COMPARISON OF THE PARTICLE MOTIONS AS CALCULATED BY TWO DIFFERENT DYNAMICS PROGRAMS*

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In the fall of 1964, we decided at MURA to prepare a program which would integrate the equations of motion of electrons or ions in the electromagnetic fields of the basic linac geometry. We named this program LINDY from the phrase "Linac Ion Dynamics." The program receives its information on the distribution of fields in the geometry directly from a magnetic tape prepared by the MESSYMESH program.

Figure 1 shows this basic geometry and electric field lines as calculated by the MESSYMESH program.

The program was prepared primarily to provide an accurate means for tracing ions and electron motions in the outer portions of the cavity and in the region between drift tube surfaces. However, there is nothing to prevent us from calculating the trajectory of a beam particle.

Figure 2 shows some results of this sort of calculation. The proton enters the cell traveling parallel to the axis and displaced from it by a half centimeter. As it passes through the cell, it experiences a small defocusing force, a small focusing force, and finally a large defocusing force, as shown by the solid trajectory.

The dashed trajectory marked PARMILA No. 1 shows the trajectory that resulted from the PARMILA program that we were using at that time. The transformations at that time were equivalent to a drift to the center, a radial impulse and a drift through the rest of the cell.

It is apparent from Figure 2 that the magnitude of the radial impulse was the same in both programs; but that, there was a discrepancy in the radial position at the end of the cell. This discrepancy prompted us to take another look at the radial equation, which can be written:

$$r_{out} = r_{in} + \int_{-L/2}^{L/2} r'(z)\, dz$$

$$= r_{in} + \int_{-L/2}^{L/2} \left(r'_{in} + \int_{-L/2}^{z} r'' d\xi \right) dz$$

$$= r_{in} + r'_{in} L + \int_{-L/2}^{L/2} \int_{-L/2}^{z} r''(z) d\xi z \ .$$

By changing the order of integration the double integral becomes

$$\frac{L}{2} \Delta r' - \int_{-L/2}^{L/2} \xi\, r''(\xi)\, d\xi$$

where $\Delta r'$ is the total change in r' suffered by the particle on crossing the cell. Since the radial forces (and hence r'') are proportional to r, the integral may be written as

$$\int_{-L/2}^{L/2} (r_o + r'_o z) z \frac{r''(z)}{r(z)}\, dz$$

where the dummy variable has been changed to z, and where r_o and r'_o are the values or r and r' at $z = 0$. Dropping the second term in this integral, the radial equation becomes

$$r_{out} = r_{in} + r'_{in} \frac{L}{2} + (r'_{in} + \Delta r') \frac{L}{2}$$

$$- r_o \int_{-L/2}^{L/2} z \frac{r''(z)}{r(z)}\, dz.$$

The first three terms correspond to terms that were in the original version of PARMILA. They are the initial radius, the change in radius on drifting to the center of the gap, and the change in radius on drifting to the end of the cell. The latter term is a radial displacement term which accounts for the fact that the radial forces are distributed over the region of the gap rather than concentrated (as a thin lens) at the center of the gap. Addition of this term to the PARMILA program, produces the PARMILA trajectory No. 2 which is in excellent agreement with the LINDA trajectory.

It is important to check the r' equation and ultimately the Jacobian of the r and r' equations to be sure that they represent a system of equations that conserve the r,r' phase space.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

The r' equations can be written

$$r'_{out} = r'_{in} + \int_{-L/2}^{L/2} r''(z)dz.$$

If the integrand is again written as

$$(r_o + r'_o z)\,\frac{r''(z)}{r(z)}\,,$$

the r' equation becomes

$$r'_{out} = r'_{in} + r_o \int_{-L/2}^{L/2} \frac{r''(z)}{r(z)}\,dz + r'_o \int_{-L/2}^{L/2} z\,\frac{r''(z)}{r(z)}\,dz.$$

If r_o and r'_o are approximated, in terms of r_{in}, r'_{in}, L, and $\Delta r/r$, the Jacobian of r_{out}, r'_{out} with respect to r_{in}, r'_{in} can be written

$$\frac{\partial(r_{out},r'_{out})}{\partial(r_{in},r'_{in})} = \begin{vmatrix} 1 - A + \frac{BL}{2} & L(1 - \frac{A}{2} + \frac{BL}{4}) \\ B(1 + \frac{A}{2}) & 1 + A + \frac{BL}{2} \end{vmatrix} = 1 - A^2 - \frac{AB^2L^2}{8} + \frac{A^2BL}{4}\,,$$

where $$A = \int_{-L/2}^{L/2} z\,\frac{r''(z)}{r(z)}\,dz$$

and $$B = \int_{-L/2}^{L/2} \frac{r''(z)}{r(z)}\,dz.$$

The Jacobian is unity to first order in A and B, which indicates that the equations conserve the r,r' phase space area to first order in A and B.

