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DESIGNING RESONANT CAVITIES WITH THE LALA COMPUTER PROGRAM*

Harry C. Hoyt

Los Alamos Scientific Laboratory
Los Alamos, New Mexico

This paper reports a few of the cavity design studies which have been carried out at the Los Alamos Scientific Laboratory using the LALA computer program. The three studies included are concerned with $\beta = 0.65$ proton linac cavities, $\beta = 1$ electron linac cavities, and the cavity in the coaxitron microwave amplifier tube. The calculations using the LALA program were done by Walter Rich, Dennis Simmonds and the author.

The essential features of the LALA computer program have been described in an article by Hoyt, Rich, and Simmonds which appeared in the June 1966 issue of The Review of Scientific Instruments. Briefly, the program is a numerical mesh calculation which determines the fields and frequency of a resonant cavity having cylindrical symmetry. The wave equation, together with appropriate boundary conditions, is solved by the use of difference equations and the method of relaxation. Only those modes which have no Θ-component for the electric field are considered. Usually we are concerned with the lowest frequency mode. Design parameters such as the transit time factor T, power loss P, Q, and the shunt impedance per unit length ZT^2 are computed from the field distribution obtained for a given cavity.

The LALA program achieves considerable flexibility by approximating the actual boundary of the cavity by a zig-zag boundary lying on or outside of the actual boundary. (See Fig. 1.) This zig-zag boundary follows mesh lines. As can be seen from the figure, symmetry is used to reduce the number of mesh points required in the calculation.

Several different relaxation methods have been used. These are point-by-point relaxation, line-by-line relaxation, and alternating direction line-by-line relaxation. These procedures are set up to go from left to right and from bottom to top across the mesh. The point-by-point method solves for the magnetic field at each point in turn, substituting new values for the old values as they are obtained. The line-by-line method solves for all values on one line simultaneously; these values then replace the old values. The alternating direction line-by-line method is similar to the line-by-line method except that it proceeds through the mesh by rows on one iteration and by columns on the next iteration. Experience to date has shown no significant difference between the results obtained with the three methods, provided we have a good initial guess for the solution. For a somewhat poorer initial guess for the solution, the alternating direction method usually is preferred. Convergence times for the three methods are very nearly the same.

Our studies on 805 MHz proton linac cavities have led to shaped cavity designs similar to that shown in Fig. 2. In these designs, the outer wall has a circular cross-section, and the drift tube has a triangular shape. The radii r_1 and r_2, together with the angle Θ, characterize the shape of the drift tube. The radius r_3 describes the shape of the outer wall. L is the length of the cavity. The wall thickness t_w and the drift tube hole radius r_H are kept constant for most studies, since they usually are determined by considerations other than the value of shunt impedance per unit length, ZT^2. Most of our 805 MHz cavity calculations have used the values of r_H, r_1, r_2, Θ and t_w given in Fig. 2. For constant frequency, the two remaining quantities, the drift tube length ℓ_{DT} and the maximum cavity radius r_{max} are not independent. A change in one requires a change in the other.

Knapp and his co-workers at the Los Alamos Scientific Laboratory have constructed a set of six $\beta = 0.65$ cavities to check the variation of ZT^2 and Q over a range of values of r_{max} from 4.5 inches to 5.75 inches. Table I presents a comparison of calculated and measured values of frequency f, transit time factor T, and the ratio ZT^2/Q for these cavities. In general the agreement is good; computed and measured frequencies agree to within 0.3 percent. Where the agreement seems poorer, this can be explained by dimensional differences between the specifications and the final machined cavity. We compare values of ZT^2/Q, rather than ZT^2 or Q, to eliminate the power P [$ZT^2/Q = (V^2T^2/PL)/(2\pi fU/P) = V^2T^2/2\pi fUL$. V is the voltage across the cavity, and U is the stored energy.]. Then impure copper and machining marks do not appear in the comparison. If the calculations are made for the exact dimensions of a cavity, we have excellent agreement. Table II presents the data for a case where this was done. Fig. 3 shows the comparison of experimental and computed values of the electric field along the axis for this cavity.

