MODES IN INFINITELY REPEATING STRUCTURES OF CYLINDRICAL SYMMETRY

Thomas Weiland

Deutsches Elektronensynchrotron DESY, Notkestr.85,2000 Hamburg 52,FRG

ABSTRACT

The computer code URMEL for calculating modes in arbitrarily shaped cavities of cylindrical symmetry has been extended to the case of infinitely repeating periodic structures. This extension now offers a simple way for evaluating the group velocity in long linac structures. URMEL now also calculates for a given phase advance over one period the resonant frequencies and fields for the m lowest modes (m=100, say). Thus a much higher accuracy can be obtained compared to multi-cell computations where one has to solve for n cells in order to obtain n+1 values of the dispersion curve. This new option has been implemented in URMEL (release 5) as a new boundary condition plus a new input variable for the phase advance.

INTRODUCTION

A typical linear accelerator structure consists of a chain of many cavity cells (of the order of one hundred and more). These structures may be modelled by an infinitely long chain of periodic cavity cells. In usual standing wave structures the number of cells is much smaller (typically between 3 and 7). For each resonant mode type in a n-cell structure there exist n+1 coupled modes. Thus a seven cell cavity has eight modes of the same basic type in each cell but with slightly different frequencies. These modes are characterized by the phase advance per cell. If there were an infinite number of cells, the resonant frequency would be a continous function of the phase advance for each mode type. These functions form the so-called Brillouin diagram. Computer codes such as URMEL [1] can calculate a given number of modes of arbitrarily shaped structures with Neumann or Dirichlet boundary conditions at the cavity walls. Thus the calculation of a five cell segment of an infinitely long chain results in six points on the Brillouin diagram. The more cells one uses the more dense the results become. In order to evaluate the group velocity of a specific mode one has to compute the derivative of the functional relationship between phase advance and frequency:

$$v_g = L \frac{\partial \omega}{\partial \phi}, \tag{1}$$

with L being the period length (see figure 1). Such an operation is inaccurate if one has only a few points of the curve $\omega(\phi)$ available. To achieve good accuracy in this calculation the usual type of mode computation should be modified such that only a single cell of the chain is taken into account, with periodic boundary conditions.

For periodic pill-box cavities connected with beam pipes, proven computer programs have existed for many years [2],[3]. In this paper we describe a new version of URMEL [1] that allows for arbitrarily-shaped cylindrical cavity cells with arbitrary periodic boundary conditions. Together with the previously available features of URMEL it is now possible to obtain not only all resonant modes but also the group velocities of each mode and Brillouin diagrams. This fact makes the code the ideal tool for designing high gradient linac structures for a TeV linear collider in which the group velocity is an important parameter.

THE METHOD

In structures of cylindrical symmetry time harmonic electromagnetic fields can be expanded into series over the azimuthal mode number m as :

$$\vec{E}(r,\varphi,z,t) = \sum_{m=0}^{\infty} Re\{\, \underline{\vec{E}}_m(r,z)\, e^{im\varphi}\, e^{i\omega t}\}, \tag{2}$$

$$\vec{H}(r,\varphi,z,t) = \sum_{m=0}^{\infty} Re\{\, \underline{\vec{H}}_m(r,z)\, e^{im\varphi}\, e^{i\omega t}\}. \tag{3}$$

For the magnitudes one has to solve Maxwell's equations in the form:

$$\begin{aligned} curl\,\underline{\vec{E}}_m &= -i\ \omega\mu\underline{\vec{H}}_m, \\ curl\,\underline{\vec{H}}_m &= \ \ i\ \omega\epsilon\underline{\vec{E}}_m. \end{aligned} \tag{4}$$

With given boundary conditions (Dirichlet or Neumann) these equations form an eigenvalue problem that can be solved by a number of mesh codes allowing for arbitrarily shaped strucures. For example URMEL [1] and URMEL-T [4] are capable of calculating many (100, say) resonant modes for each given azimuthal mode number m. Thus one can evaluate all important accelerating and deflecting modes in a cavity.

