STUDIES OF A TM012 BRIDGE COUPLER

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Abstract

The TM_{012} rf cavity mode, in a right circular cylinder, has been studied as a candidate for the bridge-coupler application in the coupled-cavity (CCL) portion of the SSC linac. The studies were made with the aid of mode charts, computer models (LOOP, DISPER, SUPERFISH, and MAFIA) and cavity models. The potential of mode mixing with other cavity modes is always a concern when considering untried structures. As the length of this type of bridge coupler increases, its diameter decreases, implying a larger range in the L/R ratio. The frequencies of other cavity modes have a strong dependence on the L/R ratio of the structure. In the range of bridge coupler lengths, required for the SSC CCL linac, mode mixing problems should be easy to avoid. The cavity model, built and tested for these studies, was close to the worst L/R ratio for the entire SSC application and no mode mixing problems were observable. Progress was made on understanding the effects introducing a third distinct geometry in the midst of a bi-periodic cavity chain and the effects of coupling two slightly different biperiodic chains. This bridge coupler reduces the effective cell count in the cavity chain and increases the mode spacing to the nearest modes in long structures. The group velocity is high, the vacuum properties are good, the structure is simple, and the fabrication costs should be low.

The TM012 Bridge Coupler

The TM_{012} mode, in a right circular cylinder, appears to be an excellent choice for the bridge coupler application. The group velocity is high, the structure is simple, and the fabrication costs should be low.

The principal quality of a bridge coupler, effecting the mode spectra of the coupled structure, is the transit time of energy through the bridge coupler, which, in turn, is equal to the bridge coupler length divided by the group velocity of the cavity mode. As the bridge coupler length is determined by other considerations, the principal figure of merit for a particular bridge coupler candidate is its group velocity -- the higher the better.

The two lower frequency TM modes were not considered because: 1) the TM_{010} mode has zero group velocity, and 2) the TM_{011} mode has a reversal of field polarity from end to end and no magnetic field at its center, where such structures are commonly driven (with magnetic coupling). Thus, the TM_{012} mode is the lowest frequency TM mode having the desired properties.

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Bridge Coupler Lengths

Bridge coupler lengths are normally constrained to be odd multiples of the average cell length ($\beta\lambda/2$). In non-relativistic linacs (most proton and ion linacs), where the particle velocities increase with energy, every bridge coupler has a different length. Strict application of this constraint would imply that every bridge coupler would also have a different diameter. These differences would keep the cost of bridge coupler fabrication from falling to the low that could be achieved with more similarity between the individual units.

However, it is not necessary for the bridge couplers to have exactly the same length as the inter-tank spacings of the accelerator which they bridge. If the bridge couplers were 10-15% longer than the minimum inter-tank spacing which they are designed to serve, relatively few bridge coupler designs could serve many different inter-tank spacings. Of course, the mating flange locations and the coupling slot dimensions would have to be tailored to the required inter-tank spacing.

Bridge Coupler Tuning

An infinitely long circular cylinder will propagate power at all frequencies above that of the cut-off mode, TM_{010} . Termination of the cylinder between parallel planes introduces discrete modes on the continuous dispersion relation of the infinite cylinder, such as the TM_{011} and TM_{012} modes. These modes still enjoy the power propagation properties and the finite group velocity of the same modes in the infinite cylinder.

The introduction of coupling slots at each end of the terminated cylinder makes if behave as one period of a periodically loaded infinite cylinder, which, in general, will have discontinuities (stop-bands) in its dispersion relation. Near these stop-bands, the power propagation properties of the mode can be seriously impaired and the group velocity can go to zero.

The goal in tuning bi-periodic accelerating structures is to tune out these stop-bands so as to restore the power propagation properties of a finite group velocity. In this context, it is usual to refer to the two different rf cavity modes that exist in the bi-periodic structure as the "accelerating" mode and the "coupling" mode. The accelerating mode is supported by the structure terminations while the coupling mode is defeated by the structure terminations. Nevertheless, the coupling mode has a great influence on the power propagation properties of the structure. The tuning goal is achieved when the geometry of the structure is modified so that the frequencies of these two modes coincide with the desired operating frequency.

It is useful to think about these two modes in the TM_{012} bridge coupler. The coupling mode is identical to the accelerating mode. but displaced from it by $\lambda/4$. The accelerating mode can be excited in the closed cylinder, while the coupling mode cannot. Without perturbations such as the coupling slots or special tuners, the frequencies of these two modes are identical, thus obviating the need for any special tuning to achieve a closed stop-band and good power flow.