The set of six problems with different values of drift tube length and maximum cavity radius enables us to determine dimensions for intermediate sets of values. Fig. 4 shows the relationship between drift tube length ℓ_{DT} and cavity radius r_{max} for $\beta = 0.65$ and frequency f = 805 MHz. The variation of the shunt impedance per unit length ZT^2 as a function of drift tube length is presented in Fig. 5. In this figure the drift tube length is given as a fraction of the half-length of the cavity. The quantity g/L, gap length divided by cavity length, is given at the top of the figure. As g/L decreases below 0.4, the value of ZT^2 falls rapidly. For the values of g/L above 0.4, ZT^2 is represented as constant, since our calculational accuracy for these particular problems is not sufficient for us to say that the apparent maximum is real.

Recently we have studied some possible cavities for a superconducting electron linac. This work has been done in collaboration with the Stanford University group. Fig. 6 shows three cavities for

β = 1 electrons, representing the limits and center of the range of configurations considered. These cavities are designed for a frequency of 952 MHz. The thick end wall indicated actually includes the region of the coupling cavity, which is to be located on-axis between adjacent accelerating cavities. This wall of the cavity has a long straight section where coupling holes will be cut. The tables show the variation of T, ZT^2 and Q as the drift tube is introduced into the cavity and lengthened. The Stanford group was interested in having the magnetic field strength a maximum at the coupling holes. The LALA program provides a plot of the value of H at the metal surface which enables one to check the field values very easily. This plot is shown in Fig. 7 for the problem with the longest drift tube. H at the metal surface is plotted against distance along the boundary from the point on the metal surface at the center of the drift tube. A second computer-generated plot, of electric field strength at the metal surface, also is very useful in designing these cavities. Fig. 8 shows the plot of E versus distance along the boundary for the same problem. The peak value of E occurs at the upper corner of the drift tube nose. The jagged appearance of the curve is due to evaluating derivatives numerically.

The LALA program also can be used to compute fields and frequencies for other types of cavities. For example, we have computed the fields in a model of the cavity in a coaxitron microwave amplifier tube. Fig. 9 shows our model of two such cavities. On the left side is the configuration representing the coaxitron which has been built by RCA for Los Alamos. On the right side is a model with an extended anode. Each of these cavities is a figure of revolution about the axis shown; the drawings show only a cross-section including the axis. This type of cavity basically is of the type called a re-entrant coaxial cavity. The calculations had several purposes: (1) To use the electric field configuration as input to a tube efficiency calculation, (2) to locate the position of zero electric field strength to check positioning of cooling and support tubes and (3) to investigate changes in shape which might reduce anode heating. Incidentally, it was necessary to learn how to calculate some higher frequency modes, since the coaxitron does not operate in the lowest frequency mode for this type of cavity. Both of the cavities shown in this figure operate at 805 MHz. Lengthening the anode did not change the efficiency of the tube, so the second model is a good candidate for such a tube. Fig. 10 shows the electric field configuration for the model with the lengthened anode. For this calculation we have made use of symmetry and have omitted half of the cavity. An aluminum model of this cavity has been constructed and tested. The comparison between measured and computed frequencies is considered to be quite satisfactory. Bead pulling experiments have shown substantial agreement between computed and actual field values. A mode of lower frequency has been computed; the aluminum model also will operate in this mode.

The LALA program has been used to calculate a wide variety of microwave cavities and has given very satisfactory results. Computer programs of this type certainly will be major design tools in the future.

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DISCUSSION

H. HOYT, LASL

YOUNG, MURA: Have you done any calculations at very low beta, say at 200 Mc structures for a cavity length of about 7 cm?