For very long chains of cavity cells the computation becomes more and more cpu-time consuming (or less and less accurate for a given number of mesh points). In this case modelling such structures by infinitely repeating chains made of a periodic section of the length L becomes more adequate.

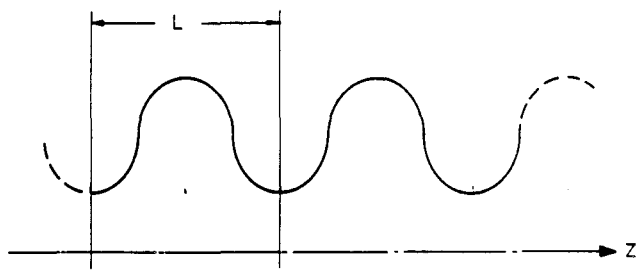

Figure 1 : Inifnitely long periodic chain of cylindrically symmetric cavities with period length L. In order to determine the field solutions it is sufficient to consider only one period imposing Floquet conditions at both ends in longitudinal direction.

In such periodic structures the boundary conditions at any position z_0 and $z_0 + L$ are replaced by Floquet conditions:

$$\underline{\vec{E}}_m(r,\varphi,z_0) = \underline{\vec{E}}_m(r,\varphi,z_0+L)\cdot e^{i\phi} \tag{5}$$

$$\underline{\vec{H}}_m(r,\varphi,z_0) = \underline{\vec{H}}_m(r,\varphi,z_0+L)\cdot e^{i\phi} \tag{6}$$

with an arbitrary phase advance ϕ.

In this paper we describe an extension to the computer code URMEL [1] that includes periodic boundary conditions (so far only for monopole modes, the extension to dipole modes will follow).

The URMEL method will not be described here but only sketched briefly; for further details see [1]. The cavity shape is first approximated in a cartesian grid with variable step sizes as shown in figure 2.

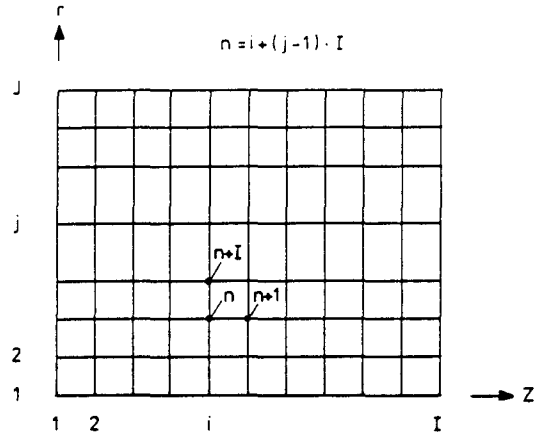

Figure 2 : Two dimensional grid used for the discretization showing the numbering system of grid nodes

In this grid Maxwell's equations are discretized in the same way as in URMEL. Changes have only to be made when unknown field components near the boundary at z_0 and $z_0 + L$ are concerned. In figure 3 we see a pictorial representation of the five point difference operator with one arm to an outside component. In the case of normal boundary conditions this unknown field component outside the mesh is replaced by equating it to inside components making use of reflection symmetry conditions which are equivalent to boundary conditions. In the periodic case each component outside the mesh on the right hand side is replaced by one located near the left hand side of the mesh multiplied by a complex phase factor (see figure 3):

$$\underline{H}_{\varphi,I,j} = \underline{H}_{\varphi,1,j} \cdot e^{i\phi} \; ; \; j = 1, \ldots, J \qquad (7)$$

This discretization of the Floquet condition adds new matrix elements to the standard URMEL-matrix which are complex and lie outside the usual bands, see figure 4.

