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A NOTE ON TRANSVERSE BEAM INSTABILITIES IN MULTISECTION LINACS*

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I. Introduction

Preliminary operation of the Stanford Two-Mile Electron Accelerator[1] has uncovered an unexpected transverse beam instability at a beam current below the design value. Various tests of the dependence of this effect on the pulse length, section length, beam current, and transverse focusing forces indicate that it is a cooperative phenomenon between transverse field oscillations in the various sections.[2]

We have in this note constructed a model for this beam blowup in the absence of large focusing forces, similar to those of Panofsky [2] and of Sessler.[3] We have further estimated the "e-folding factor" in terms of the "starting current" for runaway transverse oscillations in the first section.

II. Analysis for a Linac Consisting of Identical Sections

Let us consider $E(s,z)$ as the complex amplitude of the transverse deflecting field in a cavity. The variable z represents axial distance (cavity number times cavity length) and s represents pulse number.

A beam pulse entering a cavity off axis can couple to the transverse modes. One obtains the change in field amplitude per pulse [2,3,4] for a typical mode:

$$\frac{\Delta E}{\Delta s} \simeq \frac{\partial E}{\partial s} \simeq bx - \mu E, \tag{1}$$

where $\mu = \omega\Delta t/2Q$ is the decay constant representing the fractional ohmic loss per beam pulse. Here x is the transverse displacement at the entrance to the cavity and b is a complex parameter depending on the cavity geometry and on the beam current. Effects of motion at an angle to the longitudinal axis have been neglected since the angles correspond to transverse focusing wave lengths which are much larger than the rf wave length.

Each cavity causes a deflecting force on the beam pulse which leads to a change in slope of the trajectory. This can be expressed as

$$\frac{\Delta x'}{\Delta z} \simeq \frac{\partial^2 x}{\partial z^2} \simeq \mathrm{Re}(aE). \tag{2}$$

Here a is again a complex parameter depending on the cavity geometry.

If the transverse fields build up, the beam will be driven in the transverse direction at the frequency of the transverse field oscillations. These transverse beam oscillations can then drive subsequent cavities at the same frequency. This synchronism however does <u>not</u> require resonance between the beam frequency and the transverse oscillation frequency.

In the analysis below, the effect of synchronism can be simulated by treating all parameters as real. A more detailed calculation taking into account the phases of the various terms confirms that this is quantitatively correct. The analysis is equivalent to considering x and $\mathrm{Re}(aE)$ as the variables, with the product ab being in fact $\mathrm{Re}\,(ab)$.

We shall first discuss the solution of Eqs. (1) and (2) for constant a,b,μ (in the actual case a will be inversely proportional to momentum). In each cavity one has, after establishment of a steady state ($\frac{\partial E}{\partial s} \to 0$),

$$E = \frac{b}{\mu}\, x \ , \quad s \to \infty \tag{3}$$

in which case

$$\frac{d^2 x}{dz^2} = \frac{x}{\ell^2} \tag{4}$$

where

$$\frac{1}{\ell^2} = \frac{ab}{\mu} \tag{5}$$

The solutions of Eq. (4) is

$$x(s,z) = x(s,o) \cosh z/\ell + x'(s,o)\, \ell \sinh z/\ell, \quad s \to \infty \tag{6}$$

and the increase in transverse displacement depends on the length parameter ℓ, whose magnitude will be estimated later.

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The solution for finite s is more difficult to obtain. If one makes a Laplace transform from the variable s to the variable p, one finds from (1)

$$E_L(p,z) = \frac{b}{p+\mu}\, x_L(p,z), \qquad (7)$$

assuming that no initial fields are present. Equation (2) then becomes

$$\frac{d^2x_L}{dz^2} - K^2 x_L = 0 \qquad (8)$$

where

$$K^2 = \frac{ab}{p+\mu} = \frac{1}{\ell^2}\frac{\mu}{p+\mu} \qquad (9)$$

leading eventually to

$$x(s,z) = \frac{e^{-\mu s}}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} dq\, e^{qs}\left[x_L(p,o)\cosh Kz + x_L'(p,o)\frac{\sinh Kz}{K}\right], \qquad (10)$$

where

$$q = p+\mu\ , \quad K^2 = \frac{1}{\ell^2}\frac{\mu}{q} \qquad (11)$$

and

$$x_L(p,o) = \int_o^\infty e^{-ps}\, x(s,o)ds,$$

$$x_L'(p,o) = \int_o^\infty e^{-ps}\,\frac{\partial x}{\partial z}\, x(s,o)ds. \qquad (12)$$

