MICROWAVE UNDULATOR

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Abstract

At the present time the use of a static magnetic field has been the dominant method by which undulator synchrotron radiation has been produced. A microwave undulator utilizing a plane rectangular waveguide operating in the TE_{10n} mode presents a possible alternate means of producing this synchrotron radiation.[1,2] Also, since a typical resonator may be many guide wavelengths in length it is possible to make the undulator source tunable over a broad spectral range.

Theory

Consider a rectangular waveguide operating in the TE_{10} mode and having the dimensions shown in Figure 1. It will operate with field patterns as shown in Figure 2, where for guides operating with air or vacuum dielectric the guide wavelength λ_g is given by the relation:

$$\lambda_g = \frac{\lambda_o}{\sqrt{1 - (\lambda_o/\lambda_c)^2}} \qquad (1)$$

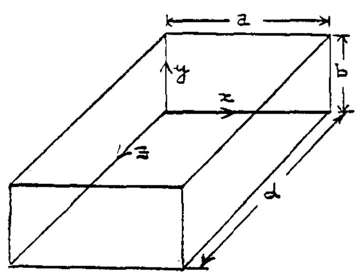

Figure 1. Rectangular Waveguide Resonator.

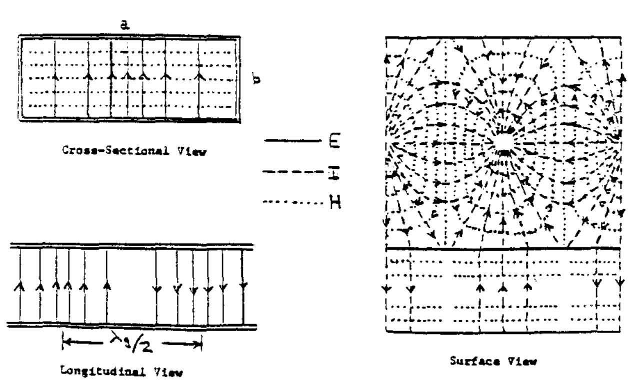

Figure 2. TE_{10n} Mode in Rectangular Waveguide.

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λ_o is the free space wavelength and in general for a $TE_{\ell mn}$ mode we have the relationship

$$\left(\frac{1}{\lambda_c}\right)^2 = \left(\frac{\ell}{2a}\right)^2 + \left(\frac{m}{2b}\right)^2 \qquad (2)$$

For the TE_{10n} mode we see that the cut-off wavelength, $\lambda_c = 2a$ and hence

$$\lambda_g = \frac{\lambda_o}{\sqrt{1 - (\lambda_o/2a)^2}} \qquad (3)$$

The electric and magnetic fields are given by the relations

$$E_y = E_o \sin \frac{2\pi z}{\lambda_g} \sin \omega t \qquad (4)$$

$$B_x = B_o \cos \frac{2\pi z}{\lambda_g} \cos \omega t \qquad (5)$$

where E_o and B_o are the peak electric and magnetic field strengths of the microwave field, respectively.

The equation of motion of a relativistic electron is given by

$$\frac{dP}{dt} = -e(E + v \times B) \qquad (6)$$

substituting equations (4) and (5) in (6) gives the transverse motion of electrons in the microwave undulator as:

$$\frac{d\beta_y}{dt} = \frac{eE_o}{2m_o c\gamma}\left(\left(1 + \frac{Z_o}{Z_w}\right)\cos\left(\omega t + \frac{2\pi z}{\lambda_g}\right) - \left(1 - \frac{Z_o}{Z_w}\right)\cos\left(\omega t - \frac{2\pi z}{\lambda_g}\right)\right) \qquad (7)$$

(where $B_o = \mu_o H_o = \sqrt{\mu_o \varepsilon_o}\, E_o$ from Maxwell's equations; the group velocity $v_z = \lambda_o/\lambda_g$; Z_w the transverse wave impedance to is given by $Z_w = (\lambda_g/\lambda_o)Z_o$ and β_y is the electron velocity in the transverse direction divided by the velocity of light.) This shows that the electron is undulated by both a forward and backward wave component of the standing microwave fields.

Each of these waves will give rise to coherent radiation but we are interested in obtaining the short wavelength radiation contributed by the backward wave component.