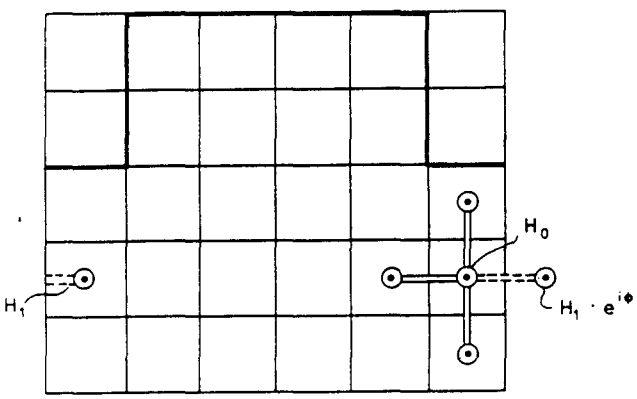

Figure 3 : The difference operator at the right hand side boundary with one arm connected to the left hand side via a phase factor (Floquet condition).

In contrast to URMEL where the matrix containing all difference equations is real and symmetric, the matrix here is complex. However, it can be shown that the resulting linear algebraic eigenvalue problem has a hermitian matrix.

$$\underline{\mathbf{A}}\,\underline{\mathbf{h}} = \omega^2 \mathbf{h}, \qquad (8)$$

$$\underline{\mathbf{A}} = \underline{\mathbf{A}}^{\dagger} \qquad (9)$$

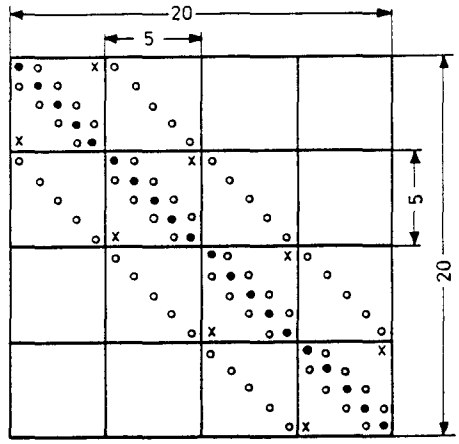

Figure 4 : Typical band structure of the extended URMEL-matrix with the standard elements (circles) and the additional complex elements due to the Floquet condition.

Thus we find that all eigenfrequencies are real but the elements of the eigensolution vector $\mathbf{h}$ containing all unknown components of the azimuthal magnetic field are complex. In order to minimize the programming effort one may rewrite the complex matrix equation into a purely real one of twice the order:

$$\underline{\mathbf{A}} = \mathbf{A}_R + i\,\mathbf{A}_I \qquad (10)$$

$$\underline{\mathbf{h}} = \mathbf{h}_R + i\,\mathbf{h}_I \qquad (11)$$

$$\begin{pmatrix} \mathbf{A}_R & -\mathbf{A}_I \\ \mathbf{A}_I & \mathbf{A}_R \end{pmatrix} \cdot \begin{pmatrix} \mathbf{h}_R \\ \mathbf{h}_I \end{pmatrix} = \omega^2 \begin{pmatrix} \mathbf{h}_R \\ \mathbf{h}_I \end{pmatrix} \qquad (12)$$

Such an algebraic eigenvalue problem can be treated with the SAP solver [5] used in URMEL without any further changes. The only difference is that all eigenvalues are doubled with two conjugate complex eigenvectors.

With this new option URMEL finds for any given phase advance ϕ the lowest modes at the price of approximately twice the computational effort compared to normal boundary conditions. It should be mentioned here that a similar extension has also been added to SUPERFISH [6] by Gluckstern and Opp [7] at the University of Maryland and Los Alamos.

EXAMPLES

As a simple test example we choose a S-band cavity similar to the SLAC structure as shown in figure 5.

