RIPPLED PLASMA WALL ACCELERATING STRUCTURES

Marco Cavenago
INFN-Laboratori Nazionali di Legnaro
via Romea n. 4, I35020, Legnaro (PD) Italy

Abstract

A concept to form a hot, pulsed, inhomogeneous plasma and to use it as a linac structure is presented. The plasma spatial distribution is controlled by an external magnetic field and by the location of thermionic emitters; microwave ECR heating at frequency ω_1 favours plasma build up and reduces plasma resistivity. A shorter microwave pulse with frequency $\omega_2 \neq \omega_1$ excites a longitudinal mode. An expression for the maximum attainable accelerating field is found. A linearized theory of accelerating modes is given.

Introduction

In this paper, we consider a plasma, whose density ripples have a macroscopic length, as a possible accelerating structure for relativistic particles; this concept belongs to the general class of plasma fibre accelerators [1,2]. The similarity with a conventional linac [3] is obvious, since both a plasma wall and a metal wall reflect waves which have a frequency ω smaller than the plasma frequency $\omega_{\rm pl}$:

$$\omega_{\rm pl}{}^2(\mathbf{x}) = \frac{4\pi e^2 n(\mathbf{x})}{m} \tag{1}$$

where n is the electron density. A copper wall is thus equivalent to a plasma wall with sharp boundaries, a huge value of $\omega_{\rm pl} = 1.6\,10^{16}$ rad/s and a resistivity $\varrho = 1.8\,10^{-8}$ ohm-m. It is well known [3,4] that, in order to accelerate relativistic electrons along the axis z, the Fourier expansion of the electric field component E_z must contain waves with wavenumbers $\mathbf{k}$ such as:

$$v_{\rm ph} = \frac{\omega}{k_z} = c \tag{2}$$

In conventional linacs, this is accomplished by corrugations with period $d \ll \lambda = (2\pi/k_z)$ on a cylindrical tube with diameter $2b$; the phase advance is defined as $\Phi = k_z d$.

In the plasma linac case, the smoother $n(x)$ imposes to consider also structures with $d \cong \lambda$ (see eq. 22). The smaller $\omega_{\rm pl}$ limits the attainable field (see eq. 16), but may be of advantage when wake fields are a concern. Wake fields are indeed reflected from plasma less effectively, so that our structure may approximate the smooth pipe impedance. The resistivity of the plasma depends from collisions and from involved wave decay processes, that are beyond the scope of this paper; anyway the collisional contribution [5]:

$$\varrho = \frac{Z\, m^{1/2} e^2 \ln\Lambda}{(3kT_e)^{3/2}} \tag{3}$$

where T_e is some effective electron temperature and $\ln\Lambda$ is the Coulomb logarithm, suggested us to heat the plasma through the Electron Cyclotron Resonance (ECR) with a microwave pulse of duration τ_1 and frequency ω_1. The typical values $kT_e = 10$ keV, $Z = 1$ and $\ln\Lambda = 15$ give $\varrho = 10^{-9}$ ohm-m.

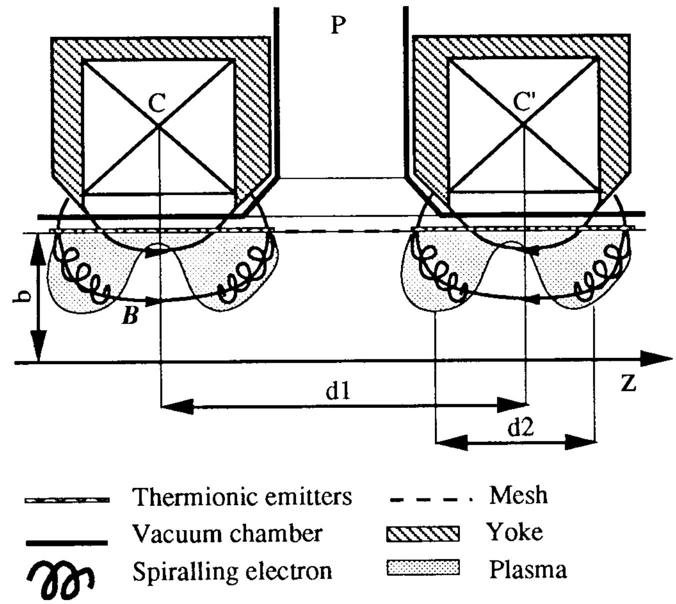

Fig 1 One cell CC' of the structure. P is the pump port.

