Three-Dimensional Cavity Calculations

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INTRODUCTION

The existence of a code that solves for the resonant electromagnetic modes of oscillation in arbitrarily-shaped three-dimensional cavities opens new possibilities in rf-structure analysis and research. The URMEL-3D code, the product of a multi-year collaboration between DESY, KFA-Jülich, and Los Alamos, has been used in some exploratory studies to determine the feasibility of using a 3-D code to calculate the properties of several practical rf structures. The results are reported here for three cases: the jungle gym, two coupled cavities, and a waveguide-cavity coupling problem.

THE JUNGLE GYM

Jungle Gym is the name given to a slow-wave rf structure long investigated as an accelerating structure. The advantages of this structure are its relatively simple, rugged construction and ease of cooling. Early studies on Jungle Gyms were carried out at the Hansen laboratories at Stanford, and a four-period structure was used for a time in the Cornell electron synchrotron[1]. A thoroughgoing analysis of this structure is not known to the authors to exist. We have performed preliminary calculations with URMEL-3D which indicate that the code is capable of modeling such a structure.

We have taken an approximation to the Jungle Gym as used at Cornell. In the interests of economy of computing and data preparation, a rather coarse description of the geometry was used. Whereas the round loading bars (vertical and horizontal) of the actual structure were tapered to reduce current densities at the joints, we have taken untapered square bars with approximately the same transverse dimension the actual bars had in the midplane. The right-circular cylindrical waveguide is approximated by horizontal, vertical, and 45 degree segments as shown in Fig. 1, which shows one-quarter of the structure. The resonant frequencies of a cavity consisting of three cells and having alternately short-circuited and open-circuited end conditions were calculated. Figure 2 gives a plot of resonant frequencies versus phase shift per cell. We interpret the slight staggering between the short and open circuit modes as resulting from the fact that the boundary conditions are not exactly those required to model a truly periodic structure. Table I gives the frequencies, shunt impedance per unit length, and r/Q values for the modes plotted as circles in the figure, calculated for traveling-wave application. These values are in good agreement with those found in the literature.

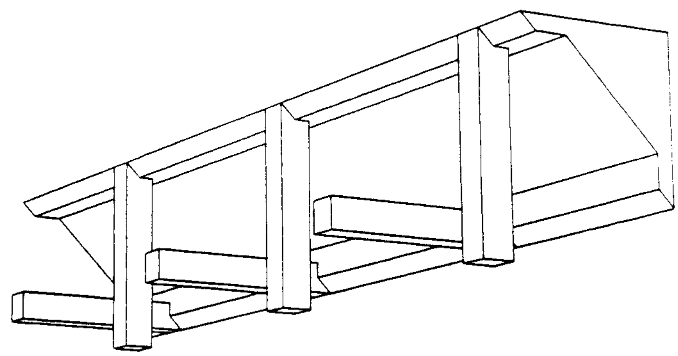

Figure 1: The Jungle Gym as modeled (one quarter of the structure)

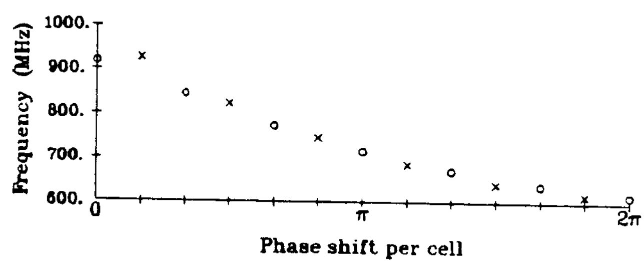

Figure 2: The calculated Jungle Gym dispersion curve

Table 1: Frequencies and parameters of Jungle Gym modes

Frequency	Phase shift per cell	Shunt impedance r(MΩ/m)	r/Q (Ω/m)
616.1	2π	0.04	2.6
638.5	$5\pi/3$	0.08	4.4
671.7	$4\pi/3$	6.02	317.2
715.4	π	26.00	1274.2
772.7	$2\pi/3$	10.86	484.2
845.1	$\pi/3$	3.42	133.6
918.2	0	0.92	23.0