It is easy to imagine that coupling slots will perturb the frequencies of the two modes differently, thus opening up a stop-band. It is also easy to conceive of tuner geometries that will counteract that detuning and close the stop-band. For example, a slug tuner on the end-wall of the bridge coupler will effect the two modes differently. Correct adjustment of the end wall location and the slug tuner position should result in a closed stop-band and good power propagation properties for this type of bridge coupler at the desired operating frequency.

Important Properties of Long Cells

The mode spacing, in the vicinity of the operating mode, is a significant figure-of-merit for accelerating structures. Comparison of two common relations for this mode spacing gives some insight into the role of the different features of the structure in this important structure property.

One relation involves a fill time, t_f, defined as the time required for power to flow from one end of the structure to the other -- not to be confused with the Q-related fill time. This fill time is made up of two parts, namely, the time of transit of power through the cells of the structure and the time of transit through the cell coupling apertures. For short cells and small coupling apertures, the latter dominates and we tend to associate the structure performance solely with the coupling geometry and forget the requirement for power propagation through the cells. When considering the effect of bridge couplers on the performance of a structure, it is important to remember both terms.

Resonance in this structure requires that power flow from end to end of this structure and back again in an integral number of rf cycles. This leads to a relation for resonant frequencies, namely, $f=N/2t_f$. The spacing to the nearest modes corresponds to the frequency difference, Δf, associated with the nearest integers, namely $\Delta N=1$, or $\Delta f=1/2t_f$.

The effective group velocity, v_g, of the structure is seen to be L/t_f. Note this group velocity depends on both terms effecting the fill time, namely the cell properties and the coupling aperture properties.

The other common relation for the mode spacing of a structure involves the number of cells, N, the phase shift per cell, Φ, and the slope of the dispersion relation, $d\omega/d\Phi$, in the vicinity of the operating mode, where $\omega=2\pi f$. Resonance in this structure requires that the accumulated phase shift from end to end of the structure and back again be an integral multiple, M, of 2π, or $2N\Phi=M*2\pi$. The nearest modes differ from the operating mode by a $\Delta\Phi$ corresponding to the nearest integer to M, namely $\Delta M=1$, or $\Delta\Phi=\pi/N$, which, in turn yields a frequency difference based on the slope of the dispersion relation, namely, $\Delta f=d\omega/d\Phi/2N$. Noting that the slope of the dispersion relation is commonly interpreted as the effective group velocity divided by the cell length, L_c, we see that the two relations for the mode spacing yield the same result, namely, $\Delta f=v_g/L_c/2N=1/2t_f$, where $L=N*L_c$.

The purpose of this is to establish that the performance of a structure, the mode spacing in the vicinity of the operating mode, and the so-called coupling constant of the structure is dependent on the properties of the cell as well as the properties of the coupling aperture. This subtle point is of little consequence when considering simple periodic or bi-periodic structures. It does, however, take on a significant when considering the effect of bridge couplers on the performance of bridge-coupled structures.

In long bridge couplers with low group velocity, the cell part of the effect predominates over the coupling part. The original LAMPF TM_{010} bridge coupler was in this category. After recognition of this problem, it was modified to a "post-coupled" bridge coupler to enhance its group velocity and the problem went away.

In long bridge couplers with high group velocity, the coupling part predominates over the cell part of the relation. The TM_{012} bridge coupler is in this category. In spite of its length, it should perform pretty much as a single cell with coupling effects commensurate with the size and shape of the coupling apertures.

Interruption of the Bi-Periodic Chain

The coupled resonator model for single- and bi-periodic chains of coupled resonators[1], with nearest and next nearest coupling, pays no attention to the distribution of electric and magnetic fields within the cavities. It involves cell properties and inter-cell properties. In the $\pi/2$ mode (the normal accelerating mode), the relation between the excitations of adjacent accelerating cells is:

$$k_{n,n+1}*X_n = -k_{n+1,n+2}*X_{n+2},$$

where the cell excitation, X_n, is defined to be the square root of the stored energy in the cell and the coupling constant, k, is the principal inter-cell quantity, as suggested by the double subscript.

It should be noted that in this mode, there is stored energy in <u>only one</u> type of cavity (the accelerating cells) and there is <u>only one</u> inter-cell geometry. That is, $k_{n,n+1}$ must equal $k_{n+1,n+2}$ and X_n must equal X_{n+2}.

The standard coupled resonator model is a superb tool for studying the properties of these regular cavity chains. Most of what we know about resonantly coupled structures comes from, or is supported by, this mathematical model.