To obtain ECR heating a periodic magnetic field is added, such that the heating condition $|B(\mathbf{x})| = B_{ecr} = m\,c\,\omega_1/e$ is satisfied on a surface, called the resonant surface: a scheme of our concept is shown in Fig. 1. A tube (with proper slots for pumping or microwave feeding) is surrounded by solenoids, spaced at a distance d_1, with alternate current flow directions (iron yoke and/or cryostats may be placed around the solenoids). Inside the tube (earth potential), a cylindrical mesh (diameter $2b$) biased to $V_1 \cong 50$ V limits the cavity region; on the mesh thermionic emitters face the solenoids and holes face the microwave feeds. A small flow of hydrogen is maintained as a buffer gas.

In the period $0 < t < \tau_1$ the thermionic electrons, perhaps reflected by the tube walls, get accelerated by microwaves and trapped along the magnetic field lines; they ionize hydrogen atoms, which provide protons for charge compensation and slow electrons, replacing the unavoidable electron losses. We have to choose a τ_1 short enough (10^{-5} – 10^{-4}s) to avoid possible magnetohydrodynamic instabilities [5]. Then the plasma follows the spatial distribution of the emitters and of the field lines to a great extent; this scheme may achieve strong inhomogeneity in $n(\mathbf{x})$, as required by eqs. 23 and 24. Note that n may contain also ripples spaced at a distance d_2, approximately equal to the yoke length; it results $d_2 \cong d_1/2$.

Detailed consideration of transport processes (for example, hydrogen ionization by plasma radiated X-rays) and experiments are of course necessary to confirm this plasma generation concept. Density and ECR microwave frequency are usually [6] related by:

$$\omega_{pl} \leq \omega_1 \tag{4}$$

At the time $t = t_1 \cong \tau_2$ a second microwave pulse of much shorter duration $\tau_2 \ll \tau_1$ and frequency $\omega_2 \leq \omega_1$ (see eqs. 4 and 24) will form the resonant wave satisfying eq. (2) condition for electron acceleration.

Field equations

From Maxwell's equation, it follows the wave equation:

$$\partial_t{}^2\mathbf{E} + c^2 \text{rot rot}\,\mathbf{E} = -4\pi\partial_t\,\mathbf{j} \tag{5}$$

let ∂_a be the notation for derivative with respect to any variable a. Here we consider the oscillations at $\omega = \omega_2$ of a plasma with density n and use a cylindrical coordinate system r, ϑ, z. The boundary conditions are $E_\vartheta = E_z = 0$ at $r = b$.

For small field amplitudes, the electric field oscillation is sinusoidal: $\mathbf{E} = \mathbf{E}(r,\vartheta,z)\exp(-\imath\omega t)$ and the plasma current may be easily expressed when $\omega \ll e|B|/mc$ as:

$$4\pi\mathbf{j} = \imath\frac{\omega_{pl}{}^2}{\omega}\mathbf{E}_{||} + \ldots \tag{6}$$

where $\mathbf{E}_{||}$ is the electric field component parallel to $\mathbf{B}$. To have an idea of the plasma modes, we use a very simplified model, considering

$$4\pi\mathbf{j} = \imath\frac{\omega_{pl}{}^2}{\omega}\mathbf{E} \tag{7}$$

which is strictly justified only when $\mathbf{E}$ is parallel to $\mathbf{B}$ or $\omega \geq e|B|/mc$. From eqs. 5 and 7 we get:

$$w^{-1}\mathbf{E} = \text{rot rot}\,\mathbf{E} \tag{8}$$

$$w(r,\vartheta,z) = c^2/(\omega^2 - \omega_{pl}{}^2) \tag{9}$$

introducing the shorthand w. Our accelerator has cylindrical symmetry, so that $\partial_\vartheta = 0$; therefore, the scalar function

$$Y(r,z) = (\text{rot}\,\mathbf{E})_\vartheta = \partial_z E_r - \partial_r E_z \tag{10}$$

allows to solve eq. 8 for E_z and E_r as:

$$E_r = -w\partial_z Y \qquad E_z = (w/r)\partial_r(rY) \tag{11}$$

From these expressions and eq. 10 a closed equation for Y is obtained:

$$Y + \partial_z(w\partial_z Y) + \partial_r(wr^{-1}\partial_r(rY)) = 0 \tag{12}$$

which is the starting point for detailed mode analysis. Indeed, the ϑ component of eq. 8 :

$$(w^{-1} + \partial_r r^{-1}\partial_r r + \partial_z{}^2)E_\vartheta = 0 \tag{13}$$

gives only the trivial solution $E_\vartheta = 0$ provided that $w > 0$.

The above linearized mode analysis of eq. 5 can be complemented with an estimate of the accelerating field at saturation of the plasma oscillating current:

$$\mathbf{j} \cong n\,e\,c\beta \tag{14}$$

where $c\beta$ is the maximum velocity of plasma electron oscillation at frequency ω_2 ; we require $\beta \leq 1/2$ to save on energy stored in the plasma.