TWO COUPLED CAVITIES

As a second example calculation we have taken two identical cavities made from 9.750" x 4.875" waveguide loaded by circular-cylindrical posts (referred to as "nose cones"). One end wall is a shorting plane, while the other wall, common to the two cavities, has a horizontal slot centered on the midplane. Figure 3 shows one-fourth of one cavity as modeled by the code. We have studied the two lowest modes of oscillation of the coupled system as a function of the length of the slot, the slot height being held constant. The two lowest modes have, repectively, even and odd parity about the plane containing the coupling slot. The odd mode is essentially the odd mode of a double cavity with the slot wall removed, perturbed only by the thickness of the wall when inserted. That is, this mode is essentially unaffected by the coupling slot. The even mode, however, corresponds to a mode of the double cavity which has an electric field maximum at the coupling plane, and is therefore severely perturbed by the insertion of the slotted plane. Figure 4 shows the even mode frequencies as a function of slot length, both computed and measured in the laboratory. Note that the difference between the computed frequencies and the measured values is less than 1 per cent. The substantial agreement indicates that the 3D code can be expected to be useful in the design of structures lacking a high degree of symmetry.

WAVEGUIDE-CAVITY COUPLING

The application of a code, designed to calculate eigenmodes of oscillation, to the solution of rf problems involving wave motion is not straightforward. The energy of an eigenmode is a constant, whereas wave motion often implies the transport of energy. In the case of the jungle gym we were able to deduce the traveling-wave properties of the structure because one envisions waves of equal amplitudes propagating in opposite directions yielding standing waves resembling those in a finite structure. In the case of a waveguide terminated in a cavity we can use the fact that at some cross-section of the waveguide removed far enough from the cavity that one can consider only a single mode to be propagating (e.g. the dominant mode), the imput impedance of the waveguide-cavity combination can be expanded as[2]

$$Z_{in} = j\omega L_o \frac{(1-\omega^2/\omega_{s1}^2)\,(1-\omega^2/\omega_{s2}^2)\,\ldots}{(1-\omega^2/\omega_{o1}^2)\,(1-\omega^2/\omega_{o2}^2)\,\ldots}. \tag{1}$$

For a lossless system, the frequencies ω_{s1}, ω_{s2},etc. are frequencies at which the input impedance vanishes, i.e. there is no transverse electric field. One can calculate these frequencies, then, by calculating the resonant frequencies of the waveguide-cavity system taken as a single cavity with a shorting plane at the waveguide cross-section at which the input impedance is to be calculated. Similarly, the frequencies ω_{o1}, ω_{o2}, etc. are frequencies for which the impedance is infinite, and can be calculated by imposing the boundary condition $\mathbf{H}_{tangential} = 0$ at the specified waveguide cross-section. To take finite Q into account one can simply take the resonant frequency ω_i, for example, and multiply it by $(1+j/2Q_i)$ in the impedance expression Eq. (1), where Q_i is the quality factor of the mode in question.

For waveguides propagating TE modes, the characteristic impedance is given by[3]

$$Z_{TE} = \frac{j\omega\mu_o}{\sqrt{k_c^2 - \omega^2\epsilon\mu}}, \tag{2}$$

where ϵ and μ are the permittivity and permeability respectively of the medium filling the guide and k_c is the cutoff wavenumber of the TE mode. For these calculations we have therefore taken the quantity L_o in Eq. (1) as

$$L_o = \frac{\mu_o}{k_c} = \frac{\mu_o a}{\pi}, \tag{3}$$

where a is the width of the waveguide, which we consider to be propagating the TE_{10} mode.

The objective in matching the waveguide to the cavity, then, is to arrange that the waveguide input impedance Eq. (1) be purely real and equal to the simple waveguide characteristic impedance Eq. (2) at the desired frequency of operation, presumably at or near a resonant frequency of the cavity alone. As a numerical example, we have modeled a cavity built for the Los Alamos free-electron laser program, and designed to operate at 1300 MHz in the TM_{110} deflecting mode. The cavity is a simple pillbox with radius .1399 m and height .105 m and is coupled to a rectangular waveguide 6.500 in x 3.250 in. The coupling aperture is 1" wide and was cut by a 1" end mill tool traveled through an 8 degree arc relative to the center line of the cavity. This structure achieved a VSWR of 1.04. This arrangement was roughly modeled with URMEL-3D. Table II gives the five lowest resonant frequencies for the system consisting of the cavity with a .14 m long waveguide coupled to it through a slot 2.6 cm x 4.7 cm.

