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STOCHASTIC ELECTRON BEAMS IN THE ION-FOCUSED REGIME[1]

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Abstract

The ion-hose instability can catastrophically disrupt a classical electron beam propagating in the ion-focussed regime (IFR). Ion hose is driven by a resonant interaction between the smooth electron-betatron and ion-betatron orbits. In a classical beam phase correlations decay secularly in time $c(t)/c(t_0) \sim (t_0/t)^n$ $(0<n\leq 2)$. In a stochastic electron beam the electron orbits are chaotic. Such a beam can be immune to resonant instabilities because phase correlations decay exponentially fast $c(t)/c(0) \sim e^{-ht}$ thus destroying the coherence of the electron response on the growth time $1/\gamma_g$ if $h \sim \gamma_g$. Using the same principles we can also envsion a stochastic damping cell in which electron phase correlations damp exponentially $c(z)/c(0) \sim e^{-hz}$ thus centering and conditioning a beam more effectively than a classical phase-mixing cell in which $c(z)/c(z_0) \sim (z_0/z)^n$. A "triple-Bennett" IFR system and the analogous "triple-wire" damping cell are analyzed. The K-entropy is introduced as a figure-of-merit for such stochastic electron beam systems.

Introduction

IFR-guiding is a proven technique for transporting short-pulse-length ($\tau_p \sim 10$ nsec) 1-10 kA, 1-50 MeV classical electron beams in sub-Torr gas or plasma.[1-8] Long-pulse-length ($\tau_p \sim 1$ μsec) classical IFR beams are easily disrupted by ion-hose instability.[9-10] Offset or aiming errors in launching a beam onto an IFR channel typically provide the initial perturbations from which the instability develops. To reduce these errors the beam may be conditioned in a phase-mix damping cell prior to launch. Further phase mixing will occur in the IFR channel due to the radially anharmonic potential. In the conditioning and launch processes it is desirable to have strong phase mixing in order to quench any growing perturbations. In classical beams this decorrelation process is a sheared laminar flow in phase space in which phases "mix" ergodically (mixing rate a power law in time). For stochastic beams[11] the corresponding flow is a nonlaminar mixing flow called a K-flow (mixing rate exponential in time) (Figure 1).

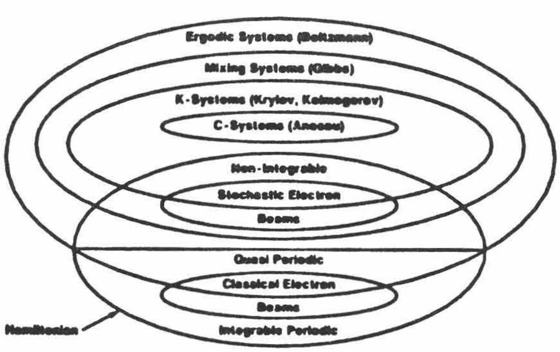

Fig. 1. Relation between classical and stochastic electron beams

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In this paper we compare the phase-mixing properties of a classical IFR/wire system with those of the stochastic triple-IFR/wire system.

Stochastic triple-IFR/wire systems

For the IFR/wire systems depicted in Figure 2 the Hamiltonian for transverse electron motion is (in appropriate dimensionless variables)

$$H = p_r^2/2 + P_\theta^2/2\rho^2 + V(\rho,\theta;t) \tag{1}$$

where the potential $V(\rho,\theta;t)$ is

$$V(\rho,\theta;t) = \frac{2}{3}\log[\varepsilon^3\rho^3 \sin 3\theta + (1+\rho^2+\varepsilon^2)^3 - 2\varepsilon^2\rho^2(1+\rho^2+\varepsilon^2)] \tag{2}$$

Here $\varepsilon \equiv d(t)/a$. The time dependent $d(t)$ enters because the IFR/wire separation is z-dependent (cf. Fig. 2) and for a fixed segment of beam we have $z = \beta ct \simeq ct$. The radius a is the IFR/wire radius and $\rho = r/a$ [the wire is treated by taking $a \to 0$ in the sense that a is much smaller than typical orbit radii].

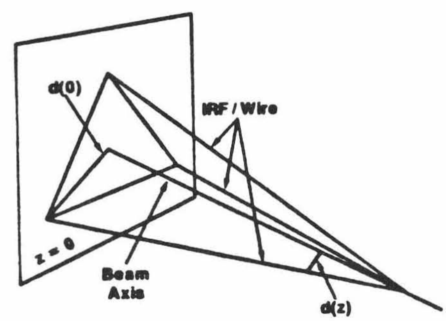

Fig. 2. Triple-IFR/wire system geometry

The corresponding classical IFR/wire systems are gotten by setting $\varepsilon=0$ so $V \to V_0(\rho)$ where $V_0(\rho)$ is a Bennett potential. Self-field effects of order $1/\gamma^2$ are neglected as we consider highly relativistic beams. Figure 3 depicts typical orbits of Eqs. (1-2). The chaotic nature of the orbits for d/a = 2 is apparent.