Although the forward wave contribution to the undulator radiation spectrum may be made small by operating in a region where λ_g is very nearly equal to λ_o there will always be a component in the radiation spectrum resulting from the interaction of this wave component. Let us investigate the nature of this radiation. If we start with the equation of motion given by expression (7) we may compare this equation with the equivalent expression for the conventional magnetic undulator which is given by

$$\frac{d\beta_y}{dt} = \frac{eB_u}{m_o\gamma} \cos \frac{2\pi z}{\lambda_u} \tag{8}$$

Let us define an equivalent forward undulator field B_{uf} as

$$B_{uf} = \frac{E_o}{2c}\left(1 - \frac{Z_o}{Z_\omega}\right) = \frac{E_o}{2c}\left(\frac{\lambda_g - \lambda_o}{\lambda_g}\right) \tag{9}$$

This may be compared with the equivalent expression for the backward wave.

$$B_{ub} = \frac{E_o}{2c}\left(1 + \frac{Z_o}{Z_\omega}\right) = \frac{E_o}{2c}\left(\frac{\lambda_g + \lambda_o}{\lambda_g}\right) \tag{10}$$

we also may define the equivalent forward and backward wave wavelengths as

$$\frac{1}{\lambda_{uf}} = \frac{1}{\lambda_o} - \frac{1}{\lambda_g} \quad \text{or} \quad \lambda_{uf} = \frac{\lambda_o\lambda_g}{\lambda_g - \lambda_o} \tag{11}$$

$$\frac{1}{\lambda_{ub}} = \frac{1}{\lambda_o} + \frac{1}{\lambda_g} \quad \text{or} \quad \lambda_{ub} = \frac{\lambda_o\lambda_g}{\lambda_g + \lambda_o} \tag{12}$$

For a conventional undulator we also define a factor;

$$K = \frac{eB_u\lambda_u}{2\pi m_o c} = 0.093\ B_u(\text{KG})\ \lambda_u\ (\text{cm}) \tag{13}$$

Note that here $K_{uf} = 0.093\ B_{uf}\ \lambda_{uf} = \lambda_o E_o/2c = 0.093\ B_{ub}\ \lambda_{ub} = K_{ub}$ i.e. the K factor for both forward and backward wave components is the same. For a single pass device there will be components in the spectrum of wavelengths λ_{uf} and λ_{ub} of amplitudes defined by the ratio $(\lambda_g-\lambda_o)/(\lambda_g+\lambda_o)$. This will give a long wavelength noise superimposed as the desired short wavelength radiation spectrum. For an undulator in a storage ring this component could result in a net deflection of the electron beam which may ultimately destroy the beam.

In order that these waves give no net deflection of the beam we require that $m\lambda_u$ backward $= \lambda_u$ forward where m is a positive integer, i.e.,

$$\frac{m\lambda_o\lambda_g}{\lambda_o+\lambda_g} = \frac{\lambda_o\lambda_g}{\lambda_g-\lambda_o} \quad \text{or} \quad \lambda_g \; \frac{\lambda_o(m+1)}{m-1} \tag{14}$$

But

$$\lambda_g = \frac{\lambda_o}{\sqrt{1 - (\lambda_o/\lambda_c)^2}} = \frac{\lambda_o}{\sqrt{1 - (\lambda_o/2a)}}$$

for the TE_{10} mode. So we require that

$$\frac{m+1}{m-1} = \frac{1}{\sqrt{1 - (\lambda_o/2a)^2}} \quad \text{or} \quad \lambda_o = 2a\sqrt{1 - \left(\frac{m-1}{m+1}\right)^2} \tag{15}$$

We may choose λ_o so that equation (15) is satisfied and then, outside of the undulator region itself, there is no net deflection of the electron beam by either the forward or backward waves. There is, however, a contribution to the undulator output spectrum from the forward wave interaction. Note that K is the same for both forward and backward wave contributions so that the wavelength and power of the undulator radiation from the forward and backward waves depends only on the undulator wavelengths λ_{ub} and λ_{uf} and equivalent undulator field strengths B_{ub} and B_{uf}. The allowable operating wavelengths and equivalent undulator wavelengths and magnetic field strengths are given in Table I for a waveguide of broad dimension(a) cms.

