WAKE FIELD CALCULATIONS WITH THREE-DIMENSIONAL BCI CODE

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Abstract

The new MAFIA code T3 is introduced, belonging to a family of fully 3-D codes for computer-aided design of RF cavities developed by the MAFIA collaboration at DESY, KFA Jülich and LANL [1,2]. T3 is a 3-D extension of TBCI, solving Maxwell's equations in the time domain using the FIT ansatz, allowing the use of structures of arbitrary shape and dielectric material insertions, and integrating the wake potential at arbitrary positions. An open boundary condition was implemented to simulate infinite beam pipes. An IBM 3081 with 5 Mbytes of main storage can handle problems up to 80.000 mesh points while a window option enables the treatment of very long structures using up to 1.000.000 mesh points. Together with its postprocessors W3COR and W3OUT and the MAFIA mesh generator M3, this code was used to calculate wake potentials for several beam pipe components (i.e. vacuum chambers, vacuum chamber junctions etc.) which required fully 3-D calculations. Comparision of T3 results with TBCI calculations in the case of a cylindrically symmetric structure (pillbox) showed agreement within a few percent.

INTRODUCTION

A bunch of charged particles passing an accelerator component or any other structure of varying geometry, such as cavities, bellows, monitors etc., excites electromagnetic fields. The resulting Lorentz force consists of decelerating and deflecting components, the strength of which is proportional to the number of particles in the bunch, and acts back on the particles inside the bunch. Single-bunch instabilities due to these so-called wakefields have been realized as a strong limitation on maximum beam current in accelerators such as PETRA and PEP, and will become even more important in the future SLC, LEP and HERA. In general these wakefield forces are functions of space and time coordinates. For particles moving in the z-direction with velocity βc these forces can be described in a comoving particle frame with the relative particle position $s = \beta ct - z$, thus yielding:

$$\begin{aligned}\vec{F}(x,y,z,t) &= \vec{F}(x,y,\beta ct - s,t)\\ &= e(\vec{E}(x,y,\beta ct-s,t) + \beta ct\vec{e}_z \times \vec{B}(x,y,\beta ct - s,t)).\end{aligned} \tag{1}$$

For, $\gamma \gg 1$ the time dependent force is a rapidly varying function compared with the particle motion (betatron oscillations for deflecting forces and synchrotron oscillations for decelerating forces), thus in most cases it is sufficient to consider the averaged forces along the passage of a structure. These integrated forces are called wake potentials:

$$\vec{W}(x,y,s) = \int_{-\infty}^{\infty} \vec{F}(x,y,\beta ct - s,t)d(\beta ct) \tag{2}$$

As a function of the longitudinal positions inside the bunch and the total charge the wake potential describes changes in momentum for each particle. For a point charge the integrated wake force corresponds to the impedance via Fourier transformation.

Further quantities which can be derived from the wake potential belonging to a charge distribution $\lambda(s)$ are the total energy lost by the bunch

$$k_{\|}(x,y) = \frac{\int_{-\infty}^{\infty}\lambda(s)W_{\|}(x,y,s)ds}{(\int_{-\infty}^{\infty}\lambda(s)ds)^2}, \tag{3}$$

and the averaged transverse kick seen by the particles:

$$\vec{k}_{\perp}(x,y) = \frac{\int_{-\infty}^{\infty}\lambda(s)\vec{W}_{\perp}(x,y,s)ds}{(\int_{-\infty}^{\infty}\lambda(s)ds)^2}. \tag{4}$$

A measurable quantity resulting from $\vec{k}_{\perp}$ is the fundamental head-tail mode tune shift in a storage ring[3]. Further applications can be found elsewhere[4]. A natural approach to calculating these wake fields and wake potentials is the solution of Maxwell's equation in the time domain[5]. For cylindrical symmetric structures this can be done by the TBCI code [6], which has already become a standard code for this purpose. The study of various components of the beam tube without any cylindrically symmetry requires a fully 3-D BCI, which will be introduced in this paper. A 3-D extension of TBCI for elliptical geometry only has been performed by Chin[7].

