FEEDBACK SYSTEM ANALYSIS FOR BEAM BREAKUP IN A MULTIPASS MULTISECTION ELECTRON LINAC

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Introduction

A recirculating electron accelerator based upon superconducting cavities technology is envisaged in different laboratories to produce a high duty cycle beam with energy in the GeV region. In all the cases the design current is in the 100 μA range. Beam break up is a severe limitation in this kind of accelerator due to the positive feedback of the returning beams. Therefore a careful analysis of the phenomenon must be made to determine the permissible characteristics of the superconducting cavities. Different analysis have been made for recirculating beam breakup by Volodin [1], Herminghaus [2], Vetter [3], Lyneis [4] and others. We present here an analysis based upon feedback system theory which takes into account the different cavities of the linac, the optics of the linac and of the recirculating path. The frequencies of the cavities may be the same or distributed in a random way. We assume that the beam position do not vary when traversing a cavity, and the bunch structure is ignored. A computer program has been written which computes the threshold current for a given configuration. The result can be checked with a simulation code which computes the transverse motion of the bunches based upon wake fields equations. An example is given for the Saclay proposal of a 2 GeV accelerator consisting of 4 passes in a 500 MeV, 100 m-long superconducting linac.

Transverse coupling impedance of a cavity.

We start with the definition of transverse coupling impedance for dipoles modes in a cavity given by F.J. Sacherer [5] :

$$V = \int_{-d/2}^{+d/2} (\vec{E} + \vec{v} \times \vec{B})_\perp \, e^{jkz} \, dz = - j \, Z_{\perp(\omega)} \, \Delta I_0$$

giving the transverse force seen by a beam of current I_0, traversing a cavity at a distance Δ from the axis, k being the wave number. In analogy with the analysis given by T. Suzuki [6] for the longitudinal coupling impedance, it can been shown [7] that the transverse impedance defined above takes the resonant form :

$$Z_\perp(\omega) = \frac{\omega_r}{\omega} \, \frac{R_\perp}{1 + j\,Q\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

ω_r being the frequency of the mode, Q the corresponding quality factor.

The impedance at resonance is given by :

$$Z_\perp(\omega_r) = R_\perp = \frac{\left|\int_{-d/2}^{+d/2} \nabla_\perp E_z \, e^{ikz} \, dz\right|^2}{2kP} \quad (\Omega/\mathrm{m})$$

Where P is the RF power dissipated in the walls.

A beam travelling at a distance y from the axis receives a transverse kick given by :

$$\Delta p_\perp = \int F_\perp \frac{dz}{c} = - j \, \frac{e}{c} \, Z_\perp(\omega) \, I_0 \, y$$

which can be written :

$$\Delta p_\perp = \frac{a}{-\omega^2 + \omega_r^2 + j\,\frac{\omega\,\omega_r}{Q}} \, y \, .$$

with :
$$a = \frac{e}{c} \, \frac{R_\perp}{Q} \, \omega_r^2 \, I_0$$

The differential equation which describes the beam-cavity interaction is then :

$$\ddot{\Delta p}_\perp = ay - \frac{\omega_r}{Q} \dot{\Delta p}_\perp - \omega_r^2 \, \Delta p_\perp \, . \qquad (1)$$

Feedback analysis of the stability.

A linac composed of N cavities is considered as a system of N oscillators coupled by the beam, with a feedback due to the recirculating paths. In order to know the evolution of this system for a small initial perturbation, in analogy with the linear control system theory [8], we define a system of n state variables $x_1(t) \ldots x_n(t)$. The evolution of the system is governed by the homogeneous equation :

$$\dot{x} = Ax$$

A being the state matrix nxn

$\dot{x}$ is the time derivative of the vector x

The differential equation (1) being of second order, we need two state variables per cavity. For the cavity i, the state variables are :

$$x_{2i-1} = \Delta p_{\perp i}$$

$$x_{2i} = \dot{\Delta p}_{\perp i}$$

The matrix A is then a 2N x 2N matrix. The transverse position of the beam in the cavity i during pass j is noted y_i^j. We assume that the beam is injected on axis :

$$y_1^1 = y'^1_1 = 0$$

The transverse momentum imparted to the beam in cavity i is given by equation (1) where we put :

$$y = \sum_j y_i^j$$

It is useful to transform the equations by using a Laplace transformation. We note X(p) the transform of x(t), x(0) being the initial conditions. For the cavity i the equation (1) is equivalent to the system :

$$p\,X_{2i-1}(p) - x_{2i-1}(0) = X_{2i}(p)$$

$$(2)$$

$$p\,X_{2i}(p) - x_{2i}(0) = -\omega_r^2 X_{2i-1}(p) - \frac{\omega_r}{Q} X_{2i}(p) + a \sum_j y_i^j(p)$$

The 2N equations are summarized in :

$$(pI - A)\,X(p) = x(0)$$

I being the unit matrix, A the state matrix previously defined.

The evolution of the system is then found by taking the inverse Laplace transform of :

$$X(p) = [pI-A]^{-1} x(o)$$

The system is stable if the poles of X(p) have a negative real part. These poles are the zeros of the determinant of pI-A, i.e they are the eigenvalues of the matrix A.

The problem is then to find the matrix A and its eigenvalues.

From the system (2) it is seen that we need to evaluate the Laplace transform $Y_i^j(p)$ of the beam position $y_i^j(t)$. The transfer matrix from cavity i to cavity i+1 during the pass j is known and depends on the energy and the focusing scheme of the linac. The angular divergence at the exit of cavity i is given by :

$$y'^{j+}_i = \frac{pz_i^j}{pz_{i+1}^j} y'^j_i + \frac{x_{2i-1}}{pz_{i+1}^j}$$

pz being the longitudinal momentum. The optical properties of the return path are summarized via a transport matrix, and finally the transit time T_j of jth complete loop from a point in the linac back to itself is included in the Laplace transform Y_i^j via a term e^{-pT_j}.

