A FEW ASPECTS OF EXCITATION OF WAKE WAVES IN ACCELERATING STRUCTURES

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The possibilities of wake-wave acceleration, proposed in the papers of Woss and Weiland [3,4], are widely discussed at present[1,2]. In the present work some aspects of excitation of wake waves in accelerating structures are considered. The latters allow to obtain higher transformation ratio $K = -E^{+}_{z\,max} / E^{-}_{z\,min}$, where $E^{+}_{z\,max}$ is the maximum of the driving longitudinal electric field, $E^{-}_{z\,min}$ is the minimum of the intrabunch brake field. For simplicity, a waveguide with dielectric filling ε is taken as an accelerating structure, which allows to obtain analytical expressions for the radiated fields, avoiding numerous computations in the case with diaphragmatic waveguides.

During the movement of particles in such decelerating structure, electromagnetic waves of electric type are emitted. For a particle with current $I = e v_z \delta(x-\bar{x})\delta(y-\bar{y})\delta(z-v_z t)$, the expression for E_z of radiated waves (Vavilov-Cherenkov radiation) has the form[5]:

$$E_z = \begin{cases} -\sum_n \dfrac{2\pi e \lambda_n^2}{\varepsilon N_n} \Psi_n(\bar{x},\bar{y})\Psi_n(x,y)\cos\omega_n\tau, & \tau>0 \\ 0, & \tau<0 \end{cases} \quad (1)$$

where (x, y, z) are the Cartesian coordinates, t is the time, e is the electron charge, v_z is its velocity, $\tau = t - z/v_z$ is the time coordinate connected with the particle, λ_n, $\Psi_n(x,y)$ are the eigenvalues and transverse functions of the waveguide, $\omega_n = v_z\lambda_n/\sqrt{\varepsilon\beta^2-1}$ is the frequency of the n-th excited mode, N_n is the normalization factor. At arbitrary distribution of current $I = Q v_z P_1(x,y) P_2(\tau)$, the expression for E_z can be obtained by integrating the (1) over all particles of the bunch:

$$E_z = -\sum_n A_n(x,y)\int_{-\infty}^{\tau} P_2(\tau')\cos\omega_n(\tau-\tau')d\tau' \quad (2)$$

where

$$A_n(x,y) = \frac{2\pi Q\lambda_n^2}{\varepsilon N_n}\Psi_n(x,y)\iint_S P_1(\bar{x},\bar{y})\Psi_n(\bar{x},\bar{y})\,d\bar{x}\,d\bar{y}$$

S is the waveguide cross section, Q is the total charge of the bunch.

High transformation ratio can be attained in two cases: at the separation of the trajectories of the driving and driven bunches - the transverse transformation ($K_\perp$), and at asymmetry of the driving bunch distribution - the longitudinal transformation ($K_\parallel$). The well-known Voss-Weiland scheme[3] with ring-shaped driving bunch gives only a transverse transformation with K of about 10-20. The optimal ratio of the longitudinal transformation, with account of the main excited mode only, reaches the value[6] of $K_\parallel = \sqrt{1+(2\pi N)^2}$, where N is the ratio of the bunch length to the radiated wave-length. At linear distribution of the driving bunch, $K_\parallel = \pi N$, and taking all excited modes into account, the optimal ratio becomes considerably larger than πN. This naturally induced us to consider a scheme with simultaneous transverse-longitudinal (TL) transformation. The total transformation ratio in this case, with account of the main excited mode only, will be equal to $K = K_\parallel \cdot K_\perp$.

A ring-shaped driving bunch with current-density linear growth over τ, moving in a waveguide with circular cross section has been considered as an example with TL transformation:

$$I = \frac{2Q\tau}{\pi(z_2^2-z_1^2)T^2} \qquad z_1 \le z \le z_2,\quad 0<\tau<T \quad (3)$$

It can be shown that in a circular waveguide such a bunch can excite only symmetrical waves E_{on} with field distribution outside the bunch:

$$E_z^-(z,\tau) = -\sum_n C_n J_0(\lambda_n z)(1-\cos\omega_n\tau)$$

$$E_z^+(0,\tau) = \sum_n C_n\left[T\omega_n\sin\omega_n(\tau-T)+\cos\omega_n\tau-\cos\omega_n(\tau-T)\right] \quad (4)$$

where

$$C_n = \frac{8Q\left[z_2J_1(\lambda_n z_2) - z_1J_1(\lambda_n z_1)\right]}{\varepsilon R^2T^2(z_2^2-z_1^2)\lambda_n\omega_n^2J_1^2(\lambda_n R)}$$

Here $z_1 \le z \le z_2 < R$, and the trajectory of the driven bunch lies along the waveguide axis. By restriction to the main excited mode for transformation ratio at $T = 2\pi N/\omega_0$, we shall obtain $K = \pi N / J_0(\lambda_n z_1)$.

Fig. 1 shows the dependence of the transformation ratio on the number of the excited modes at different radii R of the waveguide. Here $z_2 - z_1 = 2$ mm, $R - z_1 = 10$ mm, $T = 2\pi/\omega_0$. One can see from the plot that

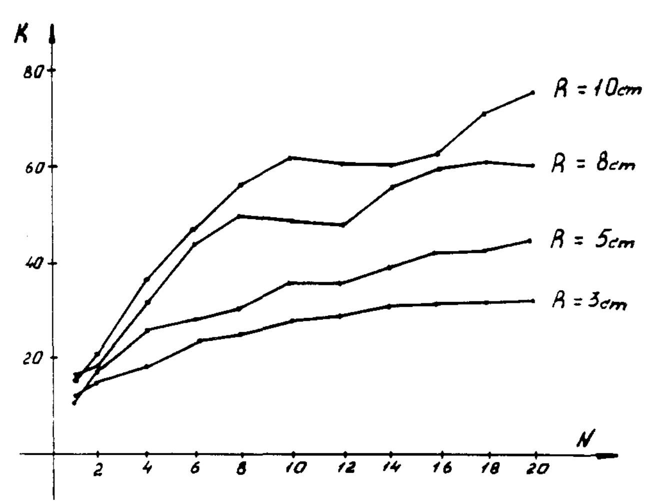

Fig. 1

the account of a great number of excited modes improves appreciably the transformation ratio.

