RF CAVITY DESIGN AND CODES

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ABSTRACT

The design of rf cavities by means of computer codes is already well established. Recently, more and more complete and versatile computer codes have been written that cover almost all possible problems that can be solved by two-dimensional meshing. Cavities without any simple symmetry (such as cylindrical or translational) have to be treated fully three-dimensionally. To date, the MAFIA CAD system by the DESY/KfA-Jülich/LANL collaboration seems to be the most highly developed family of codes. Three-dimensional computations of resonant modes have proven their accuracy in comparison with many measurements. The accuracy seems to be limited only by the size of available computers. IBM computers can handle up to 100.000 grid points, and the CRAY-2 up to 2.000.000. nodes, but no experience has been gathered so far with such large meshes. Three-dimensional computations for transient cavity wake fields have been used in designing accelerator components for some time but due to complexities, there are no measurements with which to compare. The largest meshes one can use for transient fields are much larger than for resonance calculations (over 1.000.000. on IBM, say). Under the assumption that bigger computers with memories of some ten megawords will soon become widely available, 3D codes can come into routine use as have the two-dimensional codes for years. However, even with the restricted core size, these codes can help to gain significant understanding of truly three-dimensional structures.

INTRODUCTION

In the design of rf-accelerating cavities, the major figure of merit has been the shunt-impedance, which is a measure of the efficiency of transferring power into an accelerating voltage for charged particles. However, in most accelerators the particle energy is only one of many parameters. Of similar importance in many cases is the particle beam current, which often is limited by collective phenomena such as beam break-up. Such collective effects are often caused by complicated electromagnetic interactions between the beam and accelerating cavities. Thus in designing accelerating cavities, one has to envisage the entire accelerator and optimize the system as a whole towards the design goal. In some cases it may turn out during such an optimization that a "bad" accelerating cavity with only half the optimal achievable shunt impedance is in the overall view better by a factor of two. This can happen when parasitic effects severely limit the beam current and when at the same time parasitic side effects of the cavity drop faster than the shunt impedance while lowering the latter. Figure 1 shows three accelerating cavities. The DESY-cavity (Figure 1a) is a simple set of pillboxes and was built in the 60's with a measured shunt impedance of about 10MΩ/m (Using the definition $P = V^2/2R$). The PETRA cavity (Figure 1b) built in the late 70's has the typical nose cones and optimized shunt impedance of about 11MΩ/m (measured). However, while optimizing the shunt impedance parasitic effects have been raised by about a factor of two. This can cause a loss in peak current by a factor of two which may be a very high price for saving ten percent in power. With the presently available computational tools it is possible to optimize a cavity with respect to all these effects and figure 1c shows a typical result of such a cavity (a "Single-Mode-Cavity" [1]). The shunt impedance is reduced by roughly 10 percent but the parasitic effects are reduced by more than a factor of two. Thus such a cavity may be much more economic in terms of the overall efficiency of an accelerator.

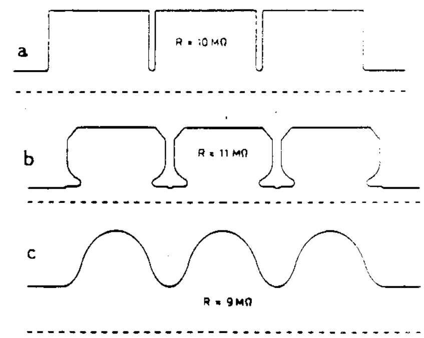


Figure 1: Three accelerating cavities
1a) DESY cavity as built in the 60's
1b) PETRA cavity with typical nose cones
(optimized with respect to shunt impedance only).
Parasitic effects are twice as strong as in cavity 1a.
1c) "Single-Mode-Cavity" optimized with respect to shunt impedance and parasitic effects.

In addition, all unwanted but unavoidable cavities along a beam line used for pumping or monitoring also contribute significantly to the collective interaction. Especially the contribution to the transverse impedance is often underestimated. Figure 2 shows three accelerator components: an accelerating cell, a short bellows and a step in a vacuum pipe. For a given set of parameters the transverse kick experienced by particles off axis is of the same order of magnitude. Obviously the volume of the object is not a good measure of the badness.