The coupled resonator model suggests that the coupling aperture serves to couple the excitations of adjacent cells, defined in terms of the square root of the stored energy in the cells. Actually, the physics of the coupling aperture knows only about the fields in the vicinity of the coupling apertures and does not know about the magnitude of the stored energy in the cells. This distinction is of little consequence in single- and bi-periodic structures, as the relationship of the square root of the stored energy in the cell to the fields in the vicinity of the coupling aperture is constant for similar cells.

The introduction of bridge couplers, however, moves us out of the strict bi-periodic domain. Here, there is stored energy in two different type of cavities (the accelerating cells and some of the bridge coupler cells) and there are two different inter-cell geometries.

The equation above shows that, in the $\pi/2$ mode, the coupling constant is inversely proportional to the square root of the energy in the excited cell. Hence, we conclude that the coupling constants associated with large bridge couplers with large stored energies will be small. However, this coupling constant should not be construed as a valid measure of the "effective" performance of the coupling aperture.

Determination of the "effective" performance of the coupling apertures between two adjacent excited cells of different cell and inter-cell geometries prompts the need for additional information on the distribution of the stored energies and fields within the cells. Let us define a new cell property, S_n, to describe the relationship of the stored energy in the cell, U_n, to the fields, E_n or H_n, in the vicinity or the coupling aperture:

$$S_n = U_n / H_n^2 = X_n^2 / H_n^2$$

This quantity is well defined for a given cell geometry and coupling aperture location, is independent of excitation, and can be evaluated from SUPERFISH or MAFIA output.

Let "a", "c", and "b" denote accelerating, coupling and bridge cells. The "effective" coupling constants, reflecting the power flow capabilities of the coupling aperture in the $\pi/2$ mode, satisfies the following equation:

$$k_{ac,eff}\ H_a = -\ k_{bc,eff}\ H_b$$

The "effective" coupling constants for bridge cells differ from the standard coupling constants for bridge cells by the square root of the ratio of the S values for the bridge and accelerating cells:

$$k_{bc,eff} = k_{bc} * (S_b/S_a)^{1/2}, \text{ and } k_{ac,eff} = k_{ac}.$$

Note that this distinction is of little consequence when the bridge cell geometry is similar to that of the accelerating cell, i.e. when $S_b=S_a$.

When the bridge cell geometry is distinctly larger that the accelerating cell geometry, suggesting a larger U_b and a smaller k_{bc}, the "effective" coupling constant, as defined here, is independent of U_b/U_a.

Avoidance of Other Cavity Modes

The resonant frequencies, f, of the rf cavity modes in a right circular cylinder of length, L, and diameter, D, are :

$$(fD)^2 = (cX_{lm}/p)^2 + (cn/2)^2*(D/L)^2,$$

where c is the velocity of light, X_{lm} is the mth root of $J_l'(x) = 0$ for TE modes and of $J_l\ (x) = 0$ for TM modes, and n is the number of half-periods in the axial field variation. Solutions of this equation yield straight lines in the $(fD)^2$ versus $(D/L)^2$ space. Graphs of this are called *mode charts*.

The bridge couplers lengths for the SSC Coupled Cavity Linac (CCL), which will operate at 1282.851 MHz, range from 0.30 to 0.48 m. The diameters of cylindrical cavities of those lengths, having their TM_{012} mode at that frequency, range from 0.285 to 0.205 m respectively. Inspection of the mode chart shows that the only modes of a right circular cylinder that could possibly cross the TM_{012} mode, in this range of parameters, are the TM_{110}, TE_{211}, and TE_{113} modes. Table I gives the L and D combinations, yielding a TM_{012} frequency at 1282.851 MHz, and the corresponding frequencies for the other 3 modes of interest.

Table I TM012 Bridge Coupler Dimensions and Other Mode Frequencies.

L (mm)	D (mm)	TM_{110} (MHz)	TE_{211} (MHz)	TE_{113} (MHz)
300	285	1283	1138	1638
320	262	1396	1207	1557
340	246	1487	1264	1503
360	235	1556	1308	1456
380	227	1611	1343	1414
400	220	1662	1377	1379
420	215	1701	1402	1347
440	211	1733	1423	1318
460	208	1758	1438	1292
480	205	1784	1456	1270
500	202	1810	1474	1251

The cross-over for the TM_{110} mode occurs at a bridge coupler length of 0.30 m. It would be possible to avoid a conflict with this mode by restricting the minimum bridge coupler length to 0.32 m. The cross-overs for the TE_{211} and TE_{113} modes occur at 0.35 and 0.47 m respectively. It would be possible to avoid conflicts with these modes by avoiding these lengths by a margin of about one centimeter.

[1] "The Coupled Resonator Model for Standing Wave Accelerator Tanks" D.E. Nagle, E.A. Knapp, and B.C. Knapp, Rev. Sci. Instr. 38, 11 (1967).