Table 1 lists the values of the transformation ratio and the maximum values of accelerating field $E^+_{z\,max}$ for the transverse, longitudinal and TL transformations with account of the first twenty excited modes.

Table 1

	$I(\tau,\tau)$	κ	$E^+_{z\,max}/Q$ (cm^{-2})	Notes
Transverse transform.	$I_0\delta(\tau-\tau_1)$	9	0.9	R = 5 cm
Longitud. transform.	$I_0 \frac{2\tau}{T}\delta(\tau)$	23	0.035	ε = 1.005
TL transform.	$I_0 \frac{2\tau}{T}\delta(\tau-\tau_1)$	45	0.175	$T = 2\pi/\omega_0$

It is evident from the Table that a considerable increase in κ and in acceleration gradient really takes place at TL transformation.

The dependence $E_z(\tau)$ in case of TL transformation (3), (4) is shown in Fig. 2, where $\tau < T$ corresponds to the brake field inside the bunch ($\tau = \tau_1$), $\tau > T$ to the accelerating field on the waveguide axis outside the bunch (τ = 0). The accelerating field peaks stand out clearly in the plot, the maximum accelerating gradient occurring near the driving bunch.

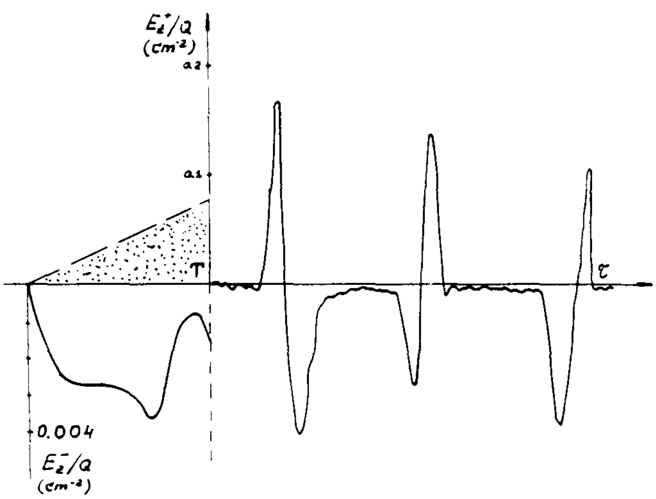

Fig. 2

As is seen from (4), the amplitudes of the radiated fields depend on the frequency of excited modes. This leads to the dependence of the transformation ratio on the ratio of the radiated wave-length to the bunch length.

Fig. 3 presents the dependence of κ on the bunch length T . The increase in κ is due to the longitudinal transformation. Fig. 4 shows κ as a function of the waveguide radius R . Here the increase in κ is due to both the longitudinal (decrease in ω_n) and the transverse transformations.

In conclusion, we'd like to note that further progress in this direction is connected with the optimization of the accelerating structure itself.

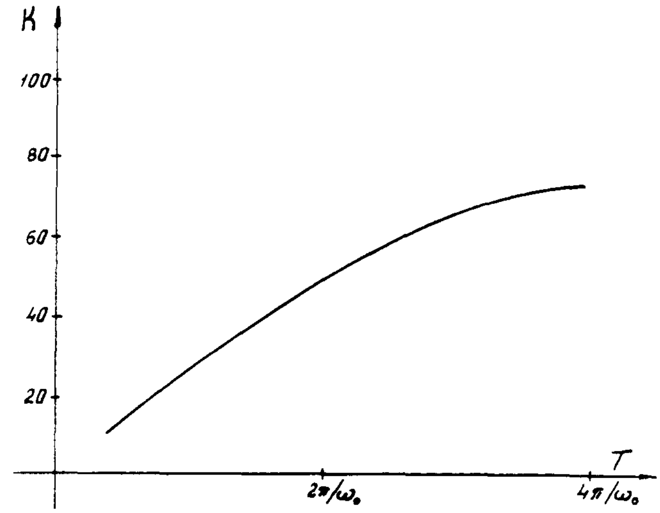

Fig. 3

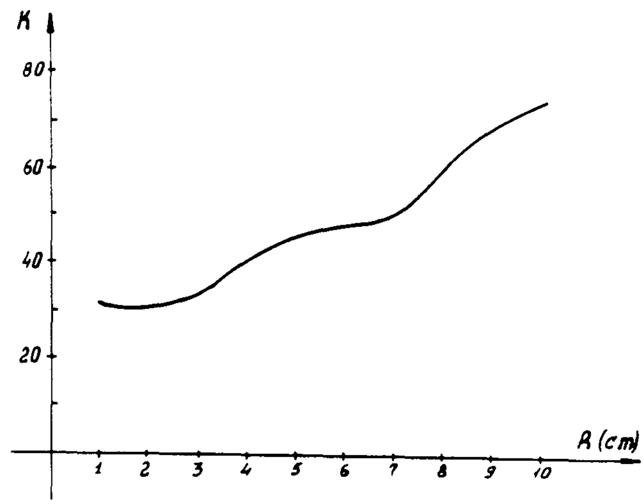

Fig. 4

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