DISCRETIZATION

Using the generalized finite integration theory [8] (FIT) the field components will be allocated as shown below. A deeper discussion about the characteristics and advantage of this method can be found in [9]. Here we will only mention the continuity of all field components used in the discretized equations at material boundaries. Putting all unknown electric field components in the grid into a vector:

$$\mathbf{e} = (\mathbf{E_{x1}}, \ldots, \mathbf{E_{xN}}, \mathbf{E_{y1}}, \ldots, \mathbf{E_{yN}}, \mathbf{E_{z1}}, \ldots, \mathbf{E_{zN}})^t. \tag{5}$$

and the magnetic field components into $\mathbf{b}$, the current densities into $\mathbf{j}$ lead us to the following matrix equations (compare [9])

$$\begin{array}{lllll}\mathbf{R}\cdot\mathbf{e} &=& -\dot{\mathbf{b}} & \oint_{(A)}\vec{E}\cdot d\vec{s} &=& -\int\int_A \frac{\partial\vec{B}}{\partial t}\cdot d\vec{A}.\\ \tilde{\mathbf{R}}\cdot\mathbf{b} &=& \mathbf{D}_\varepsilon\cdot\dot{\mathbf{e}} + \mathbf{j} & \oint_{(A)}\vec{H}\cdot d\vec{s} &=& \int\int_A\left(\frac{\partial\vec{D}}{\partial t} - \vec{J}\right)\cdot d\vec{A}.\end{array} \tag{6}$$

Breaking up the time axis in pieces with length δt, using the central difference operator for the time derivative with the notation $f^n := f(n\delta t)$ and equating $\mathbf{e}$, $\dot{\mathbf{b}}$ at half time steps and $\dot{\mathbf{e}}$, $\mathbf{b}$ at full time steps, we obtain an alternating explicit time scheme first introduced by Yee[10] (also called the leap-frog scheme of Maxwell's equation):

$$\begin{aligned}\mathbf{b}^{\mathbf{n}} &= \mathbf{b}^{\mathbf{n}-1} - \delta t\mathbf{R}\cdot\mathbf{e}^{\mathbf{n}-1/2}.\\ \mathbf{e}^{\mathbf{n}+1/2} &= \mathbf{e}^{\mathbf{n}-1/2} + \delta t\mathbf{D}_\varepsilon^{-1}(\tilde{\mathbf{R}}\cdot\mathbf{b}^{\mathbf{n}} - \mathbf{j}^{\mathbf{n}}).\end{aligned} \tag{7}$$

This algorithm fulfills the law of energy conservation [11] and requires only matrix multiplications, while the band structure of the matrices allows the use of large numbers of mesh points. A discussion of stability limitations and convergence can be found in [11,12]

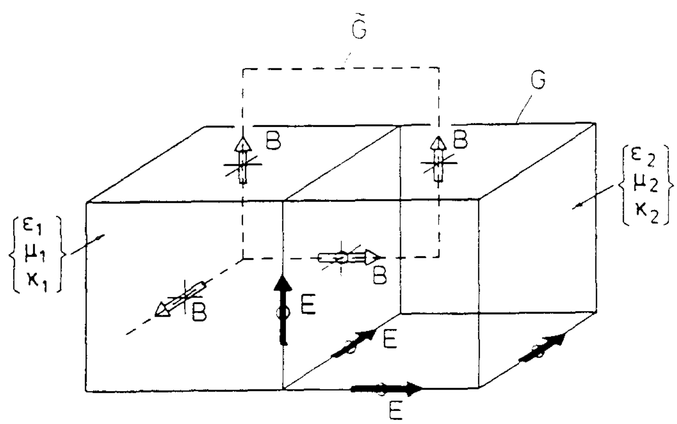

Figure 1: Allocation of field components

OPEN BOUNDARY CONDITION

Solving Maxwell's equation in a finite grid requires the simulation of long beam tubes [11]. To avoid reflection of the scattered field at the boundary of the mesh an open boundary condition has been added to simulate infinitely long beam tubes on both sides of the structure. This implementation follows the consideration in [11] and requires, unlike TBCI, the solution of a 2-dimensional boundary problem for the determination of the inhomogeneous component of the B-field. Numerical results showed a significant reduction of the reflection at the end of the tubes and stored energy after having left the structure compared with the usual boundary conditions.

WINDOW OPTION[13]

The calculation of the wake potential inside the bunch for ultrarelativistic particles makes it possible to restrict the field calculation of those components over a window containing the bunch and moving along with it. Causality requires that firstly no fields can be generated at a position preceding the first particle and secondly no disturbance in any position behind the bunch can have any influence on what will happen inside the bunch.