About a year ago we decided to undertake a more extensive comparison of PARMILA and LINDY. For this purpose, LINDY was modified into a three-dimensional program. Provisions were made for superimposing quadrupole fields, and for dealing with a series of 20 cells rather than just one cell. A special series of MESSYMESH runs were made to provide the information about the fields in the series of cells.

It was now possible to integrate the equation of motion through the first 20 cells of a linac. This was done for several sets of initial coordinates. The same initial coordinates were transformed through 20 cells of the PARMILA program.

Comparisons of the results again revealed several discrepancies between the two calculations, this time primarily in the longitudinal motion. The fact that the discrepancies had a chance to accumulate over many cells made them easier to interpret.

The magnitude of these discrepancies was reduced by a slight improvement to the cell length formula and the energy gain formula used in the PARMILA program.

The energy gain for a synchronous particle traversing a cell is

$$\Delta E_s = \int_{-L/2}^{L/2} e\,E_z(z)\,\cos\left(\omega \int_0^z \frac{d\xi}{V(\xi)} + \varphi_s\right) dz\,,$$

where $V(\xi)$ is the velocity of the synchronous particle, $E_z(z)$ is the amplitude of the z component of electric field, and φ_s is the phase of the rf fields when the synchronous particle reaches the center of the gap. This energy gain is commonly expressed in terms of a transit time factor

$$T = \frac{\Delta E_s}{e\,E_o L \cos\varphi_s}.$$

If $V(\xi)$ is approximated by its average value in the experession for ΔE_s, the expression for T becomes

$$T_{approx} = \frac{\int_{-L/2}^{L/2} E_z(z)\,\cos\frac{2\pi z}{L}\,dz}{\int_{-L/2}^{L/2} E_z(z)\,dz}$$

The transit time factors for the PARMILA calculations are based on this approximation together with the actual field distributions $E_z(z)$ as determined by the MESSYMESH program. An estimate of the error introduced by this approximation proved to be in agreement with the discrepancy in energy gain mentioned above. An expression was obtained for T in terms of T_{approx} and a correction term T_{corr}, where T_{corr} is evaluated from known quantities and varies from - 0.5% to - 0.2% over the range of geometries in a 100-MeV Alvarez linac. This correction term is now incorporated in the PARMILA program and preliminary tests indicate satisfactory results.

Figures 3 and 4 present the latest results in our effort to compare the particle dynamics as calculated by PARMILA and LINDY. They demonstrate rather good agreement between the two programs.

DISCUSSION

D. A. SWENSON, LASL

LAPOSTOLLE, CERN: About your comment on the error in transit time factor due to the progressive change in velocity, do you take into account the actual velocity or energy in the middle of the gap, or what velocity do you use in your transit time expression? What I think is that what we call the S term to compute the actual velocity in the middle is something which just takes care of this velocity variation, at least to the first order, and the rest is rather in the range of a 0.1% error.

SWENSON: The transit time factor that comes from the MESSYMESH program is based on a constant velocity through the cell corresponding to the cell length divided by the period of the rf. If we take into account the actual particle velocity, which is a function of position in the cell, we get a transit time factor that is, for the low-energy geometries, 0.5% lower than the MESSYMESH value.

OHNUMA, Yale: I would like to make a comment. A similar comparison was done at Brookhaven, and I think this is related to the question of Dr. Lapostolle, that it is very important to compute what he calls S. The energy gain in the first half of the gap and the energy gain in the last half of the gap could be quite different. For some particles, the energy really decreases in the first half and then starts increasing, and if you take this into account, the exact motion result and a simpler calculation agree very well. But if you neglect that S term, then the phase motion is very different.

MILLS, MURA: Did you vary the number of Runge-Kutta steps used to integrate the equations of motion across the gap? If so, was there a dependence of the result on this variation? If there was no dependence, then you have an exact solution to the equations of motion, using exact fields, to compare with the PARMILLA program.

SWENSON: Yes, we tend to believe the LINDY results very much. We varied the Runge-Kutta step size, and we are operating in a region where the step size has no effect.

LAPOSTOLLE: Just one additional comment about the S term which is discussed: The first expression I gave of the S term was wrong but the one which was given in the first paper of this morning is correct. The radial dependence on the S term was originally wrong; my mistake at the beginning was to make use of a non-Maxwellian field. But the new expression that I gave is right.

SWENSON: Perhaps I should say that for the T and S factors and their derivative, we use the numerical values that come from the MESSYMESH calculation in some formulas which I believe to be correct.

