10} \text{ F-m}$	$\alpha^{-1} = RC$	90 ns
Module Resistance R	300 Ω/m	Maximum growth point: $(\tau/\tau_p) z_{km}^1$.699
Ion Velocity v_0	.104 c	Amplitude at maximum growth point	$\exp(2.59 z_{km})$

code has also been exercised on this problem at Sandia Laboratory in a collaborative effort with LBL. Although it is recognized that feedforward control is probably necessary for an ICF driver, it is unclear how large a subsystem it would be. It is possible that a system designed to compensate for pulser errors alone will also be found adequate for stabilization after the realistic impedances are known. Generally, it is found that corrections must be applied within a fraction of the minimum unstable growth length to be effective.

2-d Simulation

Persistent questions about the dynamics of beam pulse ends remain unanswered. For example, it is not known to what degree a wave reflects or how wave energy converts to thermal spread there. This area is appropriate for a basic study with a 2-d simulation (WARP - r, z, t), so that the fields and particle orbits at pulse ends can be determined in a consistent manner. The study⁸ is now underway at LLNL. To date, unstable growth rates, computed with the 2-d code are in good agreement with predictions from the 1 - d codes.

Beam Dynamics Experiments

At a longitudinal instability workshop held at LBL in Spring 1990 and at a subsequent workshop in February 1992, there was considerable discussion of possible experiments, and it was emphasized that demonstrated control of the mode would be desirable prior to building a major accelerator. Unfortunately, a convincing experiment requires a combination of high currents (≥ 100 Amperes), subrelativistic velocity, system length \gg pulse length, tape cores, and a core drive system. Scaled experiments with electrons have been suggested. M. Reiser⁹ would use ~ 5 kV, 50 mA, 10 ns pulses in a short drift tube with added resistance. The cost would be low, but parasitic capacitive and radiative effects would tend to dominate over a large ($k\Omega$) imposed wall resistance. However, such an experiment could be used to check code predictions. Finally, the possibility of studying the longitudinal instability is being considered for the planned accelerator experiment ILSE.¹⁰ The ILSE induction modules will resemble those of a driver at $\sim 1/2$ scale and will allow fairly realistic impedance tests. Characterization of ILSE prototype modules is now underway at LBL.

Conclusion

In conclusion, predicted low frequency growth rates of the longitudinal instability are considerably reduced when gap capacity is taken into account, and resonant growth is suppressed by low Q values and dynamical effects. Feedforward control at low frequency is promising. A program to resolve remaining uncertainties is in place, although realistic beam

dynamics experiments appear to be very costly. However, particle-in-cell beam simulation combined with component development, module tests, and impedance calculations should provide an adequate basis for future program decisions.

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