Especially for short bunches in long structures this option helps to reduce the amount of neccessary storage and total CPU-time by a factor equal the ratio of the structure length to the window length. This can be quite large. Without the window option 5 MBytes of core allows handling up to 80.000 meshpoints while in the case of the window option structures with more than 1.000.000 meshpoints have been calculated using 5 h CPU time on an IBM 3081K. This option allows the treatment of very long structures.

INTEGRATION OF THE WAKEFIELDS AT THE BEAM TUBE SURFACE

Similar to TBCI the wakefield integration can be done only over a finite interval given by the mesh. For bunches shorter than the beam tube dimensions it was found that a significant part of the interaction between the bunch and the fields takes place in the beam tubes, due to the fact that the Fourier spectrum of such a bunch contains many frequencies above the cut off frequency of the beam ports. This requires one to extend the beam tubes with shorter bunches. Secondly it was found that a discretization noise comes into the calculation for beam positions near the axis. The second problem usually was treated by subtracting the wake potential of the empty tube from the wake potential of the structure to get the exact result.

For TBCI this problem was overcome by integrating the wakefields at the beam tube surface (where the longitudinal force vanishes), so that the infinite integration interval becomes a finite one. Making use of the knowledge of the explicitly and analytically known dependence of the integrated forces as a function of radius and azimuth it was possible to calculate the wake potential for any desired offset [14]. Following this method for T3 requires the solution of the two dimensional boundary problem for every position in the bunch frame.

First we will show how to get the transverse wake potential from the longitudinal wake potential. We use: $curl\,\vec{E} = -\dot{\vec{B}}$.
A variable change $z = \beta ct - s$ leads us for the x - component to:

$$\begin{aligned}
\frac{\partial}{\partial z}E_z(x,y,z=\beta ct-s,t) &= \frac{\partial}{\partial x}E_x(x,y,z=\beta ct-s,t) \\
&\quad + \frac{\partial}{\partial t}B_y(x,y,z=\beta ct-s,t) \\
&= -\frac{\partial}{\partial s}E_x(x,y,z=\beta ct-s,t) \\
&\quad - \beta c\frac{\partial}{\partial s}B_y(x,y,z=\beta ct-s,t) \\
&\quad + \frac{\partial}{\partial t}B_y(x,y,z=\beta ct-s,t),
\end{aligned} \tag{8}$$

which means:

$$\begin{aligned}
\frac{\partial}{\partial s}F_x(x,y,z=\beta ct-s,t) &= \frac{\partial}{\partial x}F_z(x,y,z=\beta ct-s,t) \\
&\quad - e\frac{\partial}{\partial t}B_y(x,y,z=\beta ct-s,t).
\end{aligned} \tag{9}$$

A similar relation holds for the y - component. The time integration of this expression followed by a second integration over s give us:

$$\vec{W}_\perp(x,y,s) = -\nabla_{x,y}\int_{-\infty}^{s} \vec{W}_\parallel(x,y,s')ds'. \tag{10}$$

Thus it is possible to get the transverse wake potential from the longitudinal one and we will therefore consider the longitudinal component of the electric field which can be described inside the tube by:

$$E_z(x,y,z,t) = Re\{(2\pi)^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega dk\tilde{\bar{E}}_z(x,y,k,\omega)\,e^{i\omega t}\,e^{-ikz}\}. \tag{11}$$

and the net change in longitudinal momentum for a particle at position $(x,y,z=\beta ct-s)$ is given by:

$$\triangle p_z(x,y,s) = Re\{\frac{e}{(2\pi)}\int_{-\infty}^{\infty} d\omega\,\tilde{\bar{E}}_z(x,y,\omega/\beta c,\omega)\,e^{i(\omega/\beta c)s}\}. \tag{12}$$

Applying the wave equation for the z component and performing the time integration gives finally an equation for the longitudinal wake potential:

$$\begin{aligned}
\nabla^2_{x,y}G(x,y,s) &= e\int_{-\infty}^{\infty}\beta cdt\nabla^2_{x,y}E_z(x,y,\beta ct-s,t) \\
&= e\int_{-\infty}^{\infty}\beta cdt(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2})E_z(x,y,z=\beta ct-s,t) \\
&= Re\{(2\pi\beta\gamma)^{-2}\int_{-\infty}^{\infty}d\omega(\omega/c)^2\tilde{\bar{E}}_z(x,y,\omega/\beta c,\omega)\,e^{i(\omega/\beta c)s}\}.
\end{aligned} \tag{13}$$