HOYT: We have done calculations for the three Brookhaven models, and the results that I will give you here will, in part, clear up some of the difficulties and will, in part, confuse the issue a little more. First of all, for the cavity that you mentioned, we computed a frequency of 202 MHz vs the 200 MHz that MESSYMESH and JESSY got. The transit time factors that we computed for all three of these cavities agree very well with the MESSYMESH values. This means that, for the short cavity, we do not agree with the JESSY computed value. The values of ZT^2 seem to be pretty much in agreement with the MESSYMESH values. We find that, as we go to the longer cavities, that our computed frequencies agree more and more closely to those computed by MESSYMESH and JESSY. For example, for the longest of the three cavities, which was 52 cm long (a half-length of 26 cm), all three of the calculations agreed with each other to something like 0.3 Mc. I consider that quite remarkable. I should point out here that the three programs really use slightly different methods for computing the frequency, and some of the differences that one will see can be attributed to this. Another thing that you have to keep in mind with these calculations, is that you can't tell when you have converged to an answer unless you know the answer beforehand. So each of us has to make a rather arbitrary decision as to when we feel that our calculation has converged. All of us are using more or less the same criteria, but I think that the differences do lead to slightly different answers at the time when the computer decides to quit. I also wish to comment here that I, personally, don't feel terribly disturbed about the discrepancies that one observes between the computed and measured values for these very short cavities. In particular, these cavities are short and have very long drift tubes in them, that is, the G/L ratio is very small. This makes carrying out the calculation quite difficult. All of us have had the feeling that we were spending a fortune in doing the problem for the 7-cm cavity, and so I think from a money standpoint, that we would be perfectly willing to quit on this particular problem. The two reasons that I am not terribly disturbed about this are the following:

First: Because of this small value of G/L, the calculation just isn't as good as it would be otherwise. The principal difficulty, I believe, is the communication problem between the hole of the drift tube and the upper region of the cavity. This is very difficult to fix in a numerical program. Second: I am not terribly concerned about it, because the minute somebody begins building one of these accelerators, the first thing he is going to do is to put a stem on that drift tube, and all bets are off the minute you do that, because the calculation just doesn't apply to the cavity any more. So you always have to make an experimental correction in the end, and so I say that if you are within about 1% on these frequencies, you are really fulfilling your duty as far as producing designs is concerned.

BLEWETT, BNL: Could I ask why, for a super-conducting cavity, you choose the structure that you have sketched on the board rather than a side-coupled structure?

HOYT: The choice really was made by the Stanford people. I believe that the principal reason for choosing this one had to do with plating the structure. The side-coupled structure is quite difficult to plate in the coupling cavity because the cavity is small and hard to get to, and you have difficulties in pouring solution out afterwards and so on. Use of on-axis coupling cavities improve the situation greatly.

KNAPP, LASL: I would like to make a short comment on this. This structure is the alternating periodic structure which has been developed and talked about quite a bit by Sal Giordano from Brookhaven. For this particular application I think this is probably a better way to do it. The side-coupled cavity and this cavity are essentially the same thing electrically although they look quite different mechanically.

HUBBARD, LRL: Is the coupling through the beam hole, or are there other apertures?

HOYT: There will be other apertures in the end wall; I think they are planning on having three triangular shaped slots.

TABLE I.

$\beta = 0.65$ Models

Comparison of Calculation and Experiment

r_{MAX} (cm)	11.430	12.065	12.700	13.335	13.970	14.605
ℓ_{DT} (cm)	4.3723	4.0802	3.7576	3.3893	2.9524	2.3809
$f_{calc.}$(MHz)	806.1	805.8	806.4	807.0	805.8	803.7
$f_{exp.}$	805.9	804.5	805.2	804.7	804.9	803.2
$T_{calc.}$	0.906	0.894	0.878	0.857	0.831	0.795
$T_{exp.}$	0.904	0.881	0.859	0.828	0.817	0.779
$ZT^2/Q_{calc.}$	1705	1764	1766	1718	1622	1523
$ZT^2/Q_{exp.}$	1605	1758	1816	1732	1696	1543

All models have $\frac{L}{2} = 6.0517$ cm, $r_H = 1.9050$ cm, $t_W = 0.47625$ cm, $\theta = 30^\circ$, $r_1 = 0.39878$ cm, $r_2 = 1.0008$ cm, and $r_3 = 5.5755$ cm.

TABLE II

Comparison of computed and measured values for a $\beta = 0.65$ cavity with $\ell_{DT} = 2.4206$ cm and $r_{MAX} = 14.5241$ cm.

