

# PHASE MEASUREMENT ALONG LINAC SECTIONS BY A RESONANT METHOD

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During the construction of an electron linac, it is necessary to check the amplitude and phase of the rf field along the accelerating structure, in particular along the bunching section. It is then possible with the help of a computer to deduce the behaviour of the electron beam during operation and by modifications of the structure, to optimize it.

Up to now, these measurements have been achieved by a non-resonant perturbation method which has been described by K. B. Mallory, R. H. Miller, and C. Steele from Stanford University (1). It consists in introducing a perturbation such as a metal or dielectric bead at the point where the field is being measured, the structure being fed as in normal operation; the amplitude and phase of the reflected wave are directly related to the amplitude and phase of the incident wave at the location of the bead. As simple as it seems, this method is of difficult application because of the smallness of the reflexion due to the bead, hence a low sensitivity. It requires also an almost perfect match of the structure to its load and of the generator to the structure in order for the spurious reflexions not to overshadow the one of interest.

We have tried to develop a different method which relies on resonance perturbation technique as we know by experience that such measurements can be made quite accurately.

## I- PRINCIPLE OF THE MEASUREMENT

Let us consider the arrangement of Figure 1.

The structure under test S is being fed from a UHF source G through a transition T (which need not be reflexionless). The structure is connected at the other end to a standard waveguide W by means of a transition T' which is supposed to introduce no reflexion. The waveguide is short-circuited by a movable piston at point R.

In the structure, the field is the sum of two waves, one traveling forward, the other backward.

The basic principle of the measurement lies in the fact that at some point P in the structure, each backward wave component (\*) cancels out the corresponding component of the forward wave so that the total field is zero at this point.

As the piston R is moved, the zero-field point P moves and its motion in the z direction is directly connected to the piston motion; the difference of phase between the node at the piston and the node at the zero-field point is a constant number of half-wavelengths.

This gives a method for measuring phase shifts between zero-field points in the structure. A theoretical justification of this method is given in the appendix.

The field amplitude of the traveling wave at a given node position where forward and backward waves are of equal amplitude and opposite in phase is half the amplitude of the standing wave measured at the same position when the piston R is moved by a quarter wavelength, since forward and backward waves are of equal amplitude and in phase. The input transition T is designed in such a way as to give a small coupling between generator and structure in order to achieve resonance in the structure. As a result, sensitivity and accuracy of measurements are sharply increased.

(\*) By component we mean a component of the total field such as  $E_x$ ,  $E_y$ , ...

## II- TECHNIQUE OF MEASUREMENT AND EXPERIMENTAL RESULTS

We use the arrangement of Figure 2.

The structure under test is fed by a sweep generator through a coupling loop. This is the transition T of Figure 1 which need not be reflexionless.

Transition T' is the power input for a linac structure. Here a good match is required, because a mismatch would produce a periodic shift in phase. The amplitude  $\theta^\circ$  of this shift in phase is given by:

$$\sin \frac{\theta}{2} = |\Gamma|$$

Reference (2)

(For a mismatch corresponding to  $s = 1.05$  the phase shift  $\theta$  is  $3^\circ$  -).

The transmitted power curve of the periodic resonant structure is compared on an oscilloscope to the transmitted power curve of a high Q reference cavity to determine its resonant frequency.

For each position of the movable piston R, a piston S is adjusted for resonance of the periodic structure at the frequency of its normal operation as an accelerator.

Frequency shifts of the whole resonant structure give the amplitude of the fields at the location of the perturbation P and the points where no shift occurs give the location of the nodes.

Plots of positions in phase of piston R as a function of a node position along the periodic structure give the phase law along the structure.

As already mentioned, the field amplitude of the traveling wave at a given node position is half the amplitude of the standing wave measured at the same position when the piston R is moved by a quarter wavelength. So as a practical procedure we note the positions of nodes, of maxima of fields and the frequency shifts corresponding to these maxima, for each position of piston R.