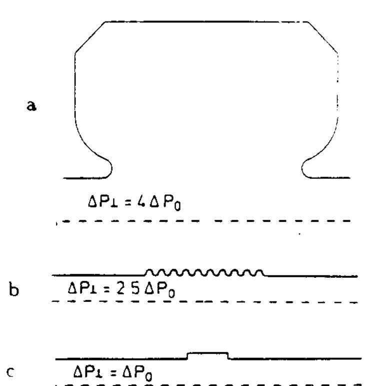


Figure 2: Three accelerator components having the same order of magnitude parasitic transverse effects on off-axis beams.
a) typical 500 MHz accelerating cavity
b) bellows
c) step in a vacuum pipe

Any cavity is a quite complicated object that leads to a manifold kind of beam-cavity interaction. This interaction can be characterized by the coupling impedance $Z(\omega)$. Below the cut-off frequency of the beam ports, $Z(\omega)$ is a collection of resonant modes with typically very high quality factors Q ($10^3..10^5$ for copper cavities). Damping of these resonances is due to wall currents and the finite conductivity of any real cavity wall. Above the cut-off frequency of the beam ports waves can travel away from the cavity and no ideal resonance can exist anymore. However, when coupling to the travelling modes is weak there may exist "quasi-resonant" modes with an appreciable Q value. In those cases damping is caused by radiation into the beam tubes and by wall currents. Figure 3a shows a typical nose cone type of accelerating cavity.

Treating the beam-cavity interaction in the time domain yields the characteristic function called the wake potential $w(s)$ (see figure 3b). This function $w(s)$ is defined as the net change in energy experienced by a test particle following a leading particle at a constant distance s, after both particles have left the interaction area and time dependences have settled. Note that the transient time dependence of the electromagnetic forces has been integrated out in this definition. The complementary characteristic function is the coupling impedance $Z(\omega)$. As can be seen, $Z(\omega)$ is a complicated function with many peaks above cut-off. All of this function may be important in the design of a cavity. In some cases only the overall behaviour is important, in other cases each single peak has to be considered.

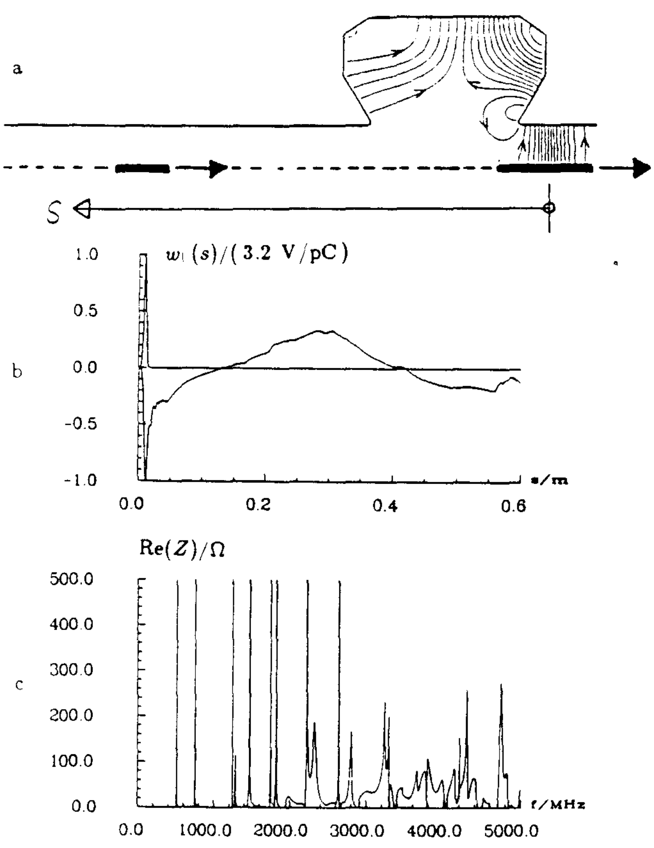


Figure 3:
a) Typical accelerating 500 MHz nose-cone cavity with fields excited by a short (σ = 1cm) bunch of particles. These fields act back on particles inside and behind the bunch at a distance s.
b) The wake potential as seen by a test particle at distance s behind a very short (σ = 0.2cm) leading bunch of 1μC charge.
c) The real part of the longitudinal coupling impedance as a function of frequency. Note the sharp peaks below the cut-off frequency of the beam port (2Ghz) and the continous behaviour above. Wake potential and impedance are related by a simple Fourier transform.

In realistic cavities one cannot calculate wake potentials for true point charges as one cannot calculate $Z(\omega)$ for infinitely high frequencies. Instead of point charges one has to use short bunches where the shortest bunch length in use corresponds to the highest frequency for which $Z(\omega)$ could be computed.