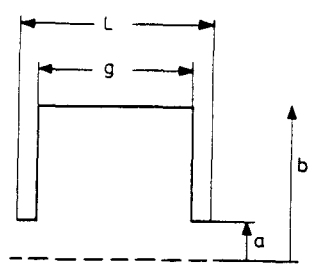

Figure 5 : Typical S-band accelerating cavity similar to the SLAC structure ($2\pi/3$ structure, a=1.163cm, b=4.134cm, g=3cm, L=3.5cm).

Calculating the modes in a set of n-cells adding Dirichlet or Neumann boundary conditions on both ends yields n+1 eigenfrequencies which represent the n+1 possible coupled mode configurations. In cavities with a weak cell-to-cell coupling these frequencies can be identified as points on the Brillouin diagram. Using URMEL for such a n-cell cavity (in this specific case where the structure has a reflection symmetry with respect to the midplane one actually needs only n/2 cells in URMEL) thus yields n+1 modes, see figure 6.

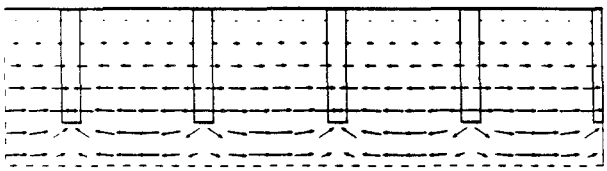

Figure 6 : Typical multi-cell mode found by URMEL in a nine cell cavity ("0-mode").

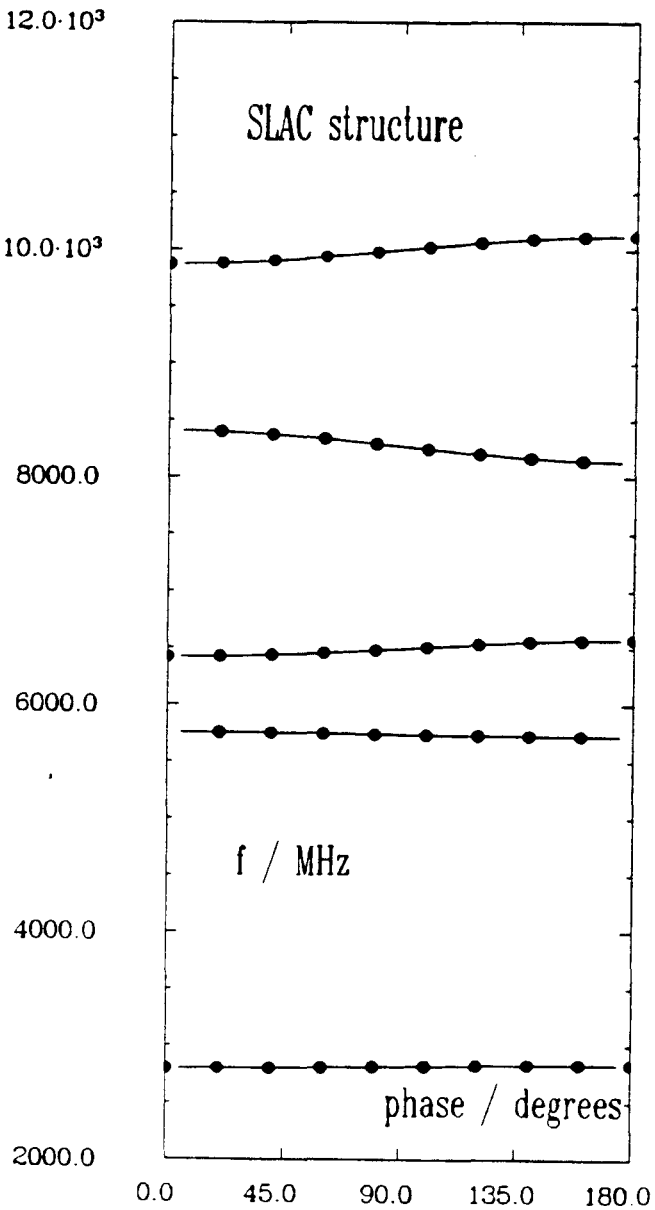

Figure 7 : Brillouin diagram of the S-band structure of figure 4. Lines show the result of URMEL calculations with periodic boundary condition, dots show the result of URMEL calcultions for a nine-cell section