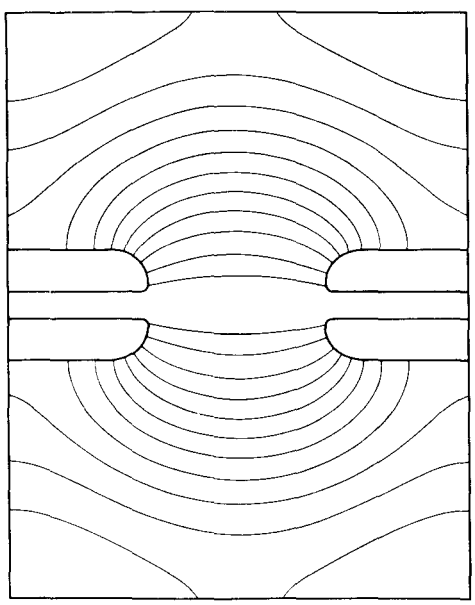


Fig. 1. Electric field lines resulting from MESSYMESH run No. 30628.

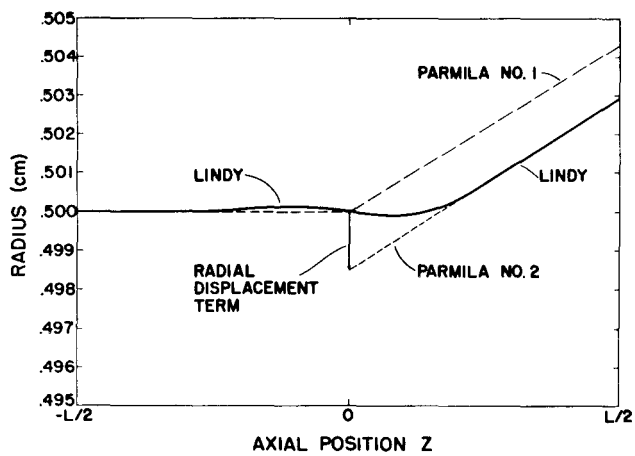


Fig. 2. Comparison of radial motion across Alvarez gap as computed by the LINDY and PARMILA programs.

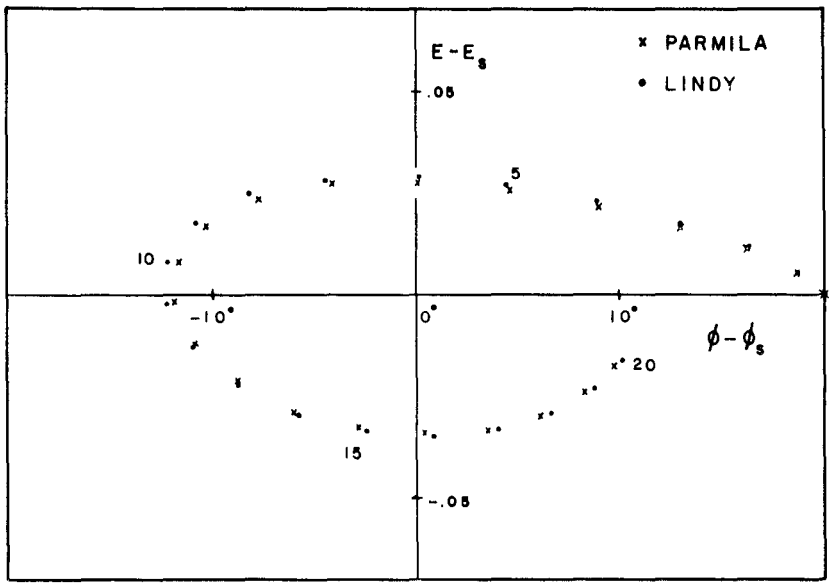
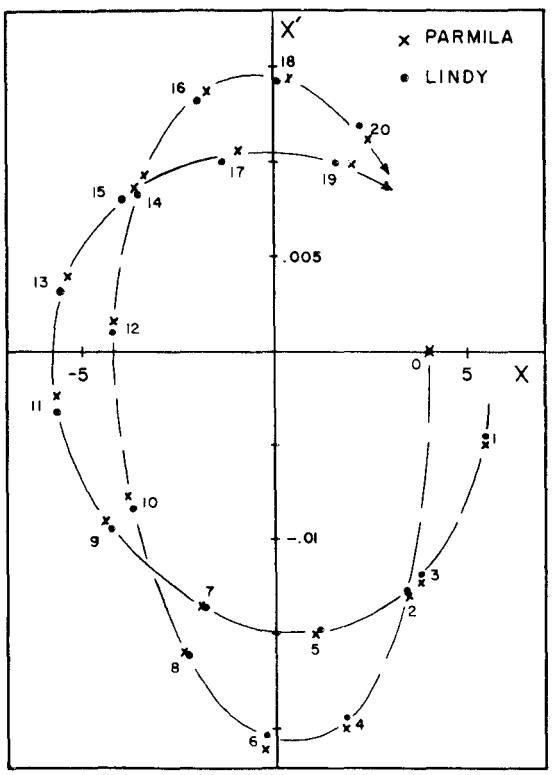


Fig. 3. Comparison of longitudinal motion as calculated by PARMILA and LINDY.



COMPARISON OF DYNAMICS PROGRAMS

Fig. 4. Comparison of transverse motion as calculated by PARMILA and LINDY.