Impedance and wake potential are simply related by [2]:

$$Z(\omega) = \int_{-\infty}^{\infty} w(s)\, e^{i\omega s} ds \qquad (1)$$

Although this relation shows that only one of the quantities $Z(\omega)$ or $w(s)$ is necessary to treat the beam cavity interaction, both are useful depending on the specific problem under consideration. When very short bunches of particles pass a cavity, the interaction evokes very high frequencies and the use of $Z(\omega)$ would require the computation of Z over a very wide range (*single bunch effects*). In these cases $w(s)$ is much more adapted to the problem since $w(s)$ has to be computed only for a short time interval.

The long range, e.g. bunch to bunch interaction is dominated by the impedance at specific frequencies. In those cases the computation of $Z(\omega)$ is much more efficient. Here one would have to compute $w(s)$ for very long time intervals in order to obtain good frequency resolution in the Fourier transform (*multi bunch effects*).

In addition to the longitudinal interaction causing a change in energy there also exist (the much more severe) transverse electromagnetic forces for which all the above arguments count as well.

From the above we conclude that in order to design an accelerating cavity as a part of a complicated system, one needs to know $Z(\omega)$ and $w(s)$ for both longitudinal and transverse effects. Furthermore one needs to know both quantities for all other accelerator components as well.

A large number of computer codes have been written during the last two decades in order to calculate these cavity properties. Historical reviews have been given at other places (e.g. [3] [4]). In this paper we will restrict ourselves to the most highly developed codes only.

One may group computer codes according to the physical dimension of the problem:

•**2D codes** treat three dimensional structures with either rotational or translational symmetry. In both cases one spatial dependence of the fields can be taken out analytically, leaving only a 2-D problem for the mesh code. For cylindrical cavities one eliminates the azimuthal dependence by expanding fields into Fourier series over $e^{im\varphi}$ (m=0 monopole, m=1 dipole, etc.) and computes one such set of modes by one for each given m saperately. For translationally symmetric cavities the dependence in one cartesian coordinate is given by an exponential function.

•**3D codes** solve for truly three dimensional structures.

A second type of classification of codes has been described above and that is frequency and time domain:

•ω – **codes** calculate resonant frequencies (below cut-off) or directly $Z(\omega)$ for a given ω (below and above cut-off).

•**t** – **codes** calculate the electromagnetic fields as a function of time and result in the wake potential $w(s)$ via subsequent integration over the time variation.

We will use one more classification in the headings of this paper and that is:

•**state of the art** we call what is in routine use at at least a few accelerator laboratories.

•**news** we call the latest news from codes finished or significantly modified recently.