For $\gamma \longrightarrow \infty$ and a smooth particle density, the right side of the last equation will go to zero, proving that for ultrarelativistic particles G is for every s a harmonic function. Thus solving for every s the boundary problem given by the wake potential at the beam tube surface gives the wake potential for postions inside the beam tube (especially near the axis), avoiding all errors by finite integration interval and discretization noise. Finally we have for $\beta \approx 1$:

$$\begin{aligned}
\triangle\vec{p}(x,y,s) &= \{-\frac{\partial}{\partial x}\int_{-\infty}^{s}G(x,y,s')ds'\vec{e}_x \\
&\quad -\frac{\partial}{\partial y}\int_{-\infty}^{s}G(x,y,s')ds'\vec{e}_y + G(x,y,s)\vec{e}_z\}.
\end{aligned} \tag{14}$$

$G(x,y,s)$ is easily computed by T3 solving $\triangle\Phi = 0$ numerically with boundary conditions at the beam tube surface. An example for G is shown in figure 4.

EXAMPLES

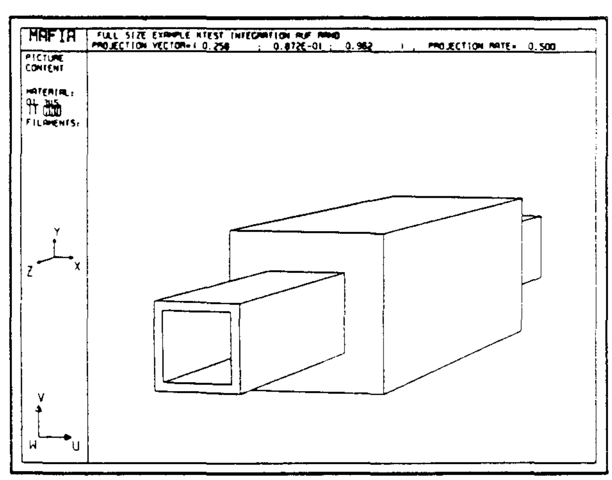

Figure 2: M3 plot of example 1 (retangular pillbox with beamports).

Example 1 (figure 2) will be used for demonstrating wakefield integration at the beam tube surface. Due to symmetry of the geometry we have to compute only one quarter of the given structure by using boundary conditions to get the complete solution. For this reason figure 3 shows only one quarter of the structure. The same will be true for all examples considered in this paper.

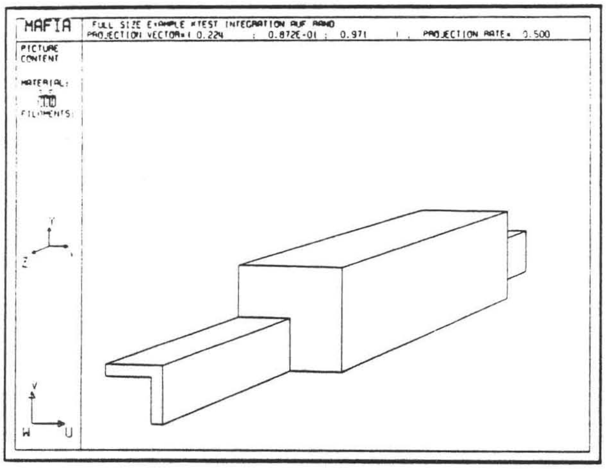

Figure 3: One quarter of structure in figure 2 as used for the computations.

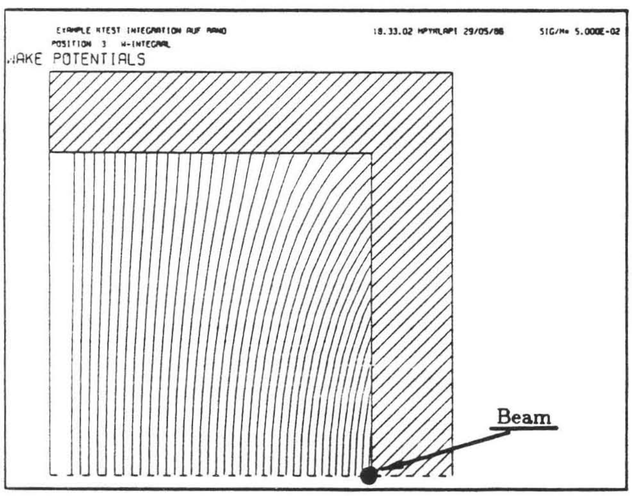

Figure 4: Scaling function $\int_{-\infty}^{\infty} ds\lambda(s)G(x,y,s)$ for $k_{\parallel}$ computed by T3 for a rectangular pillbox with beam offset in horizontal direction.