Table I. Possible Operating Wavelengths for a Standing Wave Undulator.

m	λ_o	λ_g	λ_{ub}	λ_{uf}	B_{ub}	B_{uf}	K
2	1.8856a	5.6568a	1.4142a	2.8284a	$1.3333\ \frac{E_o}{2c}$	$0.6667\ \frac{E_o}{2c}$	$0.0877\ \frac{E_o.a}{c}$
3	1.7321a	3.4642a	1.1547a	3.4642a	$1.5000\ \frac{E_o}{2c}$	$0.5000\ \frac{E_o}{2c}$	$0.0805\ \frac{E_o.a}{c}$
4	1.6000a	2.6667a	1.0000a	4.0000a	$1.6000\ \frac{E_o}{2c}$	$0.4000\ \frac{E_o}{2c}$	$0.0744\ \frac{E_o.a}{c}$
5	1.4907a	2.2361a	0.8944a	4.4718a	$1.6667\ \frac{E_o}{2c}$	$0.3333\ \frac{E_o}{2c}$	$0.0693\ \frac{E_o.a}{c}$
6	1.3997a	1.9595a	0.8165a	4.8997a	$1.7143\ \frac{E_o}{2c}$	$0.2857\ \frac{E_o}{2c}$	$0.0651\ \frac{E_o.a}{c}$
7	1.3229a	1.7639a	0.7559a	5.2913a	$1.7500\ \frac{E_o}{2c}$	$0.2500\ \frac{E_o}{2c}$	$0.0615\ \frac{E_o.a}{c}$
8	1.2571a	1.6163a	0.7071a	5.6565a	$1.7778\ \frac{E_o}{2c}$	$0.2222\ \frac{E_o}{2c}$	$0.0585\ \frac{E_o.a}{c}$
9	1.2000a	1.5000a	1.6667a	6.000a	$1.8000\ \frac{E_o}{2c}$	$0.2000\ \frac{E_o}{2c}$	$0.0558\ \frac{E_o.a}{c}$
10	1.1499a	1.4054a	0.6325a	6.3252a	$1.8182\ \frac{E_o}{2c}$	$0.1818\ \frac{E_o}{2c}$	$.0.0535\ \frac{E_o.a}{c}$
11	1.1055a	1.3266a	0.6030a	6.6330a	$1.8333\ \frac{E_o}{2c}$	$0.1667\ \frac{E_o}{2c}$	$0.0514\ \frac{E_o.a}{c}$
12	1.0659a	1.2597a	0.5774a	6.9284a	$1.8462\ \frac{E_o}{2c}$	$0.1538\ \frac{E_o}{2c}$	$0.0496\ \frac{E_o.a}{c}$
13	1.0302a	1.2019a	0.5547a	7.2111a	$1.8571\ \frac{E_o}{2c}$	$0.1429\ \frac{E_o}{2c}$	$0.0479\ \frac{E_o.a}{c}$
14	0.9978a	1.513a	0.5345a	7.4837a	$1.8667\ \frac{E_o}{2c}$	$0.1333\ \frac{E_o}{2c}$	$0.0464\ \frac{E_o.a}{c}$
15	0.9682a	1.1066a	0.5164a	7.7412a	$1.8750\ \frac{E_o}{2c}$	$0.1250\ \frac{E_o}{2c}$	$0.0450\ \frac{E_o.a}{c}$

In order to eliminate the effect of the forward wave we would need to operate with λ_o very nearly equal to λ_g or $\lambda_o \ll 2a$.

Let us consider further the effect of both forward and backward waves on the undulator spectrum. Returning to equation (7) we may integrate this equation of motion to obtain.

$$y = \frac{eE_o\lambda_o^2}{8\pi^2 m_o c^2\gamma}\left(\frac{-\lambda_g}{\lambda_o+\lambda_g}\cos\left(\frac{1}{\lambda_o}+\frac{1}{\lambda_g}\right)2\pi z + \frac{\lambda_g}{\lambda_g-\lambda_o}\cos\left(\frac{1}{\lambda_o}-\frac{1}{\lambda_g}\right)2\pi z\right) \tag{16}$$

$$\frac{dy}{dz} = \frac{eE_o\lambda_o}{4\pi^2 m_o c^2\gamma}\left(\sin\left(\frac{1}{\lambda_o}+\frac{1}{\lambda_g}\right)2\pi z - \sin\left(\frac{1}{\lambda_o}-\frac{1}{\lambda_g}\right)2\pi z\right) \tag{17}$$