Figure 3 shows the phase law at the beginning of a structure for the  $\frac{2\pi}{3}$  mode operation and particles traveling at the speed of light.

### - APPENDIX -

#### Theoretical justification of the resonant method

Let us consider the arrangement of figure 1.

We shall make the following assumptions :

1) The UHF circuit can be considered as almost lossless. This assumption is made for the sake of simplicity but is not essential.

2) The structure to waveguide transition is reflexionless. When this condition is not fulfilled a periodic shift in phase is superposed to the periodic structure law of phase.

3) The structure is assumed to carry a single mode. The frequency is supposed to be constant unless otherwise mentioned and in the analysis the factor  $e^{j\omega t}$  is systematically omitted.

We have to prove the following points :

1) The forward and backward waves have the same absolute amplitude at each point in the structure.

2) The phase of the travelling wave field component (either forward or backward) is directly related to the piston position.

We now proceed to prove the first point :

Due to the fact that the system is supposed to be lossless, both waves carry the same power in opposite directions.

If one calls  $E_i(x, y, z)$  one of the components of the electric field propagating in the forward direction, this component can be written as :

$$E_i(x, y, z) = A(x, y, z) e^{-j\varphi(x, y, z)} \quad (1)$$

A being the absolute value of the field and  $\varphi$  its phase.

$E_i$  is a particular solution of Maxwell's equations ; the system being lossless its complex conjugate  $E_i^*$  will also be a solution. It can be written :

$$E_i^*(x, y, z) = A(x, y, z) e^{+j\varphi(x, y, z)} \quad (2)$$

This solution which satisfies Maxwell's equations and the boundary conditions obviously represents a backward travelling wave. So the general form of this particular field component in the structure can be written as

$$E_{i \text{ total}}(x, y, z) = A(x, y, z) e^{-j\varphi(x, y, z)} + \alpha A(x, y, z) e^{+j\varphi(x, y, z)} \quad (3)$$

$\alpha$  being a complex constant. Because both waves carry the same energy,  $|\alpha|$  has to be unity and can be written as

$$\alpha = e^{j\psi}$$

$\psi$  being a constant, which proves our point. The final expression for the total field component is then :

$$E_{i \text{ total}}(x, y, z) = A(x, y, z) \{ e^{-j\varphi(x, y, z)} + e^{j(\psi + \varphi(x, y, z))} \} \quad (4)$$

which can also be expressed as :

$$E_{i \text{ total}}(x, y, z) = A(x, y, z) e^{-j\frac{\psi}{2}} 2 \cos(\varphi(x, y, z) + \frac{\psi}{2}) \quad (5)$$

The last expression implies that, as  $\varphi$  varies in function of  $z$ , the  $i$ th component of the field goes through zero for

$$\varphi(x, y, z) = n\pi + \frac{\psi}{2} - \psi$$

This development shows that the theory holds for any component of the field for any  $x, y$  position and for any structure even if it is not periodic in  $z$  because no assumption has been made concerning this point.

We now proceed to prove the second point :

The fact that the transition T' is matched means that between two forward components of the fields at points A et B there is a relation :

$$E_{jB} = T' E_{iA}$$

T' being the transfer coefficient which is constant. The following relation

between the corresponding backward components is valid :

$$\underline{E_{iA}} = \frac{1}{T'^*} \underline{E_{jB}}$$

The field propagating forward in the periodic structure is

$$\underline{E_z} = A(z) e^{-j\varphi(z)}$$

At point A this field becomes :

$$\underline{E_z(\ell)} = A(\ell) e^{-j\varphi(\ell)}$$

Calling  $k$  the propagation constant in the waveguide and  $s$  the distance between point B and the piston :

$$\underline{E_z(\ell)} = \underline{E_z(\ell)} \frac{T'}{T'^*} e^{-2jks}$$

if  $T' = |T'| e^{-j\theta}$

$$\underline{E_z(\ell)} = \underline{E_z(\ell)} e^{-2j(ks+\theta)}$$

Now, we can write :