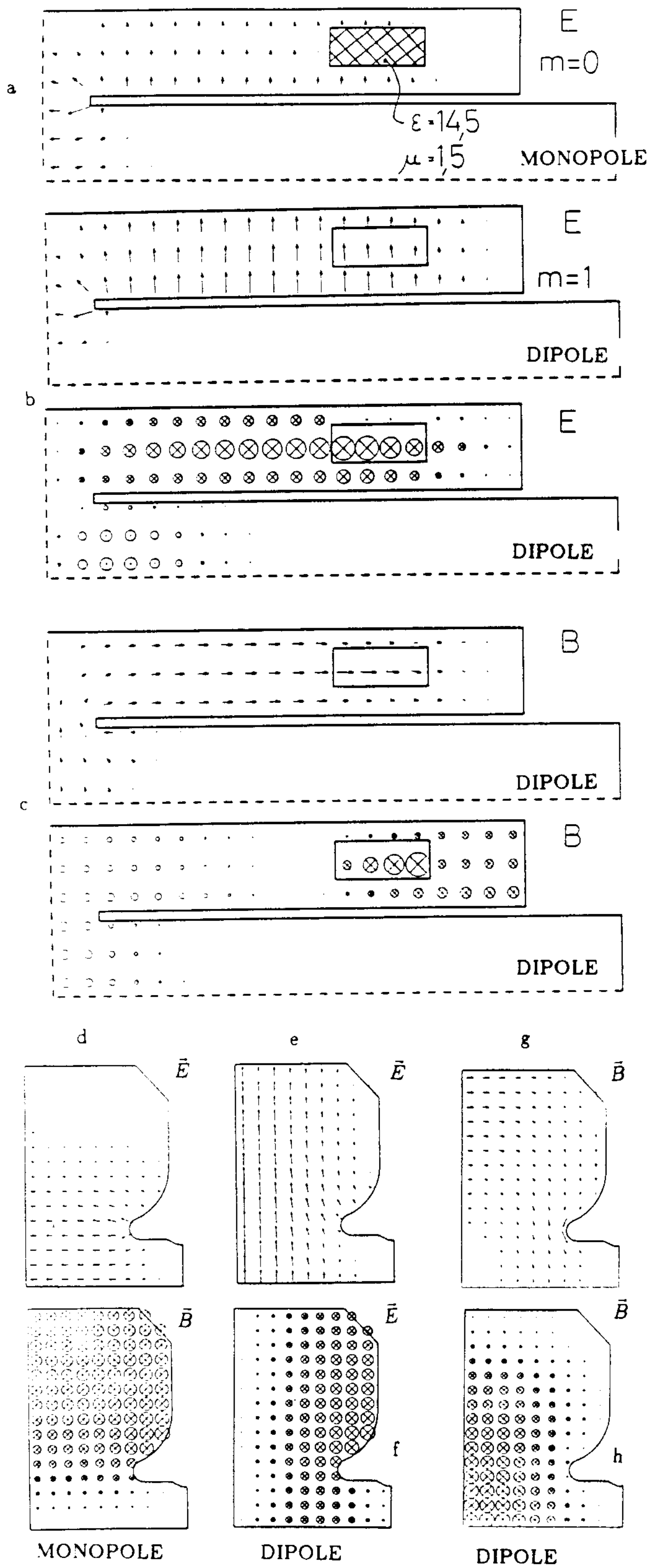


Figure 4:
a) typical nose cone cavity and fundamental TM_{010} mode, shown are the electric and magnetic field.
b) electric field of the lowest dipole mode at $\varphi = 0$ and $\varphi = 90^0$.
c) magnetic field of the lowest dipole mode at $\varphi = 0$ and $\varphi = 90^0$.
d) fundamental mode in a reentrant cavity with a ferrite insertion
e,f) electric field of lowest dipole mode
g,h) magnetic field of lowest dipole mode