From Equation (16) we see that the maximum amplitude of the backward and forward wave oscillations are given by:

$$y_b\text{MAX} = \frac{eE_o\lambda_o^2}{8\pi^2 m_o c^2\gamma}\left(\frac{\lambda_g}{\lambda_o+\lambda_g}\right) = \frac{K\lambda_o}{2\pi\gamma}\left(\frac{\lambda_g}{\lambda_o+\lambda_g}\right) \tag{18}$$

$$y_f\text{MAX} = \frac{eE_o\lambda_o^2}{8\pi^2 M_o c^2\gamma}\left(\frac{\lambda_g}{\lambda_g-\lambda_o}\right) = \frac{K\lambda_o}{2\pi\gamma}\left(\frac{\lambda_g}{\lambda_g-\lambda_o}\right) \tag{19}$$

We may also determine the maximum angular deflection of both forward and backward waves from Equation (17) as

$$\left(\frac{dy}{dz}\right)\text{MAX} = \frac{eE_o\lambda_g}{4\pi m_o c^2\gamma} = \frac{K}{\gamma} \tag{20}$$

Expression (21) is exactly that obtained for the static magnetic field undulator in reference (3). It is also possible to operate the undulator in a travelling wave mode and thus avoid the effects of the forward wave. Alternatively an external magnetic field could be applied in order to suppress the effect of the forward wave.

Radiation Characteristics

The characteristics of the radiation from the microwave undulator are exactly the same as those from a static magnetic undulator.[3] According to Hofmann[4] the wavelength and intensity of the undulator radiation are given by

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right) \tag{21}$$

$$\frac{dP}{d\Omega} = \frac{e^2c\gamma^4K^2}{\varepsilon_o\lambda_u^2\left(1+\frac{K^2}{2}\right)^3}\cdot\frac{1+2\overset{*}{\gamma}{}^2\theta^2(1-2\cos^2\phi)+\overset{*}{\gamma}{}^4\phi^4}{\left(1+\overset{*}{\gamma}{}^2\theta^2\right)^5} \tag{22}$$

where
$$\overset{*}{\gamma}{}^2 = \frac{\gamma^2}{1+\frac{K^2}{2}} \tag{23}$$

The angles θ and ϕ are observation angles defined in Figure 3 and ε_o is the permittivity of a vacuum. Integrating equation (23) over ϕ and substituting from (22) we get

$$\frac{dP}{d\lambda} = \frac{3P_o}{\lambda}\left(\frac{\lambda_1}{\lambda}\right)^2\left(1 - \frac{2\lambda_1}{\lambda} + \frac{2\lambda_1^2}{\lambda^2}\right) \tag{24}$$

where P_o is the total radiated power from an electron given by

$$P_o = \frac{1}{4\pi\varepsilon_o}\cdot\frac{e^4\gamma^2B_u^2}{3m_o^2c\left(1+\frac{K^2}{2}\right)^2} \tag{25}$$

and λ_1 is the wavelength of the on-axis radiation, i.e.

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right) \tag{26}$$

Since the particles in the beam have a statistical longitudinal distribution of a length much greater than the wavelength of the radiation, the radiation is not coherent and we can add up the powers radiated by each particle. So for a current I and an undulator length d we obtain

$$P_{o_I} = \frac{Id}{4\pi e_o}\cdot\frac{e^4\gamma^2B_u^2}{3m_oc\left(1+\frac{K^2}{2}\right)^2} \tag{27}$$

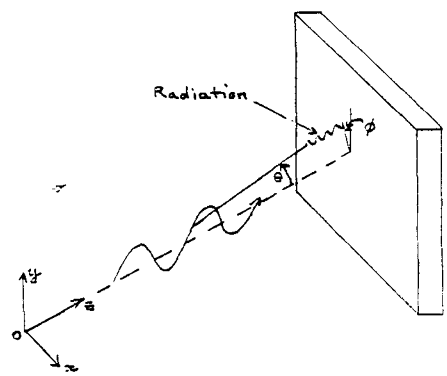

Figure 3. Definition of Observation Angles θ and ϕ. (Electrons undulate in the $\phi = 0$ plane.)