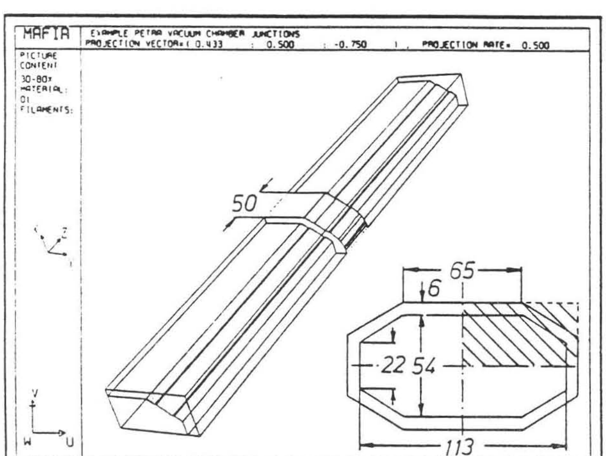

Figure 5: One quarter of a PETRA vacuum chamber junction.

Example 2 (PETRA vacuum chamber junction) was found to contribute significantly to the transverse impedance for short bunches[3].

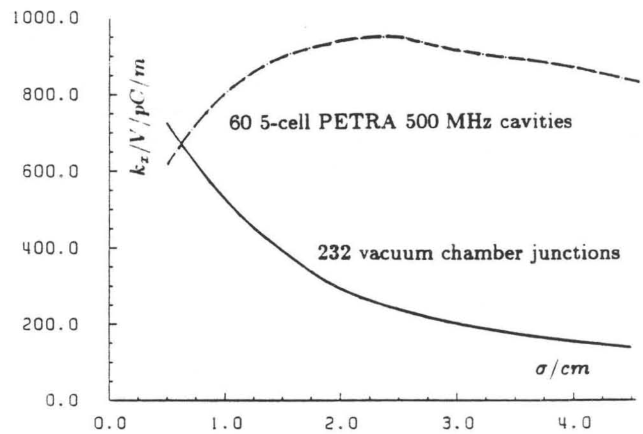

Figure 6: Vertical kick (averaged over the bunch) for 232 vacuum chamber junctions (figure 5) in PETRA in comparison with the kick due to 60 PETRA cavities.

For short bunches (σ less than 1 cm) the contribution from the cavities has the same order of magnitude as the contribution of the vacuum chamber junctions.

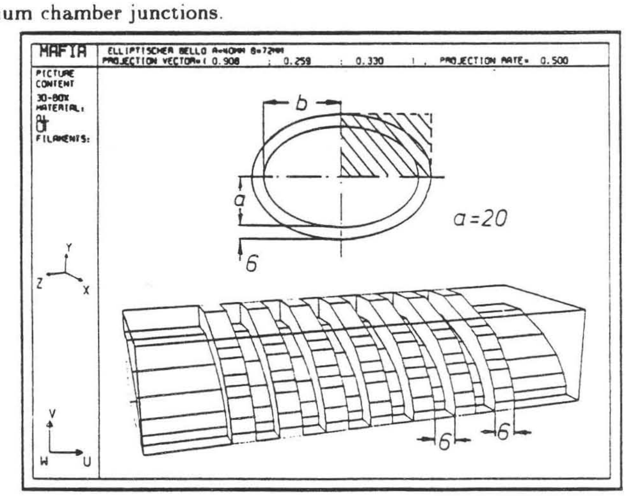

Figure 7: One quarter of an elliptical bellow.

Example 3 (elliptical bellow) shows the influence of the ratio of major axis and minor axis on the vertical kick averaged over the bunch. The calculation was done for a bunch length σ = 1.2cm and gives a maximum for a circular bellow. For this special case the TBCI result is given in figure 8.