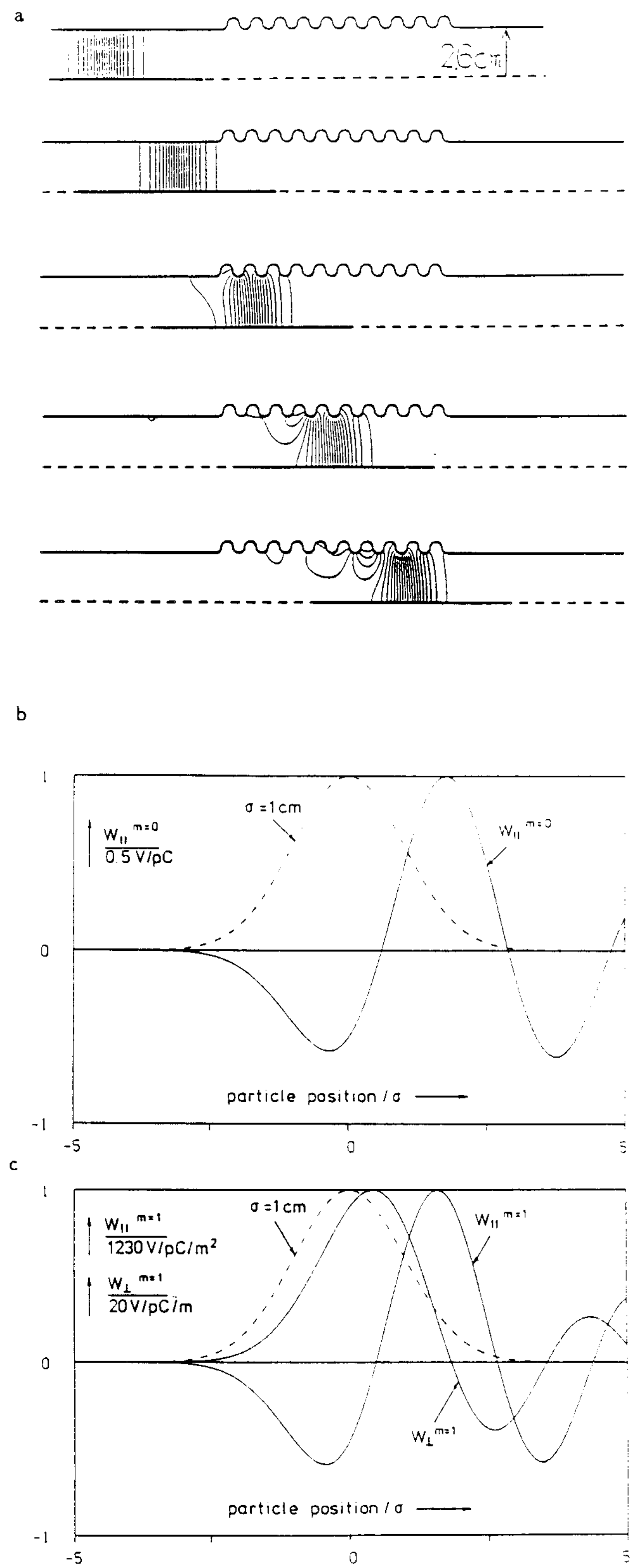


Figure 5:
a) Transient electric fields excited by a bunch of charged particles traversing a bellows.
b) The net effect of these fields yields the longitudinal wake function $w_{\parallel}(s)$ in the usual units of V/pC.
c) For a unit offset from the axis the dipole transverse and longitudinal wake functions are shown. Shown are the longitudinal and transverse dipole wake potentials for a bunch per unit offset from the axis.

2D-CODES : ω DOMAIN : STATE OF THE ART

SUPERFISH [5] is probably the most widespread code for design of cylindrical cavities. However, SUPERFISH can handle only cylindrically symmetric modes of type TM_{0mn} (monopole modes with m=0). For the study of the transverse deflecting resonant modes URMEL [6] and URMEL-T [7] are probably the most versatile codes in this domain. URMEL uses a rectangular grid and URMEL-T uses a much more flexible triangular mesh. The latter code also allows for permeable and permittive insertions. Both codes enable $(r - \varphi - z)$ and $(x - y - z)$ coordinates. In the cylindrical case they calculate the lowest N (50, say) resonant modes for any given azimuthal mode number m, i.e. for monopole, dipole etc.. In the cartesian system they calculate the waveguide propagation constant in the z-direction for a given frequency.

In contrast to SUPERFISH which looks for the zeros of a determinant, the URMEL codes yield a linear algebraic eigenvalue problem, the eigenvalues of which are the resonant frequencies. The algebraic eigenvectors represent the (r-z) or (x-y) components of the electric field. Thus the eigenvalue problem can be solved much more easily by the SAP processor [8] which yields a large number of modes in one run without missing any.

Both codes have undergone extensive testing and comparison with measurements. All these tests have proven the reliablity and accuracy of the programs [9] [10]. Figure 4 shows typical output from these codes.

2D-CODES : $t-$ DOMAIN : STATE OF THE ART

TBCI [11] is the most widespread code for calculating wake potentials and has been in routine use for several years. TBCI solves the time dependent Maxwell equations in a time stepping procedure. At each time step fields are evaluated from fields at earlier time steps by simple matrix multiplication. Figure 5 shows a typical result.

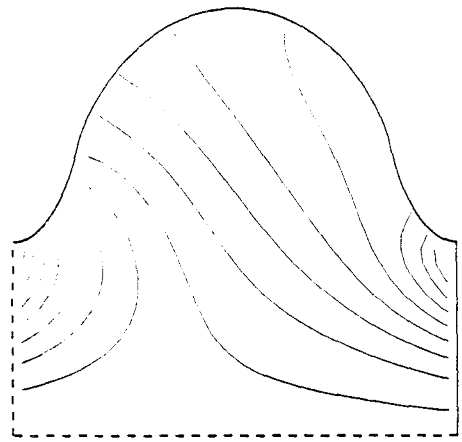
2D-CODES : ω DOMAIN : NEWS

As mentioned in the introduction, $Z(\omega)$ is calculated below the cut-off frequency of the beam pipe by searching eigensolutions by e.g. URMEL. Above cut-off such eigensolutions no longer exist. Here one has to excite the cavity by a current which is given by the Fourier-transform of a point charge traversing the structure. The inhomogeneous Maxwell equations have to be solved. From the calculated fields one easily obtains the impedance.

For pill-box cavities with side tubes a computer program has been written [12] based on Bessel function series. For arbitrarily shaped cylindrical cavities URMEL-I [13] can now be used. Similar work on extending SUPERFISH [14] is still in progress. Figure 3c shows a typical output of URMEL-I. Note the quasi resonances above cut-off which may be quite important for superconducting cavities when higher order mode losses are concerned. So far, losses into quasi-resonances above cut-off have been mostly neglected since there was no computational tool available for this regime.

For long linac structures a numerical computation may become less accurate if the number of mesh points is fixed since the number of nodes available for one cell becomes less and less the more cells are considered. In such cases it may be advantageous to model the long chain by an infinitely long one. Then only one cell has to be taken into account in the computation. Boundary conditions are then replaced by Floquet-conditions. Such extensions have been added to SUPERFISH [15] and to URMEL [16]. The latter one now computes for any given phase advance over one cell the N (50, say) lowest synchronous modes. Figure 6 shows a typical output. With this new version of URMEL one may now evaluate the group velocity much more accurately. Such application can be quite important for the design of future high energy accelerators in the TeV region.