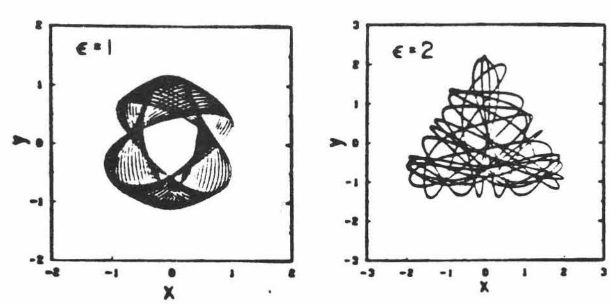

Fig. 3. Orbit of triple-IFR system (ε=1,2).

K-entropy and phase decorrelation rates

In order to quantify the effectiveness of the stochastic IFR/wire systems we consider the global stability of orbits. Consider an orbit of H which we specify as $\eta(t) = [\rho(t),\theta(t),p_r(t),P_\theta(t)]$. At each point on this orbit the linearized version of Hamilton's equations for Eqs. (1-2) evolves according to

$$\Delta\dot{\rho} = \Delta p_r \quad , \tag{3a}$$

$$\Delta\dot{\theta} = (1/\rho^2)\Delta P_\theta - (2P_\theta/\rho^3)\Delta\rho \ , \tag{3b}$$

$$\Delta\dot{p}_r = -V_{\rho\rho}\Delta\rho - V_{\rho\theta}\Delta\theta + (2P_\theta/\rho^3)\Delta P_\theta - (3P_\theta^2/\rho^4)\Delta\rho \tag{3c}$$

$$\Delta\dot{P}_\theta = -V_{\rho\theta}\Delta\rho - V_{\theta\theta}\Delta\theta \tag{3d}$$

where subscripts on V indicate partial derivatives evaluated at $\eta(t)$. Here $\Delta\eta = (\Delta\rho,\Delta\theta,\Delta p_r,\Delta P_\theta)$ is a vector in the tangent space at $\eta(t)$ and the coefficients are also evaluated at that point. The eigenvalues $\lambda_\eta(t)$ of Eqs. (3) describe the local properties of the system at the point $\eta(t)$. Locally, we cannot properly distinguish between the stochastic and regular cases. For example, the axisymmetric Bennett potential has an inflection point at $\rho=1$ and thus has orbits which separate exponentially (but, do so only locally). For this reason, we proceed to look at the global properties, or the "tangent map" defined by these equations. The solutions of Eqs. (3) are of the form $\Delta\eta(t) \sim \exp(\lambda_\eta(t))\Delta\eta(0)$. Geometrically, the tangent map $\exp(\lambda_\eta(t))$ maps vectors from the tangent space at $\eta(0)$ into the tangent space at $\eta(t)$ and it is in this sense that it describes global properties of the orbit η.[14] The derivatives $d\lambda_\eta/dt = \Lambda_\eta(t)$ are determined as the roots of the quartic characteristic polynomial of Eqs. (3). The eigenvalues $\lambda_\eta(t)$ are then the time integrals

$$\lambda_\eta(t) = \int_0^t d\tau \ \Lambda_\eta(\tau) \quad . \tag{4}$$

Figure 4 depicts the positive real part of $\lambda_\eta(t)$ for a representative orbit $\eta(t)$. For this particular orbit the real part of $\lambda_\eta(t)$ is zero for all four eigenvalues unless $\varepsilon > 1.3$ (approximately).

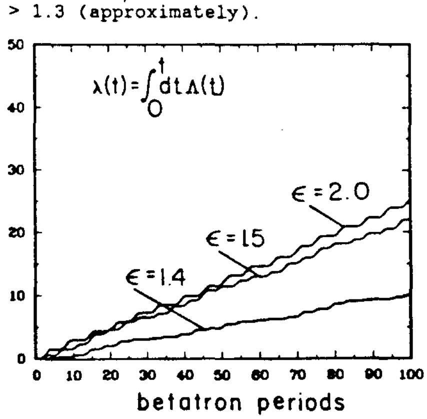

Fig. 4. Positive real part of $\lambda_\eta(t)$ for a representative orbit.

The quantity $|\Delta\eta(t)/\Delta\eta(0)|$ will grow exponentially during those times when at least one eigenvalue $\lambda_\eta(t)$ has a positive real part. To globalize we introduce the mean exponential growth rate $h(\eta)$

$$h(\eta) = \lim_{\Delta\eta(0)\to 0} \lim_{t\to\infty} \frac{1}{t} \log |\Delta\eta(t)/\Delta\eta(0)| \tag{5}$$

This quantity will certainly vanish (or be negative) unless one eigenvalue $\lambda_\eta(t)$ has a positive real part for a sufficiently large fraction of the time. Eigenvalues of autonomous two-degree-of-freedom Hamiltonian systems are always of the form $(-\lambda,0,0,\lambda)$ where λ may be pure real, pure imaginary, or complex. Thus, looking only at the one eigenvalue with a positive real part (if it exists) we may write

$$h(\eta) = \lim_{t\to\infty} \frac{1}{t} \mathrm{Re}[\lambda_\eta(t)] \quad . \tag{6}$$

When $h(\eta) > 0$ the orbit is "unstable in the global sense". Figure 5 is a plot of the ratio $\mathrm{Re}[\lambda_\eta(t)]/t$ for the orbit of Fig. 4. We see that the ratio converges to a positive constant, indicating strong orbit instability.