Assuming that any field f approximately behaves like $\exp(-\imath\omega t + \imath k_z z)f(r)$ and using eq. 2 to simplify the z-component of eq. 5, we get

$$\triangle_\perp E_z = (4\pi/c)\,\text{div}_\perp\,\mathbf{j} \tag{15}$$

where the suffix $_\perp$ means taking the transverse part (that is, ignoring the terms which contain ∂_z). From dimensional analysis and according to eq. 19, each r-derivative gives a factor $\eta = 2.405/b$; so we get:

$$E_z < \beta\,\frac{4\pi}{2.504}\,n\,e\,b = \beta\,\frac{m\,b\,\omega_{pl}{}^2}{2.504\,e} \qquad ; \tag{16}$$

for $\omega_{pl} \cong \omega_1 = 14$ GHz (that is $n = 2\;10^{12}$ cm^{-3}) and $b = 2$ cm we obtain a satisfying $E_z \lesssim 144$ MV/m.

Zero phase advance mode

We consider now the class of simple problems $n = n(z)$, which offers a rough approximation of our acceleration field. In a context of a conventional copper wall linac, this would mean no coupling between cells, since the irises are then closed; for plasma linacs, fields can still pass from cell to cell in part. Note that the complicate class $n = n(z)$ for $a < r < b$ (peripheral plasma) and $n = 0$ for $r < a$ (central high vacuum) is also desirable for the sake of beam propagation (see later).

The assumption on n implies that w is also a function of z only and that eq. 12 is separable with $Y(r,z) = \chi(r)\zeta(z)$; we get:

$$\zeta - \zeta w\eta^2 + \partial_z(w\partial_z\zeta) = 0 \tag{17}$$

$$\eta^2\chi + \partial_r(r^{-1}\partial_r(r\chi)) = 0 \tag{18}$$

where η^2 is the separation constant. From eq. 11 and 18 we easily get:

$$E_z(r,z) = J_0(\eta r)w(z)\zeta(z) \qquad ; \tag{19}$$

the boundary condition $E_z(b,z) = 0$ requires ηb to be a root j_n of the Bessel function J_0 ; in particular, let us consider the fundamental mode $\eta = j_0/b = 2.504/b$.

Eq. 17 may be transformed to a Mathieu equation for ζ, which is well known in circular accelerator theory.

As alternative, we discuss here the inverse method: setting ζ with a desired periodicity and solving for w The analytic solution for w of eq. 17 is then:

$$w(z) = -\frac{1}{\zeta,_z(z)\,I(z)}\int_0^z \zeta(t)I(t)dt \tag{20}$$

where $\zeta,_z$ is the z-derivative of ζ and $I(z)$ is

$$I(z) = \exp\left(-\eta^2\int^z \frac{\zeta(t)}{\zeta,_z(t)}dt\right) \tag{21}$$

The $\Phi = 2\pi/3$ mode may apparently be obtained by

$$\zeta = A\,[\cos(kz) + (B/4)\cos(2kz) + (C/16)\cos(4kz)]$$

but here we consider the $\Phi = 0$ mode, which has no correspondence in usual linac theory. This mode is given by:

$$\zeta = A\,[1 + B\cos(k_z z)] \qquad (22)$$

where B is a positive constant; profiles of w are shown in Fig. 2 and 3, respectively for $B = 0.25$ and $B = 0.1$; in both plots, we have chosen $k_z = \eta\sqrt{1.5}$ so that the parameter $R = \eta^2/k_z^2$ has the nearly optimum value 2/3 .

Indeed, from eq. 20 the maximum w_+ and the minimum w_- of w are:

$$w_\mp = \frac{1 \pm B}{k_z^2(R \pm B(1+R))} \qquad (23)$$

The maximum is finite provided that $B < R/(R+1)$. Moreover, we find that a cusp develops at each maximum point when $B > R/(R+3)$.

The eq. 2 requirement for electron acceleration $k_z c = \omega$ and the definition of w in eq. 9 imply:

$$k_z^2 c^2 = \omega^2 = {\omega_{\rm pl}}^2 + \frac{c^2}{w} > \frac{c^2}{w} \qquad (24)$$

since ${\omega_{\rm pl}}^2$ must be positive. Therefore: $k_z^2 \min w > 1$, which using expression 23 for w_- gives:

$$B < \frac{1-R}{R} \qquad (25)$$

Requiring no cusp in the w profile, the maximum $B = 0.215$ is attained for $R = R_m = 0.823$. Accepting cusps in w gives $B = 0.414$ at $R = 0.707$. Using again $b = 2$ cm, the frequency $\omega_2 = c\eta R^{-1/2}$ is, respectively 6.6 GHz and 7.1 Ghz, for these two R values.

