Short RF pulse propagation in a traveling wave structure

L.Ferrucci,C.Pagani and L.Serafini
Istituto Nazionale di Fisica Nucleare and Universitá di Milano, Milano, Italy

Introduction

We present in this paper some numerical results on the propagation of RF pulses inside an accelerating structure of the iris loaded, constant impedance, type.

Our aim is to evaluate the possibility to fill an accelerating structure with a train of short RF pulses (shorter than the filling-time τ_f), instead of a single RF pulse whose time-duration is of the order of τ_f.

The electric field $E_q(t)$ at a certain position q along the waveguide, as a function of time, is given by the convolution integral:

$$E_q(t) = \int_0^t E_0(t-\tau)G_q(\tau)d\tau \qquad (1)$$

where:
- $E_0(t)$ is an arbitrary input waveform (train of rectangular RF pulses or train of Gaussian-shaped RF pulses, etc.)
- $G_q(t)$ is the response, at a position q, to a delta-function input at the point $q = 0$. This function, which completely determines the pulse transmission inside the waveguide, can be deduced from the dispersion curve of the accelerating structure.

From the literature [1] we know that the expression for the function $G_q(t)$ for the case of a SLAC type structure (S-band, 2856 MHz) is the following:

$$G_q(t) = 2q\frac{J_{2q}(\omega_c t)}{t}e^{i(\omega_0 t-\pi q)}e^{-\frac{\alpha_0}{2}\omega_c t} \qquad (2)$$

where:
- $2q$ is the position in number of cells.
- ω_0 is the mid-band frequency of the pass-band
- ω_c is the half pass-band width
- α_0 is the attenuation coefficient of the structure at mid-band frequency.

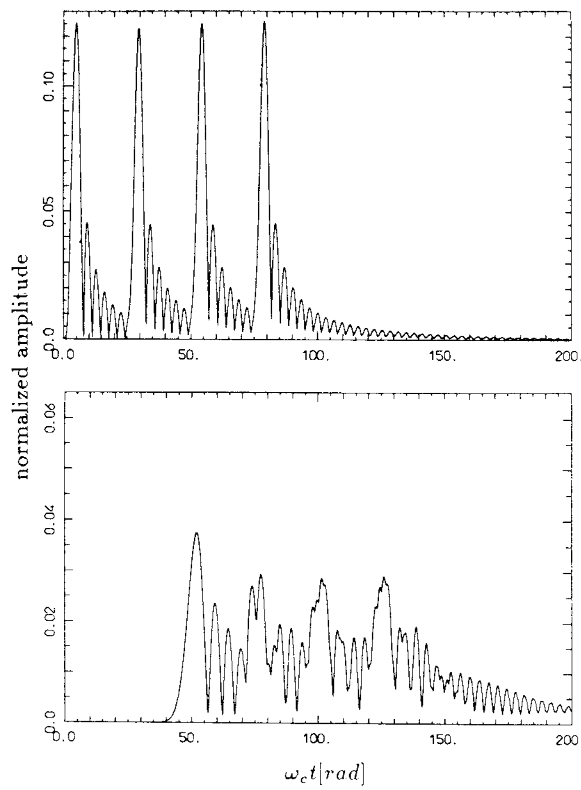


Fig. 1 a and b : normalized amplitude of the electric field at the 5^{th} and 50^{th} cell as a function of the time for a train of rectangular RF pulses in a SLAC-type structure. (see text)

RF pulses propagation

In fig. 1 a and b we present some numerical results concerning the propagation of a train of four rectangular RF pulses through a SLAC-type accelerating section. The four pulses have initially unitary amplitude, with a lenght of 3.5 ns (10 RF cycles), and they are separated by 0.2 μs in order to have a total time-duration equal to the structure filling- time (0.8 μs). The plots are obtained by numerically integrating the eq. (1) and they show respectively the normalized amplitude of the electric field at the 5^{th} and 50^{th} cell, as a function of $\omega_c t$ (being t=0 defined by the entrance of the first pulse into the waveguide). The strong spreading of the pulses, due to the dispersive behaviour of the waveguide, is quite visible.

The normalized amplitude and the real part of the electric field, for the same train of pulses, is shown in fig. 2 at a given time ($\omega_c t$ = 82 rad corresponding to about 4/5 of the filling time), as a function of the position $2q$ (in number of cells). From the plot of the real part of the electric field is evident that the 2/3 π phase relationship between adiacent cells is not anywhere satisfied, expecially at the head and tail regions of each pulse.