For a constant driving displacement at the first cavity, $x(s,o) = x_o, x'(s,o) = 0$, and $x_L(p,o) = x_o/p$. This leads to

$$x(s,z) = \frac{x_o}{2\pi i}\, e^{-\mu s}\int_{\gamma-i\infty}^{\gamma+i\infty}\frac{dq}{q-\mu}\, e^{qs}\cosh\frac{z}{\ell}\sqrt{\frac{\mu}{q}} \qquad (13)$$

The evaluation of Eq. (13) separates into three regions according to whether

$$\mu s << \frac{\ell^2}{z^2}\ , \qquad (14a)$$

$$\frac{\ell^2}{z^2} << \mu s << \frac{z}{\ell}\ , \qquad (14b)$$

$$\frac{z}{\ell} << \mu s. \qquad (14c)$$

The region (14c) corresponds to the steady state solution ($s \to \infty$), the region (14a) to some initial build up, and the region (14b) to an intermediate region, which turns out to be the one of interest. The three solutions are:

$$\frac{x(s,z)}{X_o} \simeq 1 + \mu s\frac{z^2}{\ell^2}\frac{1}{2!} + \left(\mu s\frac{z^2}{\ell^2}\right)^2\frac{1}{2!4!} + ---- \qquad (15a)$$

$$\frac{x(s,z)}{X_o} \simeq \frac{e^{-\mu s}}{2\sqrt{3\pi}}\, e^{3\left(\frac{z^2\mu s}{4\ell^2}\right)^{\frac{1}{3}}}\left(\frac{z^2\mu s}{4\ell^2}\right)^{-1/6} \qquad (15b)$$

$$\frac{x(s,z)}{X_o} \simeq \cosh\frac{z}{\ell} \qquad (15c)$$

where (15b) has been obtained by a saddle point approximation.

We can now modify the foregoing analysis to take into account a variation of a with z according to

$$a = a'\,\frac{L}{z} \qquad (16)$$

where a' is constant and L is the cavity length. (The parameter a' is the value of a for the first cavity). Setting

$$\frac{1}{\ell^2} = \frac{a'b}{\mu}$$

one eventually obtains the saddle point solution

$$x(s,z) \simeq \frac{K_o'}{2\pi\sqrt{3}}\left(\frac{\ell^4 z}{L^2\mu^2 s^2}\right)^{\frac{1}{3}}\exp\left\{-\mu s + 3\left(\frac{Lz\mu s}{\ell^2}\right)^{\frac{1}{3}}\right\} \qquad (17)$$

valid in the region $\frac{\ell^2}{Lz} << \mu s << \frac{\sqrt{zL}}{\ell}$, with x_o' being the assumed (constant) initial angle of the beam to the longitudinal axis.

III. Numerical Estimate for the Stanford Linac

The growth is controlled by the exponent in Eq. (17)

$$F = 3\left(\frac{Lz\mu s}{\ell^2}\right)^{\frac{1}{3}} \qquad (18)$$

which is the "e-folding factor" of Panofsky.[2] (The parameter ℓ^2 is inversely proportional to the current.) The dependence of F on the parameters is in agreement with that of Panofsky for the constant energy gain case and also agrees qualitatively with the observed dependence of the blowup on the beam current, accelerator length, and time.