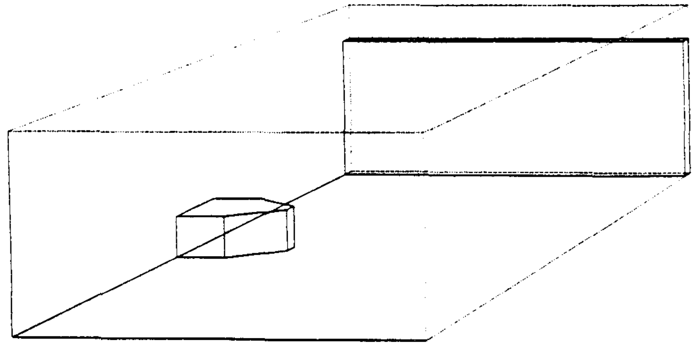

Figure 3: One fourth of one of two cavities coupled by a slot

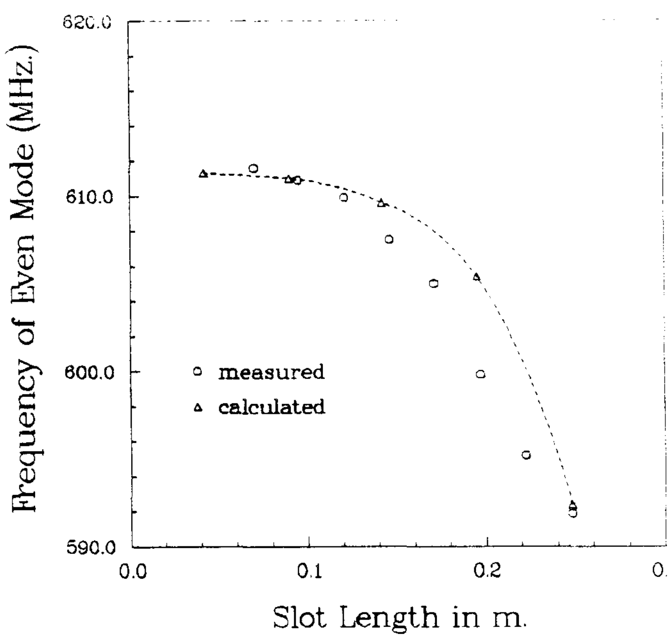

Figure 4: Computed and measured values of the lowest mode of the coupled-cavity system

Figure 5 is a plot of the input impedance Eq. (1) near the TM_{110} resonance using the results tabulated in Table II, along with the waveguide characteristic impedance. A matched situation is obviously not achieved. Figure 6 shows the same impedances resulting from moving the corresponding frequencies further apart by 20kHz. A much better matching condition results. It is worrisome to have such a large effect result from such a small difference; it may be that the code can in fact produce relative frequencies more accurately than absolute frequencies, but accuracies of 1 part in 10^4 seem at present difficult to achieve. We are presently investigating whether this close spacing is a natural concomitant of the method.

SUMMARY

In addition to the quantitative results produced by URMEL-3D, the field plots generated by P3 give one a qualitative feeling for the structure of the modes. Figure 7 shows the electric and magnetic fields of the 2π mode of the Jungle Gym, shown in a plane containing the horizontal bars. Figure 8 shows one quarter of the waveguide-cavity soupled system, as well as the electric field arrow plot of the TM_{110} mode in the cavity midplane. Figure 9 shows the electric and magnetic field plots of the TM_{210} mode.

ACKNOWLEDGEMENT

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References

[1] M. Tigner, Bar Loaded Waveguide for Accelerator Service, *IEEE Trans. Nucl. Sci.*,**NS-18** No. 3 (1971).

[2] C.G. Montgomery, R.H. Dicke, and E.M. Purcell, *Principles of Microwave Circuits*, M.I.T. Radiation Lab. Series, McGraw-Hill, 1948, p 216.

[3] C.G. Montgomery, R.H. Dicke, and E.M. Purcell, *op. cit.* p 31.

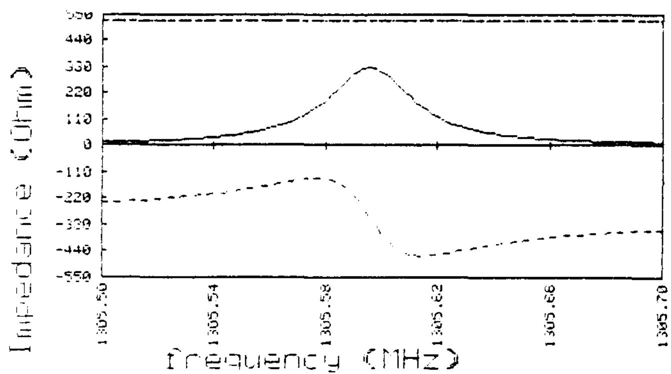

Figure 5: Input impedance to waveguide as a function of frequency near the TM_{110} resonance of the pillbox cavity. The real part of the impedance is shown solid, the imaginary dashed. The waveguide characteristic impedance is shown with a dot-dash pattern near the top of the figure.