Higher Order Modes

For a resonator installed in a storage ring other possible resonant modes are of concern since the beam may excite them. For a plane guide of dimensions a,b,d as shown in Figure 1 there are many possible modes with resonant wavelength, λ_o, given by the relation

$$\lambda_o = \frac{2}{\sqrt{\left(\frac{\ell}{a}\right)^2+\left(\frac{m}{b}\right)^2+\left(\frac{n}{d}\right)^2}} \tag{28}$$

for either transverse electric $TE_{\ell mn}$ or transverse magnetic $TM_{\ell mn}$ modes. Those with the longest cut off wavelength are the TM_{11n} and TE_{11n} families of modes both of which have a longest cut-off wavelength, λ_c given by

$$\lambda_c = \frac{2a}{\sqrt{1+\left(\frac{a}{b}\right)^2}} \tag{29}$$

For a standard guide with a/b = 2 this gives λ_c = 0.894a so that the useful wavelength region for operation of a resonator in the TE_{10n} mode is given by $0.895a < \lambda_o < 2a$. Note that for the TE_{20n} family the cut-off wavelength λ_c = a, so it may be desirable to restrict the operating wavelength region to $a < \lambda_o < 2a$. This clearly conflicts with the need to make λ_g very nearly equal to λ_o or $\lambda_o \ll 2a$.

Possible Undulator Configuration

Let us consider a standard waveguide of internal dimensions a = 2 cm, b = 1 cm and a length of 100 cm. The lowest practical operating wavelength, λ_o, is given by equation (15) which defines the cut-off wavelength for the TE_{11n}, TM_{11n} mode families at 1.95 cm. Let us choose λ_o = 2 cm which is well below the cut-off wavelength of 4 cm for the TE_{100} mode.

The resulting undulator parameters are as follows:

Operating Frequency, fo	15 GHz
Undulator Length	100 cm
Guide Wavelength, λ_g	2.31 cm
Equivalent Undulator Field, Bu	466 Gauss
Undulator Quality Factor, K	0.05
Undulator Wavelength, λ_n	1.07 cm
Shunt Resistance, R/b	306 MΩ/m
O Value	14750
Number of Undulations, n	87
Peak Electric Field Strength, E_o	15 MV/m
Power	367 KW

We are interested in attaining undulator wavelengths in the order of 0.5 cm to 1 cm so let us consider a waveguide half filled with a ceramic dielectric or a ferrite material as shown in Figure 4.

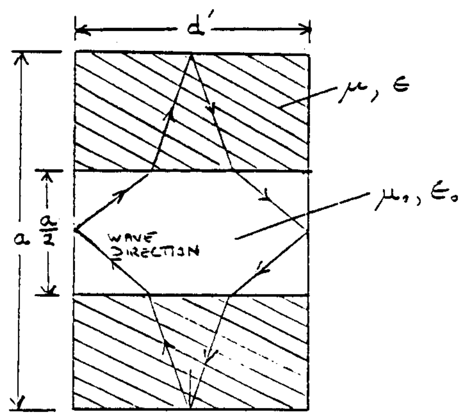

Figure 4. Cross-section of a dielectric loaded resonator of length 'd'.

The resonant wavelength for a fully filled waveguide of material of permeability μ and permittivity ε is given by:

$$d = \frac{\lambda_g}{2} = \frac{\lambda_o \sqrt{\left(\frac{\mu_o \varepsilon_o}{\mu\varepsilon}\right)}}{2\sqrt{1 - \left(\frac{\lambda_o}{2}\sqrt{\frac{\mu_o\varepsilon_o}{\mu\varepsilon}}\right)^2}} \tag{30}$$

For a guide which is half dielectric and half vacuum we have

$$d' = \frac{\lambda_g}{2} = \frac{\lambda_o}{4\sqrt{1 - \left(\frac{\lambda_o}{2a}\right)^2}} + \frac{\lambda_o \sqrt{\left(\frac{\mu_o\varepsilon_o}{\mu\varepsilon}\right)}}{4\sqrt{1 - \left(\frac{\lambda_o}{2a}\sqrt{\frac{\mu_o\varepsilon_o}{\mu\varepsilon}}\right)^2}} \tag{31}$$

For a 2 cm x 1 cm waveguide half filled with a dielectric which had a value of $\sqrt{\frac{\mu\varepsilon}{\mu_o\varepsilon_o}} = 3$ operated at a wavelength of 2 cm we obtain an undulator wavelength $\lambda_u = \lambda_g/2 = 0.746$ cm.

Conclusion

The microwave undulator represents a viable option for undulator wavelength down to about 1 cm where peak voltage and available microwave power considerations limit its effectiveness. For undulator wavelengths in the 5 to 10 cm region a pulsed microwave undulator is an attractive proposition for either storage ring or linac, single pass, operation.

References

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