These two new extensions of URMEL have so far been implemented only for monopole modes as a first test. Extension to m=1,2,.. (dipole, quadrupole,..) is in progress.



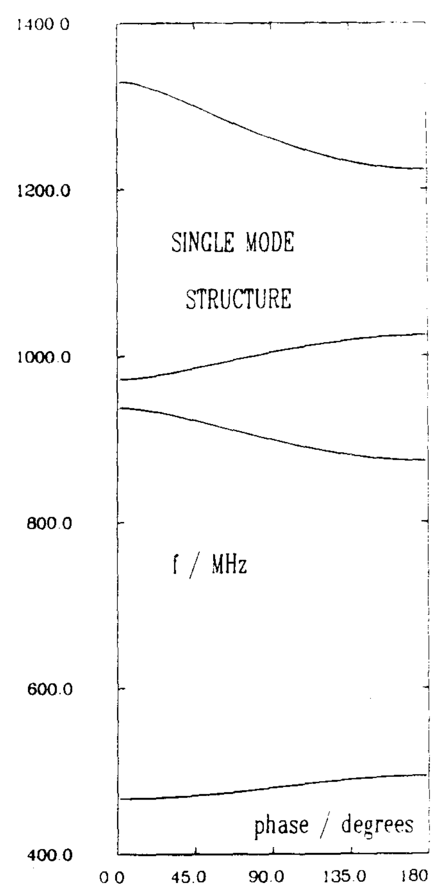


Figure 6: One cell of an infinitely long chain of accelerating cavities of "Singke Mode Type" [1]
a) Second lowest monopole mode at $\varphi = 60^0$ phase advance.
b) dispersion diagram of the lowest five modes.

2D-CODES : $t-$ DOMAIN : NEWS

Thanks to the users at some laboratories running TBCI for many different examples the (hopefully) last few and minor errors could be found and corrected [1].

New features that have been added allow for launching TM_{01}-waves or TEM-waves in a structure instead of a bunch of charged particles. Thus one may study the transient fields when filling a cavity or wave propagation along a linac structure. Figure 7 shows a typical result.

[1] K.Bane (SLAC) and B.Zotter(CERN) have been especially diligent at spotting errors

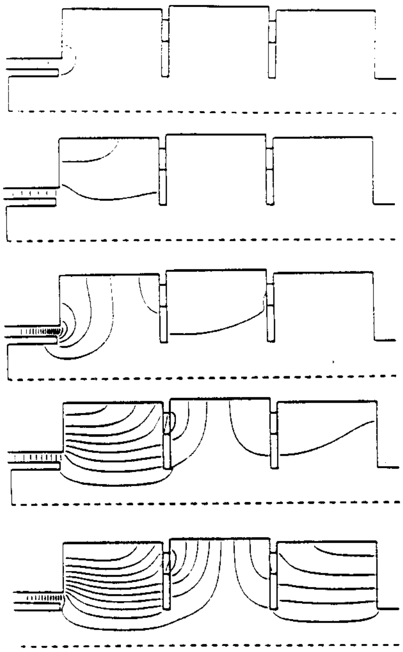


Figure 7: TEM wave launched on a coax cable into a three-cell cavity with coupling slots between the cells.

3D-CODES : ω– DOMAIN : STATE OF THE ART

The three dimensional electromagnetic CAD system MAFIA [17] is in use at a few laboratories and comparison with measurements have been done for the resonance program (3D-URMEL) [18]. Fully 3D resonators with permeable and permittive insertions can be treated. The code finds the N (50, say) lowest resonant modes in one run. The special way of discretizing Maxwell's equations guarantees that only truly resonant solutions are found and that there are no "ghost" modes. Final checks of computed eigensolutions sort out automatically eventually occuring unphysical fields. Figure 8 shows typical output of this code. For more examples see [19].