The computer code for stability study computes the elements of the matrix A for a given configuration of the linac and of the return paths and for given characteristics of the deflecting modes in the cavities (ω_r, Q, $\frac{R}{Q}$). The eigenvalues of A are then computed by successive iterations since the elements of A contain p. The limit for stability corresponds to the solution with a nul real part :

$$p_s = j\omega_s$$

Application to a simple case

We consider the case where the linac is supposed to be concentrated into one cavity and compute the threshold current for two passes. In that case the matrix A is :

$$A = \begin{bmatrix} 0 & 1 \\ a\dfrac{R_{12}}{p_{z2}} e^{-pT_1} - \omega_r^2 & -\dfrac{\omega_r}{Q} \end{bmatrix}$$

R_{12} being the coefficient of the transfer matrix from the cavity back to itself in TRANSPORT notation. p_{z2} is the longitudinal momentum at the exit of the cavity. The limit for stability is obtained for an eigenvalue $p = j\omega_s$. The characteristic equation becomes :

$$-\omega_s^2 + j\,\omega_s \frac{\omega_r}{Q} + \omega_r^2 - a\frac{R_{12}}{p_{z2}} e^{-j\,\omega_s T_1} = 0$$

The frequency at which instability occurs is given by :

$$\frac{\omega_s\,\omega_r}{Q} \cos\omega_s T_1 + \left(\omega_r^2 - \omega_s^2\right) \sin\omega_s T_1 = 0$$

and the threshold current is :

$$I_s = -\frac{p_{z2}\; c/e}{R_\perp R_{12} \sin\omega_s T_1}\,\frac{\omega_s}{\omega_r}$$

giving a result similar to the one given by Herminghaus [2].

Application to a 2 GeV accelerator.

We apply our results to a possible scheme for a 2 GeV accelerator composed of 4 passes in a 500 MeV linac. We take as an example the 1 GHz, nine-cell superconducting cavity studied at DESY [9] and assume an electric field of 5 MeV/m. The linac is composed of 72 cavities in 18 groups of 4 units. The focusing uses a FODO scheme with a spacing between quadrupoles of 10 m and constant gradient (corresponding to a focal length of 5 m at 50 MeV). The harmonic number is 1403 and is the same for all turns, the transport matrix for the return path is the unit matrix.

We study the effect of one of the dipole modes given in [9] whose frequency is 1.436 GHz. The shunt impedance given by the code URMEL, computed at d = 44.5 mm from the axis is R/Q = 38.5 Ω/m. The coupling impedance used in equation (1) is given by :

$$\frac{R_\perp}{Q} = \frac{L}{2kd^2}\left(\frac{R}{Q}\right)_U = 436\ \Omega/m$$

L is the length of the cavity. For a group of 4 cavities the impedance is :

$$\frac{R_\perp}{Q} = 1.75\ 10^3\ \Omega/m$$

We assume that the Q of that mode has been reduced to $Q = 10^6$. As the frequency of the modes is not known exactly, the BBU threshold shown in table 1 for 2, 3 and 4 passes are the lowest values calculated with the feedback theory, sweeping the frequency around 1.436 GHz.

Table 1

Energy (GeV)	1	1.5	2
Nb of passes	2	3	4
Threshold current (μA)	299	205	115
BBU frequency	1.43594	1.43600	1.43604

In the case of no focusing in the linac the currents are respectively 84 μA, 33 μA and 17 μA for 2, 3, and 4 passes.

Stability study using a tracking code.

A tracking code has been written following the formulations given by R. Glückstern [10], but including multipass effects. The focusing in the linac, and the optics of the recirculating paths are included.

When a bunch of charge Q_b moves at a distance a from the axis, the transverse integrated force seen by a test charge following the bunch at a distance l, is given by :

$$\int_{-d/2}^{+d/2} F_\perp\, dz = e\, a\, Q_b\, W(\tau)$$

where $\tau = \frac{l}{c}$. W(τ) is the transverse wake potential which can be expressed as a Fourier-transform of the coupling impedance :

$$Z_\perp(\tau) = -\frac{\tilde{W}(\tau)}{j} = -\frac{1}{j}\int W(\tau)\, e^{-j\omega\tau}\, d\tau$$

For the case of a transverse resonant mode we obtain :

$$W(\tau) = \frac{R_\perp}{Q} \frac{\omega_r^2}{\omega'_r} e^{-\frac{\omega_r}{2Q}\tau} \sin\omega'_r \tau \quad \text{with } \omega'_r = \omega_r \left(1 - \frac{1}{4Q^2}\right)^{1/2}$$

The transverse momentum imparted to the bunch n passing in cavity i at time T is the sum of all the wakes left by the bunches which passed at time between 0 and T.

In order to test the stability of the system composed of N cavities coupled by the beam with several passes, we apply a perturbation to the stable system and look for the evolution of the free oscillations (damping or growth). The perturbation may be a transverse momentum imparted to the first bunch when it passes in the first cavity. When the complete system is described (cavities, transverse modes and optics) the threshold current is found by iterations on the charge Q_b to find the limit of stability. This method was tested and gave the same threshold current than the matrix method described in the first part. However it should be noted that a very large number of bunches must be tracked before steady-state is reached in the case of high Q (superconducting case) and large accelerator, thus leading to impracticable computing times.

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