An important question that has to be pointed out is the dependence of the dispersion effects on the group velocity of the accelerating structure: in principle a higher group veloc-

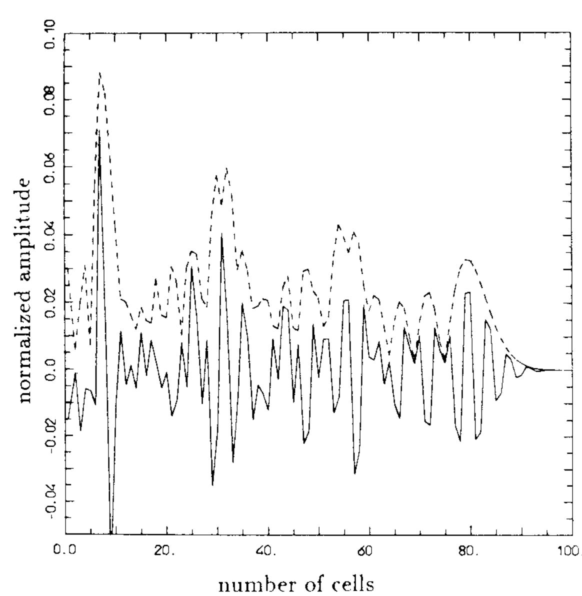


Fig. 2 : normalized electric field amplitude (- - -) and real part (——) at a given time $\omega_c t$ = 82 rad, as a function of the longitudinal position in number of cells, for a train of four RF rectangular pulses in a SLAC-type structure. (see text)

ity, corresponding to a larger pass-band , must correspond to a lower deformation of the pulse pattern. [2,3] To verify this effect we have considered an accelerating structure of the SLAC-type with a group velocity of approximately 0.1 of the speed of light instead of 0.012, which corresponds to a half-pass-band width of 163.83 MHz instead of 19.66 MHz.

The results are shown in fig. 3 *a* and *b*, for equivalent conditions as fig. 1 *a* and *b*, i.e. for a train of four RF pulses,each one 10 RF cycles long, spaced by 27 ns. As we can see, the pattern is nearly the same as for the narrow-band structure and we can conclude that the dispersive effects produced by the two structures on very short pulses are similar.

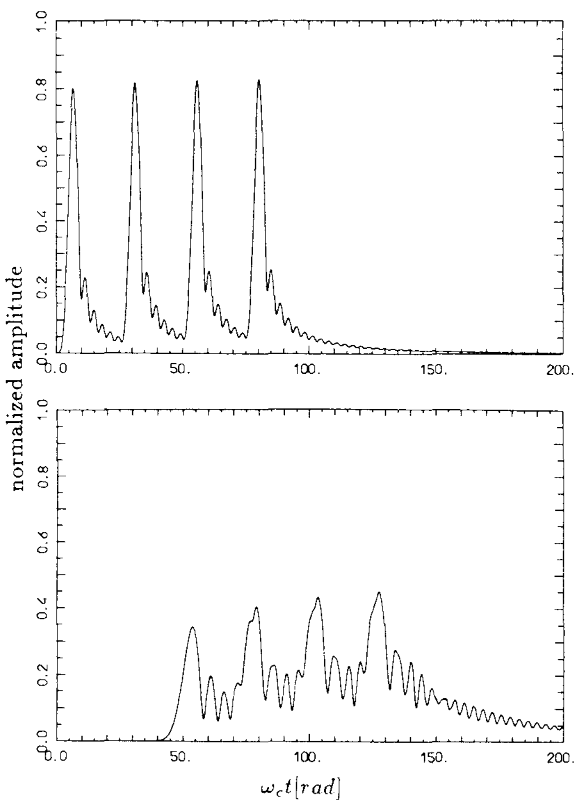


Fig. 3 *a* and *b* : normalized amplitude of electric field at the 5^{th} and 50^{th} cell as a function of the time for a train of four rectangular RF pulses in an iris-loaded structure with a group velocity $v_g = 0.1c$.

Electron energy gain

The question now arising is: what about the energy gain for a relativistic electron passing throughout the field pattern produced by a RF pulse train?

Following again the method in ref. 1 we have numerically integrated the equation for the energy gain $W(t)$ of a fully relativistic electron (v=c) passing through the waveguide, without taking into account the beam-loading effect. The energy gain is a function of the injection time, measured from the entrance of the first pulse into the waveguide:

$$W(t) = \int_0^{q_m} E_q(t + q\frac{(\pi + \psi_0)}{\omega_a})dq \qquad (3)$$

where:
- q_m is equal to the half of the total number of cells
- $E_q(t + q\frac{(\pi+\psi_0)}{\omega_a})$ is the electric field seen by the syncronous electron and is given by the following equation:

$$E_q(t+q\frac{(\pi + \psi_0)}{\omega_a}) = \int_0^{t+q\frac{(\pi+\psi_0)}{\omega_a}} E_0(t+q\frac{(\pi + \psi_0)}{\omega_a} - \tau)G_q(\tau)d\tau \qquad (4)$$

- $q\frac{(\pi+\psi_0)}{\omega_a}$ is the transit time for the electron to point q. ψ_0 is a phase depending on the operating mode of the accelerating structure, which is equal, in this example, to $\frac{\pi}{3}$.