It is convenient to estimate F by comparison with the calculation of starting current for runaway oscillations in the first cavity.[5] In this case

$$\frac{E}{s} = fE + bx - \mu E \qquad (19)$$

where f can be expressed in terms of the ratio of actual to starting current as

$$\frac{f}{\mu} = \frac{I}{I_s} \tag{20}$$

Neglecting phase factors in Reference 2, one can write

$$\frac{f}{a^2 b} = \frac{2\,\ell^2 W}{\pi^3} \tag{21}$$

where W is of order 1. One then has

$$\frac{\ell}{L} = \frac{1}{L}\sqrt{\frac{\mu}{a^2 b}} \sim \frac{1}{4}\sqrt{\frac{I_s}{I}}\ . \tag{22}$$

According to Reference 1, the starting current is approximately 600 mA. For a current I = 15 mA, one then has

$$\frac{\ell}{L} \sim 1.6\ , \tag{23}$$

justifying the use of Eqs. (14b) and (15b) for $\mu s \sim 1$, and z = 2 miles. In fact, for $\mu s \sim 1$, L = 3 meters, one has from Eq. (18)

$$F \sim 20$$

which is in excellent agreement with the factor Panofsky finds is necessary to obtain with observations.[2]

IV. Multi-Section Proton Linacs

The foregoing discussion of the mechanism for beam growth suggests that the seriousness of the effect in the Stanford Linac depends crucially on the fact that all cavities possess modes with the same frequencies and phase velocities. A field oscillation in a given cavity increases the transverse modulation of the beam at the same frequency. The beam then drives the field in the next cavity which amplifies the modulation, which grows exponentially with distance as described by Eq. (6).

In a proton linac the value of β and the cavity geometry will vary from cavity to cavity, as will the transverse mode structure. If the typical variation from section to section in the frequencies of the transverse modes which cause blowup is $\Delta\omega/2\pi$, then the phase relations appropriate to the resonance will be maintained for about $\omega/\Delta\omega$ rf cycles or $2\pi/(\Delta\omega)\Delta t$ beam pulses, where Δt is the time between beam bunches. The value of μs to be used in (18) is therefore

$$\mu s \simeq \frac{\omega \Delta t}{2Q}\,\frac{2\pi}{\Delta\omega\Delta t} = \frac{\pi(\omega/\Delta\omega)}{Q}\ , \tag{24}$$

For values of $Q \sim 10^4$, $\omega/\Delta\omega \sim 10^2$, the current at which a serious effect is observed will be increased by a factor $\sim 10^2$, for a two-mile machine. If the LASL proton linac is designed so that deflecting modes with phase velocities near β in the successive sections have frequencies different by this much (i.e., the dispersion curves for the various sections are well separated), then it should not be affected by cooperative beam blowup.

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REFERENCES

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2. W.K.H. Panofsky, Internal SLAC Report TN-66-27, June 7, 1966; See also G. A. Loew, "Beam Break-Up Experiments at SLAC." 1966 Linac Conference, Los Alamos, N. Mex., p.267.

3. A.M. Sessler, private communication.

4. See, for example, Transverse Beam Blow-Up in Standing Wave Linacs, R.L. Gluckstern, BNL Report AADD-38, July 1964.

5. See, for example, Gluckstern and Butler, IEEE Transactions in Nuclear Science, Vol. 12N, No. 3, p. 607, June 1965.

DISCUSSION

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COURANT, BNL: With respect to that last point: What you say about the proton linacs not being subject to resonant blowup is certainly true, but we should remember what Dr. Panofsky mentioned in the discussion after the last paper--that, in addition, there is a wake field which is a combination of all frequencies, including very low ones. This decays as the inverse square root of time. The over-all intensity of that effect is about one order of magnitude, but no more than that--weaker than the resonant effect. Therefore, it might be worth while to see whether wake field phenomenon might be relevant to long-proton accelerators. A few months ago I did a very rough calculation for this sort of thing and found that it might, indeed, be somewhat worrisome for the large separated-orbit cyclotrons being proposed. I think the Los Alamos linac is shorter and therefore better than an SOC.

GLUCKSTERN: Is there also a dependence on the wave length which makes it less serious for the lower-frequency machines or not?

COURANT: I'm not sure. I don't think so.