Using periodic boundary conditions and varying the phase advance ϕ one obtains a set of curves, each of which gives the frequency of one specific mode type as function of ϕ. Choosing $\phi = i \cdot \pi / n; i = 0, 1, ..., n$ should yield exactly the same frequencies as obtained from URMEL for a n-cell cavity structure. Since results of URMEL have been extensively compared with measurements and are well proven [1] [8] we may accept such a comparison as a proof of the correctness of the new periodic version. Figure 7 shows results from URMEL with usual and periodic boundary conditions. Figure 8 shows the same results as figure 7 but only for the fundamental mode, showing the excellent agreement between both computations.

The second example shows the dispersion curves of a "single mode cavity" [9] which is designed such that only the fundamental accelerating mode out of all m=0 modes is kept in the cavity. All higher m=0 modes have frequencies higher than the cut off frequency given by the inner radius of the iris, so that these modes will travel away. This cavity as shown in figure 9, was considered (for some time) to be a candidate for superconducting structures at DESY. Figure 10 shows the dispersion curves for the five lowest modes and figure 11 again only the fundamental mode with its large dispersion step.

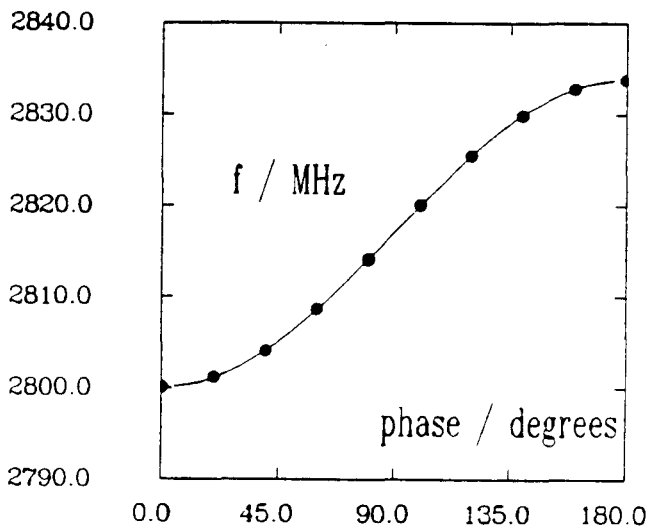

Figure 8 : Brillouin diagram of figure 7 but only for the fundamental mode showing the excellent agreement

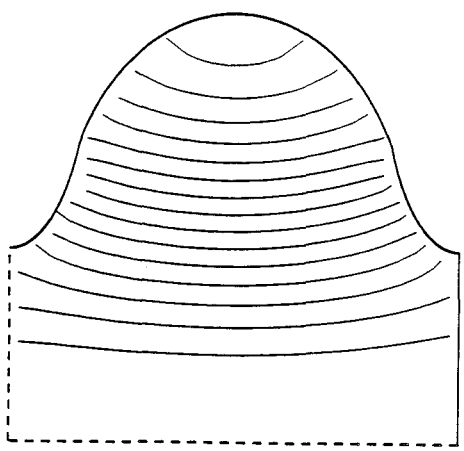

Figure 9 : "Single Mode Cavity" with fundamental mode with phase advance $\phi = 60^0$