The electric field seen by the syncronous electron traveling down to a 30 cells SLAC-type structure, of the narrow-band type, is plotted in fig. 4 *a*. The structure is filled with a train of four rectangular RF pulses, with unitary amplitude, 10 RF cycles long and with a spacing of about 1/4 of the section filling-time.

The results for the wide-band structure ($v_g = 0.1c$) are shown in fig. 4*b*.

The difference between the two cases becomes now more evident: the field seen by the electron passing through a wide-band structure is never decelerating, which corresponds to a better acceleration efficiency.

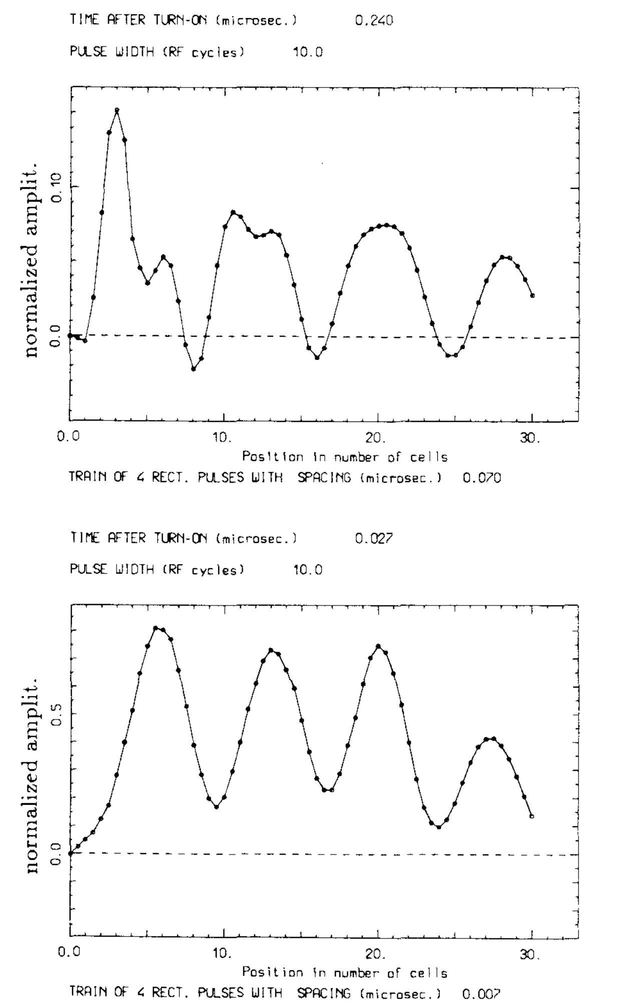


Fig. 4 *a* and *b* : normalized electric field seen by a syncronous electron traveling through the accelerating structure filled with a train of four rectangular RF pulses. They refer to a group velocity $v_g = 0.012c$ and $v_g = 0.1c$ respectively. (see text)

The energy gain for the two cases, as a function of the injection time of the electron after the entrance of the first RF pulse into the waveguide, is shown in fig. 5 *a* and *b*. The energy is measured in arbitrary units such that the total energy gain for a $v = c$ electron is just equal to the total number of cell, in absence of dissipation and dispersion in the guide.

Let us now compare the energy gain of an electron for the two cases of one RF pulse τ_f long and of a train of very short RF pulses, to give a figure of the efficiency of how much of the RF energy put into the waveguide with short pulses is useful for acceleration.

Let us consider first a rectangular unitary-amplitude RF pulse with a lenght equal to the filling time for a 30 cells SLAC-type structure (300 ns). The energy gain W_{rect} for a v=c electron that enters the waveguide at a time equal to the filling time is: $W_{rect} = 26.2$, where the difference with respect to nominal value of 30 is due to the attenuation in the waveguide. Consider also a train of four rectangular RF pulses with a lenght of 10 RF cycles (3.5 ns), a spacing of 70 ns and an amplitude E_{train} choosen to have the same total area of the rectangular unitary-amplitude pulse:

$$E_{train} = \frac{300}{4 \times 3.5} = 21.43$$

The energy gain W_{train} for a v=c electron that enters the waveguide at a time equal to the filling time is: $W_{train} = 23.5$. This value is less than the energy gain with a single pulse by only about 11%, this means that the loss in efficiency due to the dispersion effects for the 30 cells accelerating strucure filled with short pulses is quite low.