Quantity	Computed Value	Measured Value
f	804.33 MHz	804.24 MHz
T	0.795	0.778
ZT^2/Q	1518	1502

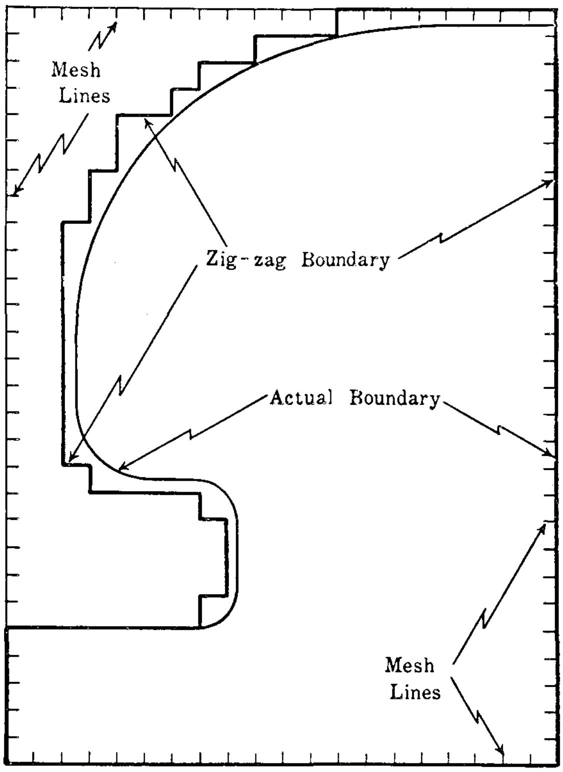


Fig.1. Actual boundary and zig-zag boundary

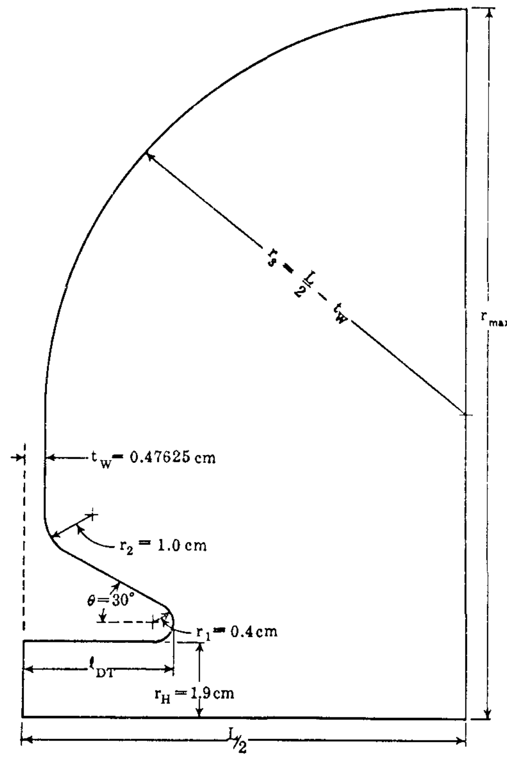


Fig.2. Shaped 805 MHz Proton Linac Cavity

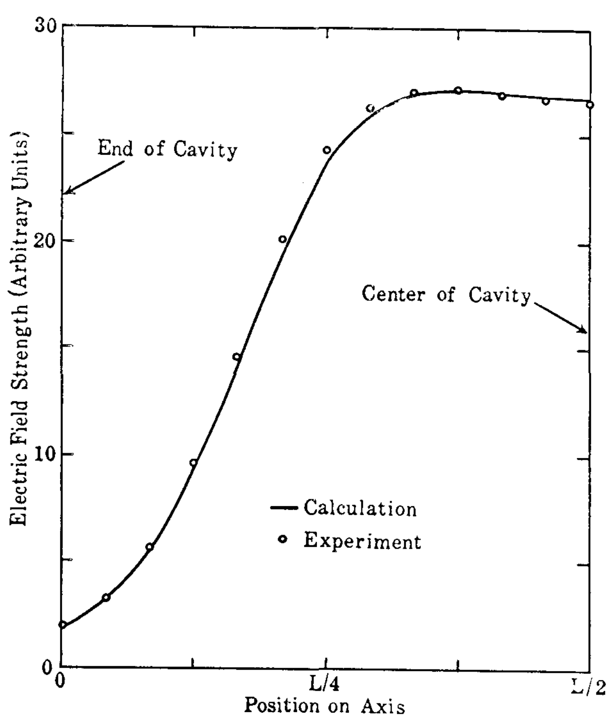


Fig.3. Electric field on axis of cavity of Table II.