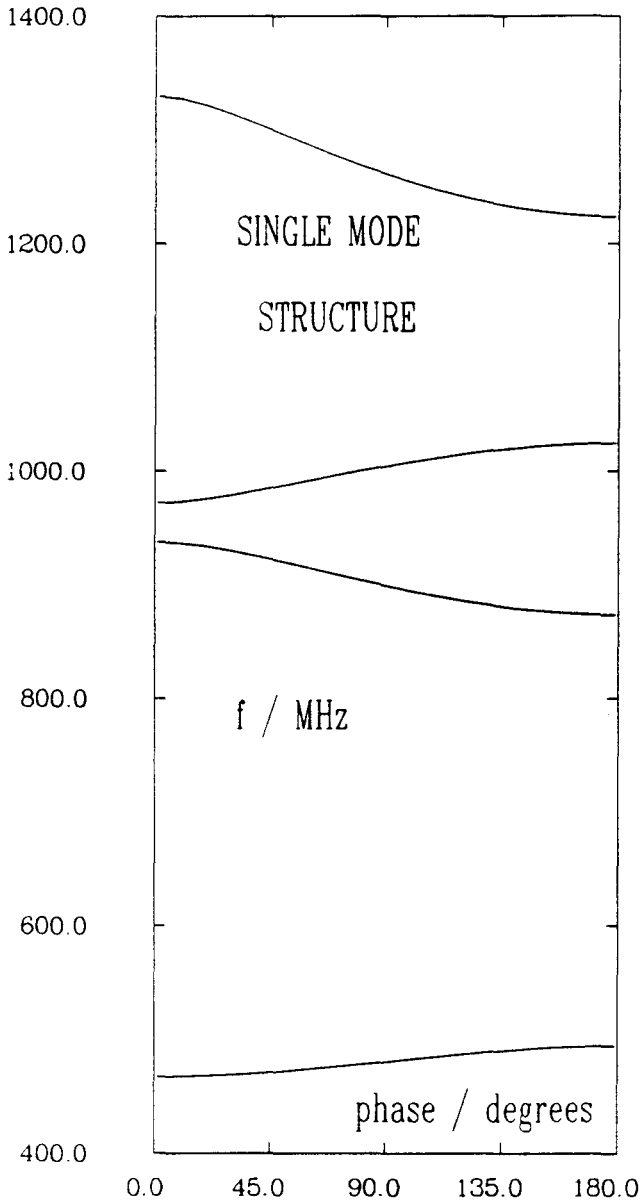

Figure 10 : Brillouin diagram for a "Single Mode Cavity" showing the five lowest modes

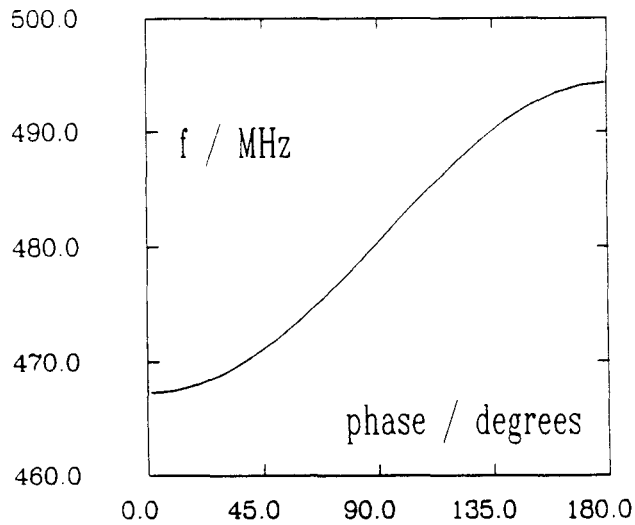

Figure 11 : Fundamental mode of figure 10 showing the large dispersion step in the "Single Mode Cavity"

SUMMARY

The extension of URMEL to periodic boundary conditions has proven to work correctly for accelerating monopole modes. Thus this extension may become a useful tool in the design of linac structures wherever the group velocity is of interest.

As a next step the periodic conditions will be extended to transverse modes and finally we intend to implement this option into the triangular version URMEL-T, which will result in a much more flexible and versatile cavity code.

Together with the new broadband impedance option [10] the complete range from single cells to infinitely periodic structures and from low frequency resonances to the non-resonant part of the impedance above cut-off is covered.

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