The strong difference is between the amount of peak power P_p that the RF generator must supply for the two cases; that is proportional to the square of the amplitude of the RF pulse:

$$\frac{P_{p\ pulse}}{P_{p\ train}} = \frac{1}{(21.43)^2} = \frac{1}{460}$$

The RF energy put into the waveguide is equal to the RF power times the time width of the pulse, so that the ratio between the RF energy in the waveguide for the two cases is:

$$\frac{P_{p\ pulse} \times \tau_{pulse}}{P_{p\ train} \times \tau_{train}} = \frac{1}{460 \times (300/14)} = 0.047$$

A "figure of merit" M_t for the train of pulses can be introduced, which gives the acceleration efficiency with a certain train of pulses, in respect of a single RF pulse with a length equal to the filling time.

$$M_t = \frac{W_{train}}{P_{p\ train} \times \tau_{train}} \Big/ \frac{W_{pulse}}{P_{p\ pulse} \times \tau_{pulse}}$$

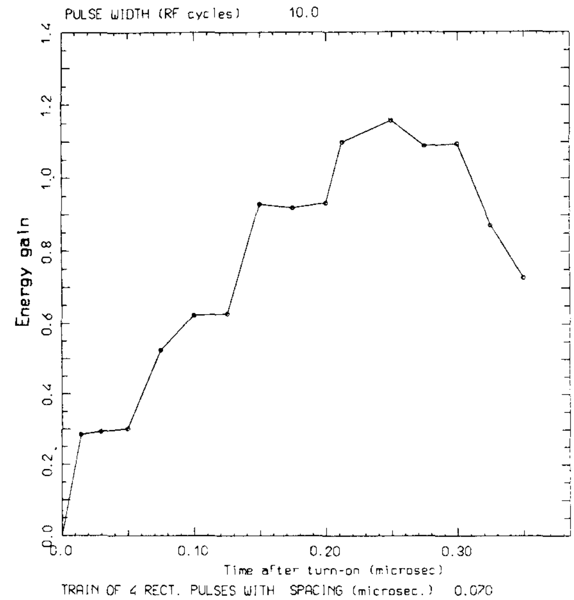
In the case discussed above we have: $M_t = 0.042$.

Results very similar can be obtained with a train of short Gaussian-shaped pulses, as it is shown in ref. 4.

Conclusions

We can conclude that the acceleration efficiency for a train of RF pulses as described is much less than the one with a single RF pulse, with a length of the order of the filling time. In particular the dispersion of the waveguide affects only a little the overall efficiency (11% and 5% for $v_g = 0.012c$ and $v_g = 0.1c$ respectively),the latter being mainly determined by the duty cycle of the pulse train.

It is also straightforward that with short RF pulses the accelerating structure must have an higher peak power handling capability.



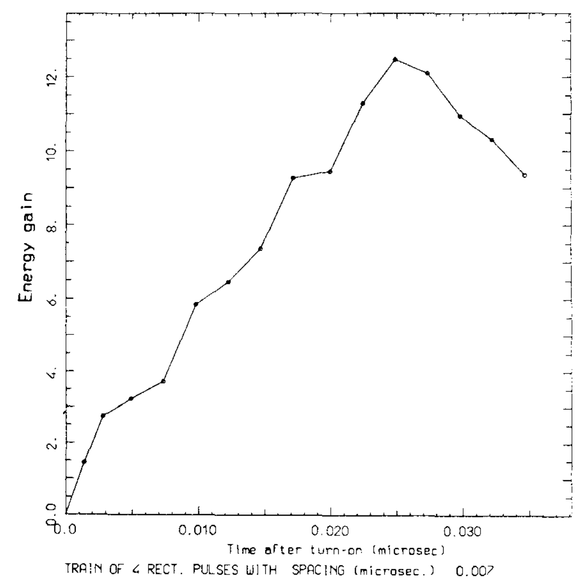


Fig. 5 *a* and *b* : energy gain for a v=c electron traveling in a 30 cells SLAC-type accelerating structure having respectively low and high group velocity. (see text for details)

References

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4) L.Ferrucci, C.Pagani and L.Serafini, Propagation of a short RF pulses train in an iris-loaded waveguide, to be published in the proceedings of the " INFN International School on E.M. Radiation and Particle Beams ", Varenna, 1988.