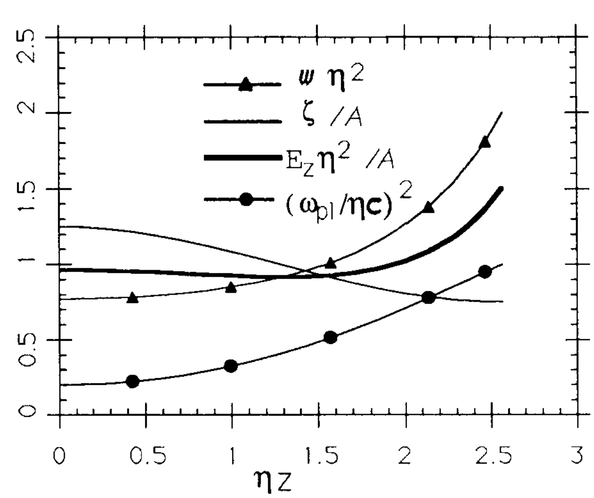

Fig 2 Electric field and other quantities (normalized) for B=0.25 and R=2/3 ; half a period is shown.

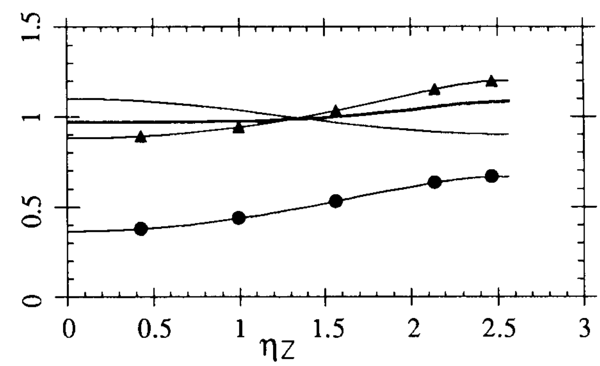

Fig 3 Same quantities of Fig. 2 when B=0.1 and R=2/3

Residual gas and plasma duty cycle

The quality of vacuum where the beam propagates is a concern for every accelerator. The most optimistic assumption is that the electron stay frozen to **B** lines (for $t < \tau_1$) and that the previous pulse left no residual atom inside the tube.

Then the plasma electron density in the centre of the tube $n(r < a)$ may well be negligible in comparison to the density at tube periphery $n(r > a)$, equal to $10^{12} - 10^{14}$ e/cm^3 ; the central ion density n_i (mostly protons) is also negligible, since it is equal to the electron density thanks to the quasi neutrality condition $n_i \cong n$. Neutral gas generation in the peripheral plasma and transport gives a contribution n_n to the central density:

$$n_n(r<a) = \frac{2}{\pi^2}\frac{\sigma_{\rm cap}}{\sigma_{\rm ion}}\, n_i(r>a) \qquad (26)$$

where $\sigma_{\rm cap}$ is the cross section of electron capture from a singly ionized atom and $\sigma_{\rm ion}$ is the ionization cross section of neutral gas. The large value of $\sigma_{\rm ion}/\sigma_{\rm cap}$ (for $kT_e \gg 1$ keV) makes the plasma opaque to neutrals and eq. 26 contribution negligible. Since the plasma acts as an ion pump on the outgassing from the tube walls, vacuum better than 10^8 atom/cm^3 may be expected for $t < \tau_1$.

A more careful consideration of the afterglow $t > \tau_1 + \tau_2$ is then needed. We require that the recombined gas reaches the pumping port to be cleared by the external pumping. It may be necessary to taper the ω_1 microwave power off, so that the plasma forming the accelerating structure can still provide some ion pumping, while it is being gradually released. Note that the plasma must not be held on continuously, to prevent build up of large Z impurities, accumulating from wall sputtering.

References

[1] T.Taijma, p. 470 in "Proceedings of the Twelfth Int. Conf. on High-Energy Accelerators", (eds. F.T.Cole, R. Donaldson), Fermilab, Batavia (1983).

[2] T.Tajima, M.Cavenago, Phys. Rev. Lett., **59**, 1440 (1987).

[3] G.A.Loew, R.B. Neal, "Accelerating structures", pg 39-113 in "Linear Accelerators" (eds. P.M. Lapostolle, A.I. Septier), North Holland, Amsterdam, (1970).

[4] W.Scharf "Particle Accelerators and their uses", 2 vol., Harwood Academic Publ., Chur, (1986).

[5] K.Miyamoto, "Plasma physics for nuclear fusion", in particular chapters 4.6 and 7, revised ed., The MIT press, Cambridge, (1989).

[6] R.Geller, B. Jacquot, Nucl. Instr. Meth, **184**, 293 (1981).