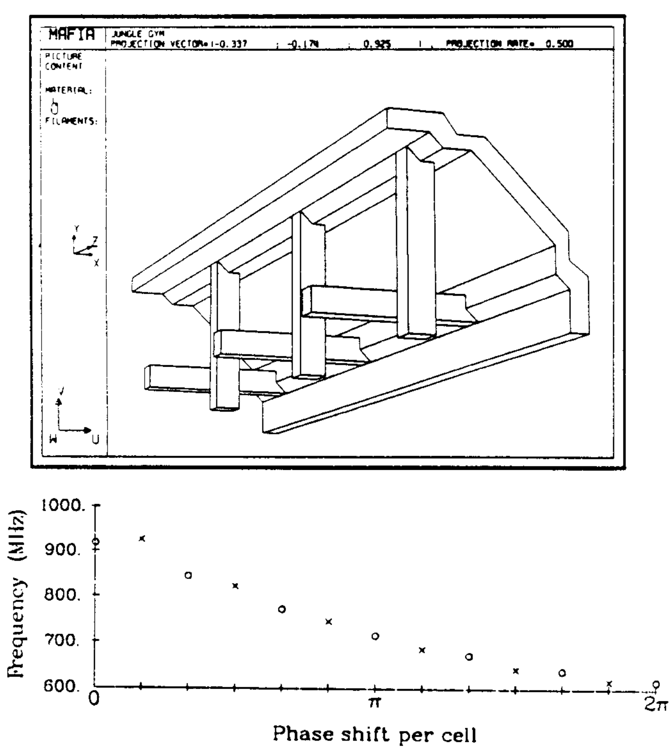


Figure 8: (A quarter of) a 3-cell Jungle-Gym cavity and the dispersion curve as computed by the 3D-URMEL of the MAFIA CAD system.

3D-CODES : ω– DOMAIN : NEWS

A new release of the 3D-URMEL of MAFIA enables now three times as many nodes with the same core size as before. On an IBM one can have about 14.000 nodes per Megabyte. On a CRAY this corresponds to 55.000 nodes per Megaword. Furthermore, a new eigenvalue solver has been written [20] which is specialized for very large meshes.

3D-CODES : t– DOMAIN : STATE OF THE ART – NEWS

Three-dimensional versions of BCI have hardly been used yet. This is due to various reasons. Some codes [21] were restricted to specific geometries. No code could give reliable results since the number of meshpoints that could be taken into account was much too small. Only very recently significant progress was achieved [22].

The major problem with 3D wake field computations was found to be discretization noise and core limitations. When bunches traverse a vacuum pipe with small changes in the cross- section, waves are excited and may travel for quite a while with the particles downstream the pipe. Thus one must calculate the electromagnetic field for long sections. Firstly this length is severely limited by memory capacity. Secondly, discretization noise becomes a problem the smaller the effects are that one is investigating and the longer the region of field computation is. Both problems could be overcome by a relatively simple "trick" similar to the one used in TBCI [23]. A separate paper [22] describes this new version in more detail. The new 3D-BCI is now also available as part of MAFIA. Figure 9 shows a typical result for the sliding contacts as used in PETRA. The result shows that even such small steps in a vacuum pipe can cause transverse impedances of the same order as relatively big accelerating cavities.

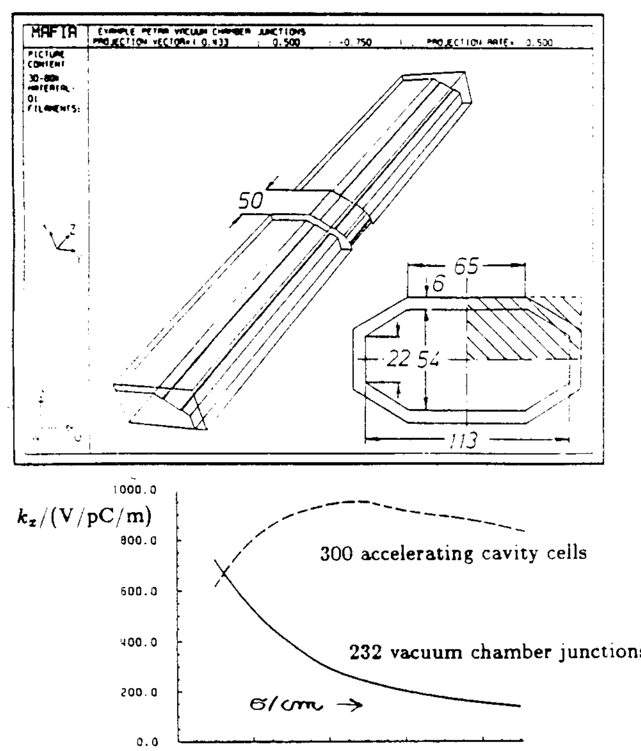


Figure 9:
a) Geometry of the PETRA sliding vacuum chamber joints and the quarter used in the computation.
b) Vertical average kick k_z experienced by a bunch passing PETRA vacuum chamber joints off axis as function of bunch length in comparison to the contribution of 300 accelerating cavity cells. Note that for short bunches 232 small steps of only a few millimeter in height cause as much transverse kick as 300 cavity cells, each of which has a diameter of 40 cm and a gap of 20 cm.