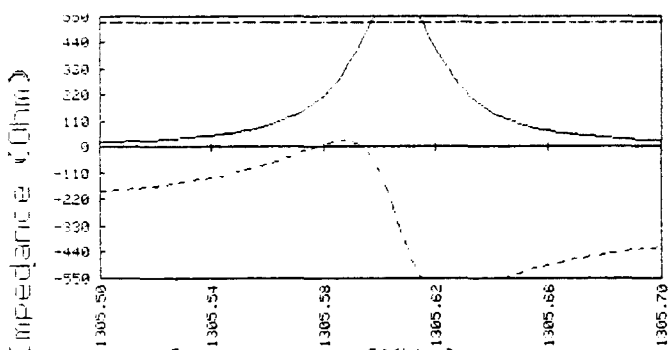

Figure 6: Same plot as previous figure, with open and short circuited frequencies moved 20 kHz further from each other.

Table 2: Resonant frequencies of waveguide-cavity coupled system

waveguide opened			waveguide shorted		
mode	frequency	Q	mode	frequency	Q
TM_{010}	827.609	26843	TM_{010}	827.610	26844
WG	1092.194	21819	TM_{110}	1305.576	35461
TM_{110}	1305.596	35465	WG	1494.168	28360
TM_{210}	1723.992	38565	TM_{210}	1724.057	38571
TM_{020}	1883.788	41717	TM_{020}	1884.276	41580

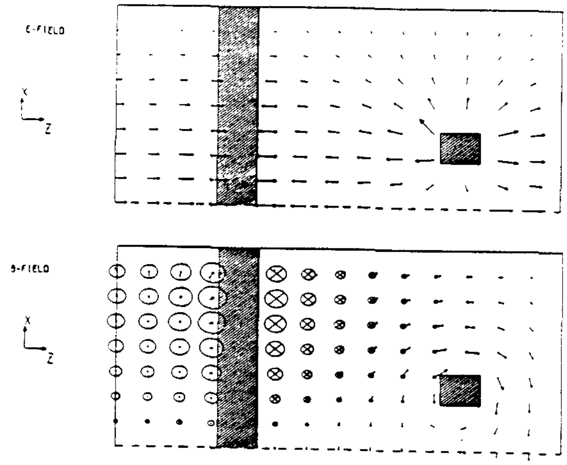

Figure 7: Electric and magnetic field plots of the 2π mode of the Jungle Gym

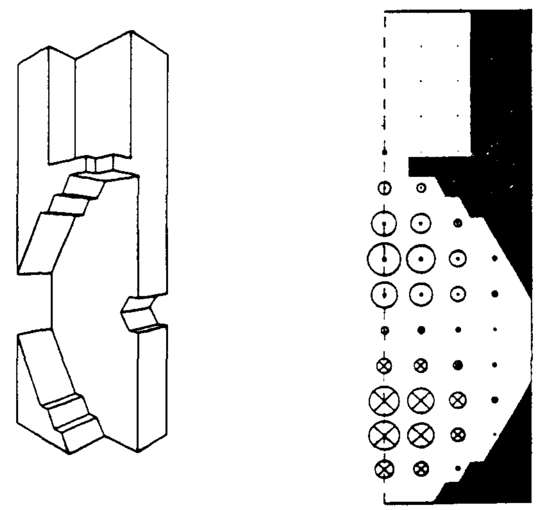

Figure 8: One quarter of the waveguide-cavity system and the electric field plot of the TM_{110} mode

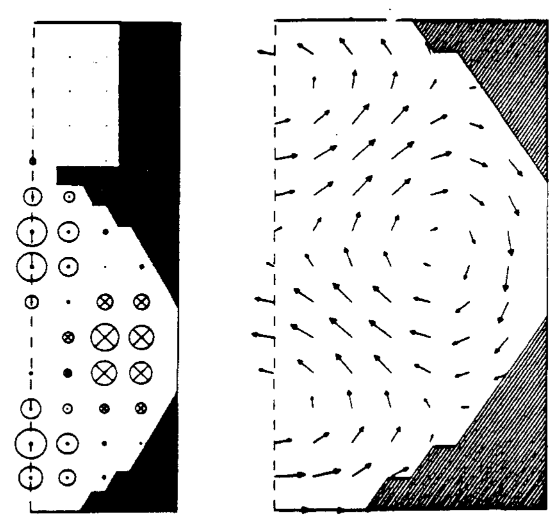

Figure 9: Electric and magnetic field plots of the TM_{210} mode