CALCULATIONS ON PERMANENT-MAGNET QUADRUPOLES WITH NONRECTANGULAR CROSS SECTION*

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Abstract

The current trend toward higher frequencies to power drift-tube linacs (DTLs) and coupled-cavity linacs (CCLs) reduces the space available for quadrupole focusing magnets. Similarly, the space available for matching sections between linac sections is limited, and often the matching section bunchers are designed in odd shapes to make them fit. This shaping further restricts focusing magnet space. One approach to attaining sufficient quadrupole strength in such situations is to use rare-earth permanent-magnet quadrupoles (PMQs) with cross sections tailored to fill as much of the available space as possible. In this paper, we describe some techniques we have developed to calculate the properties of such magnets both singly and when other magnets are nearby.

Discussion

Recently, linear accelerators have been proposed that operate close to or in the gigahertz region. These designs make use of rare-earth permanent-magnet material to supply the strong focusing. In current designs of linacs that operate below 500 MHz, there is barely room for these magnets in the low-energy drift tubes of a DTL. At higher frequencies, the space is further limited for two reasons. First, the structure size in general must be smaller and, second, the bore size cannot be reduced proportionally. In another area, it is proposed to use CCLs for acceleration at lower energies where stronger transverse focusing is needed. For these designs, it is assumed that the focusing magnets will be installed in the webs between cells where there is very little space. Similarly, the space in matching sections for these linacs is smaller and may be further restricted by longitudinal matching bunchers. One cure for the problems associated with limited space is to use magnets that are unconventionally shaped. Such magnets, by using heretofore wasted volume, can give higher strength than conventionally shaped magnets. As an example, consider a quadrupole magnet with the cross section shown in Fig. 1 and with rotational symmetry around the z axis. This magnet might be used as the entrance quad for a DTL when the entrance quad requires extra strength, because it also acts as a matching element between the DTL and the preceding section. Or it might have this odd shape in order to fit underneath a buncher cavity.

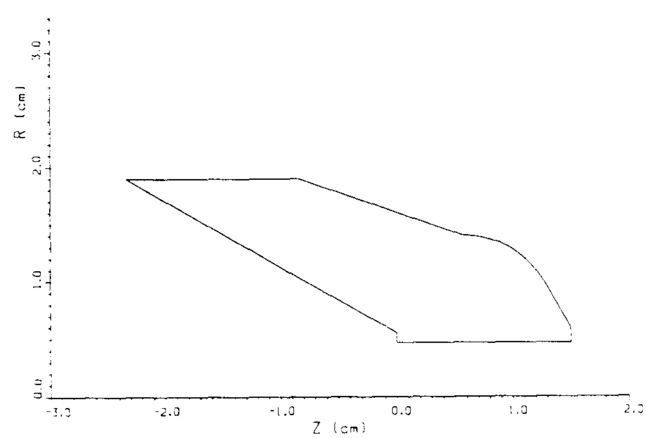

Fig. 1. Hypothetical quadrupole cross section, upper half.

To select a quadrupole for use in a system and to predict its effect on a beam, the properties of the quadrupole must be known. Some of the properties we are interested in include the strength (GL product), the gradient as a function of position, and the location and size of the maximum pole-tip field. Because we often want to use a hard-edged model for the quadrupole, we also want to know the hard-edged effective length and the hard-edged effective gradient. In a periodic or nearly periodic focusing system, the field that a particle experiences depends not only on the nearest magnet but on other nearby magnets. When the modeling is hard-edged, the effect of nearby magnets must be taken into account by changing the local magnet's properties. We need to know what changes to make. In the following paragraphs we describe some of the methods we use to calculate the quantities we need to know in order to use nonrectangular cross-section quadrupoles.

Our methods are merely applications of Klaus Halbach's results.[1,2] As he points out, the properties of rare-earth magnets are such that superposition of fields can be used with negligible error, and we take full advantage of this principle. For the required calculations, we need a general first-order expression for the gradient as a function of axial distance for a quadrupole that has an arbitrary $r-z$ cross-sectional shape. This gradient expression is derived below.

To first order, Halbach gives the scalar potential of a semi-infinite quadrupole extending from $z=-\infty$ to $z=0$ as [see Ref. 2, Eq. (4)]

$$V = B_r C_2 (\frac{1}{r_1} - \frac{1}{r_2})(x^2 - y^2) F(z) \quad .$$

The quantity B_r is the remanent field of the magnet material and C_2 is a constant that depends on the quadrupole's construction [see Ref. 2, Eq. (4f)]. In this, r_1 and r_2 are the inner and outer magnet radii, respectively, and

$$F(z) = \frac{1}{2}\Big[1 - \frac{z}{8}(\frac{1}{r_1} - \frac{1}{r_2}) \times \left(\frac{\nu_1^2\nu_2^2(\nu_1^2 + \nu_1\nu_2 + \nu_2^2 + 4 + 8/\nu_1\nu_2)}{\nu_1 + \nu_2}\right)\Big]$$