$$\underline{E_z(z)} = A(z) e^{-j\varphi(z)}, \quad \underline{E_z(z)} = A(z) e^{j(\varphi(z)+\psi)}$$

$$\underline{E_z(\ell)} = A(\ell) e^{-j\varphi(\ell)}, \quad \underline{E_z(\ell)} = A(\ell) e^{j(\varphi(\ell)+\psi)}$$

and

$$\underline{E_z(\ell)} = \underline{E_z(z)} \frac{A(\ell)}{A(z)} e^{-j(\varphi(\ell)-\varphi(z))}$$

hence

$$\underline{E_z(z)} = \underline{E_z(\ell)} \frac{A(\ell)}{A(z)} e^{j(\varphi(\ell)-\varphi(z))}$$

$$\underline{E_z(z)} = \underline{E_z(z)} e^{-2j(ks-\varphi(z)+\varphi(\ell)+\theta)}$$

So that the total field at point  $z$  becomes 0 when

$$\cos(ks - \varphi(z) + \varphi(\ell) + \theta) = 0$$

or

$$ks - \varphi(z) = n\pi + \frac{\pi}{2} - \varphi(\ell) - \theta$$

If the piston is moved from  $S_1$  to  $S_2$ , the position of the zero field point will move from  $z_1$  to  $z_2$ , according to the relation

$$\varphi(z_2) - \varphi(z_1) = k(S_2 - S_1)$$

#### - REFERENCES -

(1) "On non resonant Perturbation measurements", K.B. Mallory and R.H. Miller, IEEE-MTT, february 1966.

"A Non-resonant perturbation theory, C. Steele, IEEE-MTT january 1966

(2) "Microwave measurements", E.L. Ginston, Mc Graw-Hill 1957.

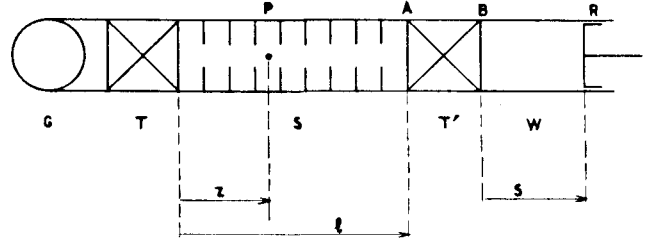


Fig. 1.

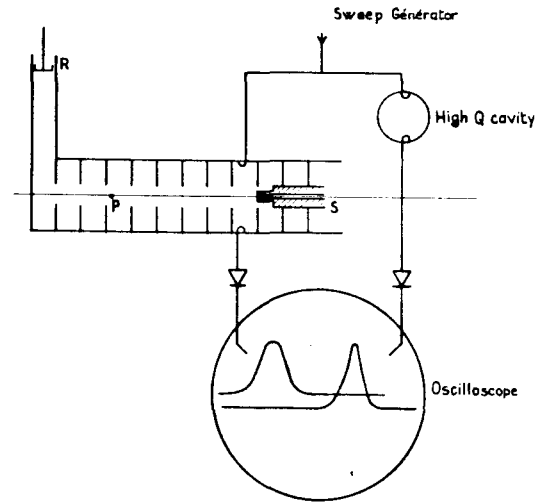


Fig. 2.

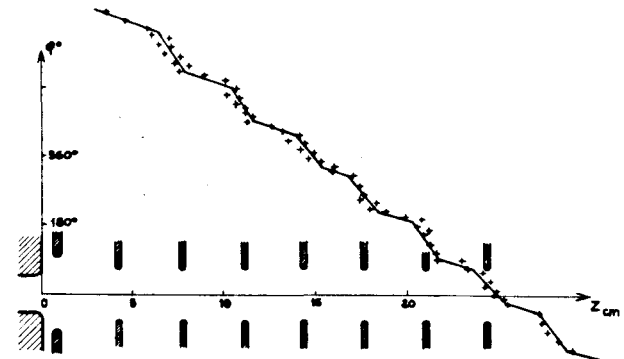


Fig. 3.