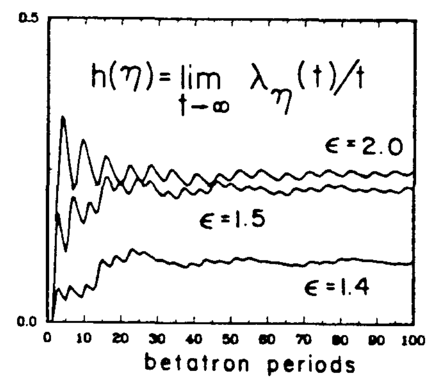

Fig. 5. Ratio $\mathrm{Re}[\lambda_\eta(t)/t] \to h(\eta)$ as $t\to\infty$.

A positive $h(\eta)$ indicates exponential instability of the orbit η. A related quantity is the Kolmogorov dynamical entropy or "K-entropy".[13] Strictly speaking, the K-entropy should be taken as an average of $h(\eta)$ over the triple-infinity of distinct orbits

$$h = \int d\mu(\eta)h(\eta) \quad . \tag{7}$$

This would account for the fact that different regions of phase-space can develop exponential instability at different rates (as a function of ε). For simplicity we have identified $h=h(\eta)$ assuming homogeneity.

Next, consider a pair of phase points (1,2) which are infinitesimally separated at z=0. In Figure 6 we plot x2(z)-x1(z) for the orbit of Fig. 4. For $\varepsilon=2$ the exponential separation rate is apparent.

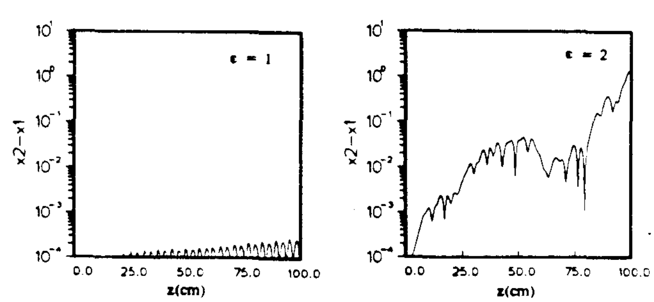

Fig. 6. Separation x2(z)-x1(z).

The autocorrelator c(z) describes how rapidly an orbit loses memory of its past For an orbit $[x(n\Delta z),y(n\Delta z)]$ $n=1,\dots,N$ we define an m-order correlator

$c_m = \Sigma_n$ sign{x(nΔz)x[(n+m)Δz]}/(N-m) for m=1,...,N/2. Note, $c_m = \pm 1$ is perfect correlation/anticorrelation. The further an orbit is from regular the closer c_m is to zero. In Fig. 7 we plot c_m versus m for $\epsilon=1$ and $\epsilon=2$ (same orbit as Fig. 4). Note the strong decorrelation for $\epsilon=2$.

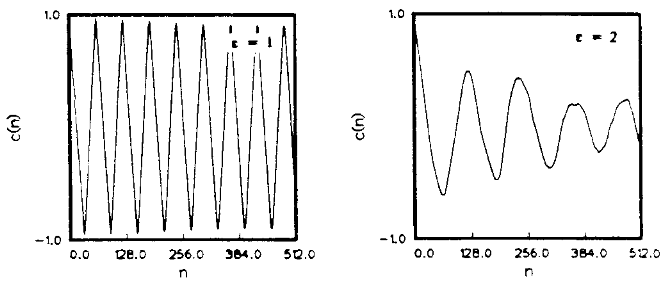

Fig. 7. Autocorrelator c_m versus m.

Conclusions

The K-entropy enables us to make statements about decorrelation rates.[14] In fact, when h=0, as for a classical beam, one obtains a power-law for correlation decay $c(t)/c(t_0) \sim (t_0/t)^n$ ($0<n\leq 2$) [The Bennett potential is maximally anharmonic and has n=2.[15]] On the other hand, when h > 0 one obtains an exponential law for correlation decay $c(t)/c(0) \sim e^{-ht}$. In a fixed segment of beam the ion-hose instability grows exponentially with a growth rate γ_g scaling roughly as $\gamma_g \sim (2T_0/\gamma_b m_e a^2)^{1/2}$.[9-11] Growth requires a coherent response of the electrons within a given beam segment. Phase decorrelation results in a competition between the growth and decay of this coherence. To get an idea of the degree of stochasticity required to suppress growth we equate the growth and decorrelation rates. We thus find the requirement $h > 1/(2^{3/2}\pi)$. As indicated in Fig. 5 this is a value of h which is not difficult to achieve. Thus, we have a possible mechanism to suppress hose growth. Likewise, by employing a triple-wire (or some other array of wires) we may be able to condition a beam more efficiently than with a single wire. A converging wire array has another advantage (unrelated to stochasticity) in that it provides a more adiabatic transition of a badly centered beam to the single channel; thus, the emittance growth is less than with a single wire.[16]

Acknowledgements

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