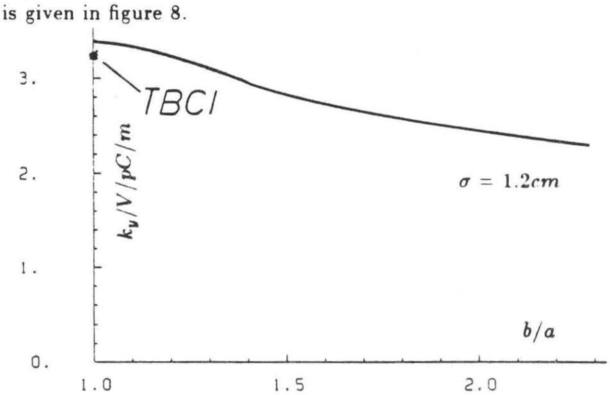

Figure 8: Vertical kick (averaged over bunch) against ratio major axis to minor axis.

Example 4: In HERA vacuum chambers with horizontal and vertical slots will be used (vacuum reasons). We studied for a simple model (square vacuum chamber) the contribution of the slots to the transverse impedance. First calculations showed that the part resulting from horizontal slots can be neglected compared with the part resulting from vertical slots (though the first type has a length of 4.1km and the second type only 0.4km). The second type with vertical slots requires further examinations.

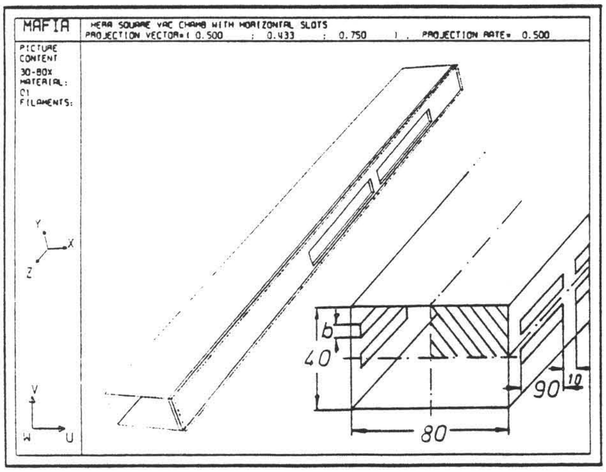

Figure 9: One quarter of HERA sqare vacuum chamber with horizontal slots.

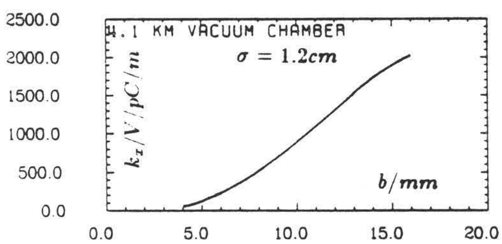

Figure 10: Vertical kick (averaged over bunch) against slot high b for 4.1km structure (figure 9).

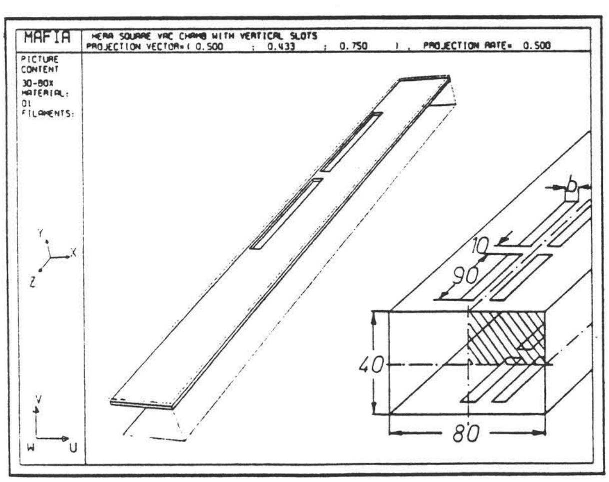

Figure 11: One quarter of HERA sqare vacuum chamber with vertical slots.

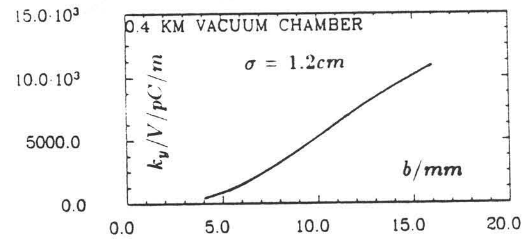

Figure 10: Vertical kick (averaged over bunch) against slot high b for 0.4km structure (figure 11).

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