SUMMARY

Two-dimensional cavity codes are well developed and cover a very wide range of problems. Although one still may be able to improove the one or other codes in some detail, the area of two-dimensional field computation can be considered a solved problem.

3D codes are just now becoming used more widely and the future performance will depend highly on the number of users and their cooperation with the program authors. 3D cavity modes can be computed using reasonable amounts of cpu time. The maximum number of mesh points ranges from 100.000 (IBM) to up to 2.000.000 (CRAY2).

The 3D transient wake field codes have finally been settled and can now be used for even small objects in the vacuum system. The maximum number of meshpoints is almost unlimited. On an IBM computer one already can handle about 1.000.000 nodes.

The MAFIA collaboration now provides a coherent set of 3D codes with only one common input processor and one post processor. Together with the 2D codes URMEL, URMEL-T and TBCI they are now accessible in the USA on the MFE Cray. For users that have no access to the MFE computer center DESY provides source codes. (Please contact the author for more deatils).

ACKNOWLEDGEMENT

The author wishes to express his thanks to R.Cooper for careful reading of the manuscript.

REFERENCES

[1] T. Weiland, Single Mode Cavities - A Possibility for Fighting Collective Beam Instabilities, DESY Vol.83(073), September 1983

[2] K.Bane et al., Wake Fields and Wake Field Acceleration, American Institute of Physics, AIP Vol.127(1983),pp.875-928

[3] T.Weiland, Design of rf Cavities Lecture Notes in Physics, Vol.215(1983), pp.21-32

[4] T.Weiland, Computer Modelling of Two- and Three-Dimensional Cavities. IEEE NS-32(1985), p. 2738

[5] K.Halbach, R.F.Holsinger, SUPERFISH - A Computer Program for Evaluation of RF Cavities with Cylindrical Symmetry, Particle Accelerators 7 (1976), 213-222

[6] T.Weiland, On the Computation of Resonant Modes in Cylindrically Symmetric Cavities, Nuclear Instruments and Methods, NIM 216 (1983), 329-348

[7] U.van Rienen, T.Weiland, Triangular Discretization Method for the Evaluation of RF-fields in Waveguides and Cylindrically Symmetric Cavities, Particle Accelerators (1986), in print and this conference

[8] J. Tückmantel, Application of SAP in URMEL, CERN/EF/RF 85-4, July 1985

[9] J.Peters and T.Weiland, Mode Computation and Measurement of the DESY Superconducting Cavity, DESY M-83-31(1983), October

[10] J. Peters. T. Weiland, Mode Computation and Measurement of the DESY Nine Cell Superconducting Cavity, DESY M-85-02. February 1985

[11] T. Weiland, Transverse Beam Cavity Interaction. Part I: Short Range Forces, Nuclear Instruments and Methods (NIM), Vol.212(1983), pp.13-34

[12] H.Henke, Point Charge Passing a Resonator With Beam Tubes, CERN-LEP-RF/85-41,November 1985

[13] U.van Rienen, T.Weiland, Impedance of Cavities with Beam Ports Above Cut-Off ,this conference

[14] R.L.Gluckstern and F.Neri, Longitudinal Coupling Impedance for a Beam Pipe With a Cavity, IEEE NS-32(1985), pp.2403-2404

[15] R.L.Gluckstern and E.N.Opp, Calculation of Dispersion Curves in Periodic Structures, IEEE MAG-21(1985),p.2344-2346

[16] T.Weiland Modes in Infinitely Repeating Structures of Cylindrical Symmetry, this conference

[17] R.Klatt et al., MAFIA - A Three Dimensional Electromagnetic CAD System for Magnets, RF Structures and Transient Wake Field Calculations, this conference

[18] T.Weiland, On the Unique Solution of Maxwellian Eigenvalue Problems in Three Dimensions, Particle Accelerators Vol.00(1985). in press

[19] M.J.Browman, R.Cooper, T.Weiland, Three-Dimensional Cavity Calculations. this conference

[20] B.Steffen, Incorporation of Multi-grid Methods in Accelerator Software. Proc. of the 2nd europeen conference on multi-grid methods,Köln 1985(FRG), in press

[21] Y.Chin. The Wake Field Acceleration Using Cavities of Elliptical Cross Section ,KEK 83-19. November 1983

[22] R.Klatt, T.Weiland, Wake Field Calculation with Three-Dimensional BCI Code, this conference

[23] T.Weiland. Comment on Wake Field Calculations in Time Domain. Nuclear Instruments and Methods (NIM). Vol.216(1983), pp. 31-34