$$\nu_i = (1 + (z/r_i)^2)^{-1/2} \quad .$$

From this expression for the potential, the first-order gradient magnitude is

$$G(z) = 2B_r C_2 (\frac{1}{r_1} - \frac{1}{r_2}) F(z) = G_o F(z) \quad .$$

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For an actual quadrupole with its right end situated at z_r and its left end at z_l, the gradient is found as the difference between two semi-infinite quadrupoles, the first extending from $-\infty$ to z_r and the second from $-\infty$ to z_l. The result is

$$G(z) = G_o\Big(F(z - z_r) - F(z - z_l)\Big) \quad .$$

The above expression applies to a quadrupole that has a rectangular cross section $z_r - z_l$ long and $r_2 - r_1$ thick. Specifically, it applies to an infinitesimally wide quadrupole of radial width dr where dr is centered on $r = \frac{1}{2}(r_1 + r_2)$. The gradient of this quadrupole can be written

$$dG(z, r) = dG_o(r)\Big[F\Big(z - z_r(r)\Big) - F\Big(z - z_l(r)\Big)\Big] \quad .$$

By expanding the expressions for G_o and F with respect to dr and by keeping terms to first order in dr, it is easy to show that $dG_o(r) = 2B_rC_2 dr/r^2$ and, also to first order, that

$$F\Big(\zeta(r)\Big) = 1/2\Big\{1 - \frac{\zeta}{4r\Big(1 + (\zeta/r)^2\Big)^{5/2}}\Big[1 + 8\Big(1 + (\zeta/r)^2\Big)^2\Big]\Big\} \ ,$$

where $\zeta(r)$ stands for either $z - z_r(r)$ or $z - z_l(r)$. From this comes the general expression for the gradient of an arbitrary $r - z$ cross-section, rare-earth PMQ, namely,

$$G(z) = \int_{r_1}^{r_2} dG_o(r)\{F(z - z_r(r)) - F(z - z_l(r))\} \quad .$$

The integration extends from the smallest cross-section radius r_1 to the largest cross-section radius r_2. Notice that the right and left ends of the quadrupole are now explicitly assumed to be functions of radius.

The integration is easy to carry out numerically. A standard integration routine needs only a function that supplies the values of the integrand as a function of the independent variable. In this case, the function routine must also be supplied with functions that give z_r and z_l as functions of r. This method was used to calculate the gradient as a function of z for the quadrupole whose $r - z$ cross section is shown in Fig. 1. The result is given in Fig. 2. The code that did the calculation also calculates the maximum pole-tip field and its position, the GL product (by double integration), a hard-edged effective length, and a hard-edged effective gradient.

The hard-edge values are somewhat arbitrary. We use one of the standard definitions for the effective length;[3] that is,

$$G_{\mathrm{max}} L_{\mathrm{eff}} = \int_{-\infty}^{\infty} G(z)dz \quad ,$$

where G_{max} is the maximum value of the gradient on the axis. For long quadrupoles, this is the usual definition.

One of our major applications of these formulae is to determine the effective hard-edged gradients to be used by PARMILA in modeling a DTL. The method for a single quadrupole used above is sufficient at higher energies where the individual quadrupoles are separated by enough distance axially that the fringe fields do not substantially overlap. However, at low energies, drift tubes are close together and the gradient near a given quadrupole is the sum of its gradient and the gradients of its nearby neighbors. For instance, Fig. 3 shows a plot of the on-axis gradient that results from using a $(FO)^3(DO)^3$ focusing scheme for the first nine quadrupoles in a 2- to 3-MeV, 850-MHz DTL. The considerable fringe-field overlap is evidenced by the fact that the gradient is quite far from zero between quadrupoles of the same sign.

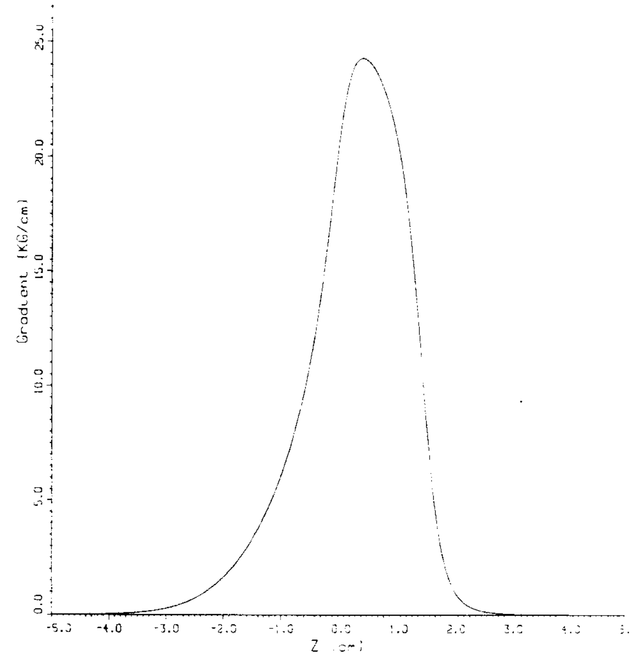

Fig. 2. Plot of the gradient for the quadrupole that has the $r - z$ cross section shown in Fig. 1.

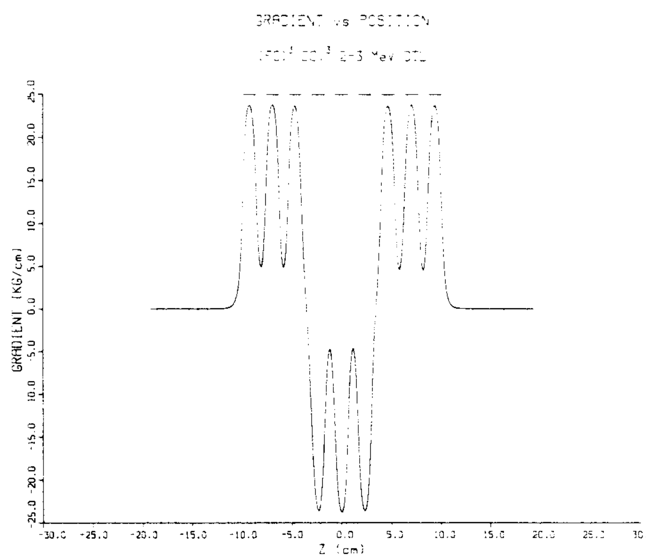

Fig. 3. Plot of the gradient produced by the first nine quadrupoles in a low-energy DTL.

The quadrupoles have different "free-space" GL products that can be determined by measurement, and the center-to-center spacing increases to the right. We need to assign to each quadrupole a hard-edged effective length and gradient to represent them in the PARMILA calculation. We can do this as follows: First, calculate the free-space effective length for each quadrupole and assign it as the hard-edged effective length of the corresponding quadrupole. Then assign to each quadrupole the total gradient from halfway between it and its nearest neighbor on the left to halfway between it and its nearest neighbor on the right. Then calculate, by double numerical integration, the GL product over this distance and divide the product by the previously determined effective length to obtain a hard-edged effective gradient.

Figure 4 plots the effective gradient magnitudes obtained this way for the 2- to 3-MeV $(FO)^3(DO)^3$ DTL. Although this is an example, we have assumed that the quadrupole strengths vary randomly $\pm 0.5\%$ about the design strength, which accounts for the scatter in the figure. The calculation was made by summing the contributions of each quadrupole along with its four nearest neighbors. We have also used the six nearest neighbors but see negligible differences from the four nearest-neighbor result. It is easy to explain the results shown in the figure. The upper line corresponds to the center quadrupoles in the groups of three of the same sign. These quadrupoles are aided by the contribution of like sign by the nearest quadrupoles on each side. The next quadrupole on each side is of opposite sign but these have less effect because they are farther away. The lower group of points corresponds to those quadrupoles that have quadrupoles of opposite sign on one side. The lower line rises because the quadrupole centers are spaced $\beta\lambda$ apart and $\beta\lambda$ increases with acceleration. The quadrupoles of the upper line do not increase because they are essentially at their free space gradient values.

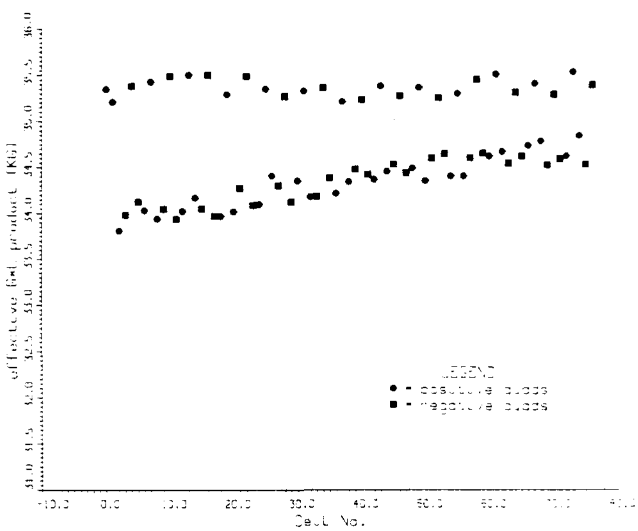

Fig. 4. Plot of the effective gradient magnitude versus cell number for a 2- to 3-MeV 850-MHz DTL with $(FO)^3(DO)^3$ focusing.

The code that made the plot shown in Fig. 4 requires an input file that gives, for each quadrupole, its measured GL product, its physical dimensions, its serial number, and the distance from its center to the center of the nearest upstream quadrupole. For each quadrupole, the code prints, as output, a reduced GL product, the percentage reduction below the measured free space GL product, the measured value of the remanent field, the hard-edged effective length, the free space hard-edged effective gradient, and the reduced hard-edged effective gradient.

References

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