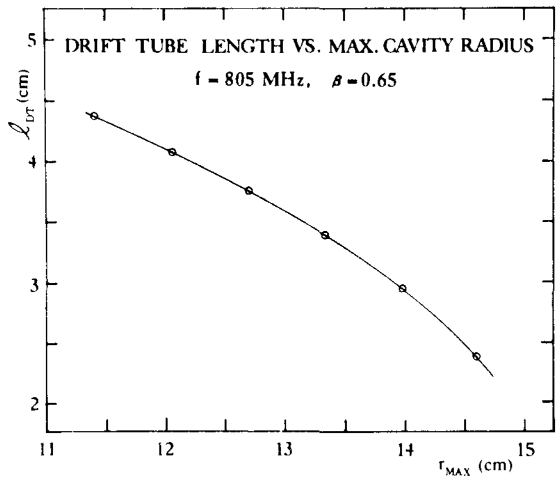


Fig.4. Relationship between ℓ_{DT} and r_{MAX} for cavities of Table I.

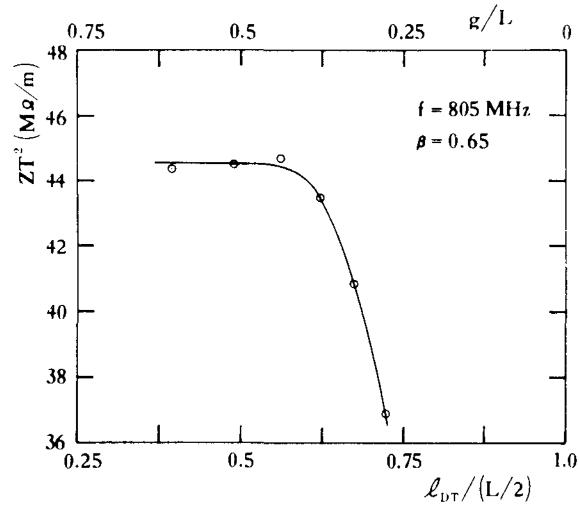
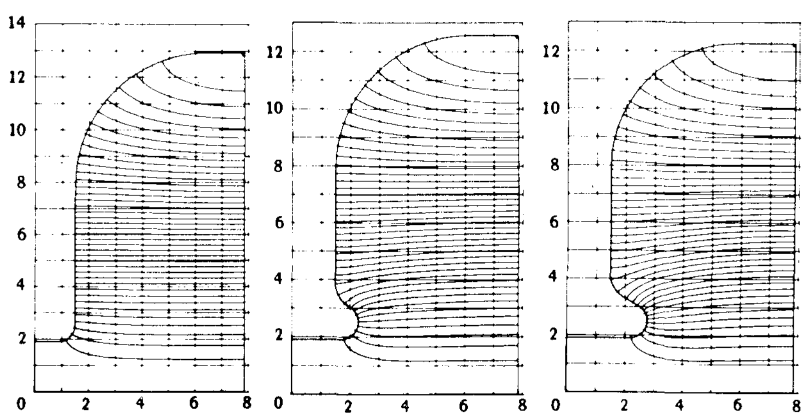


Fig.5. Variation of Shunt Impedance ZT^2 with drift tube length for cavities of Table I.

$\beta = 1$ ELECTRON LINAC CAVITIES



Problem No.	69.35	69.38	69.18
r_1 cm	0.6135	0.6135	0.6
r_2 cm	———	1.0	1.0
r_3 cm	4.975	4.6	4.3
ℓ_{DT} cm	(1.5)	2.3065	2.79
r_{MAX} cm	12.930	12.555	12.255
f MHz	950.6	953.5	950.6
T	0.738	0.762	0.788
ZT^2 MΩ/m	43.3	47.3	49.3
Q	34300	33000	31700

$L/2 = 7.8727$ cm, $r_H = 1.9$ cm, $t_W = 1.5$ cm.

Fig.6. Comparison of electron linac cavities.

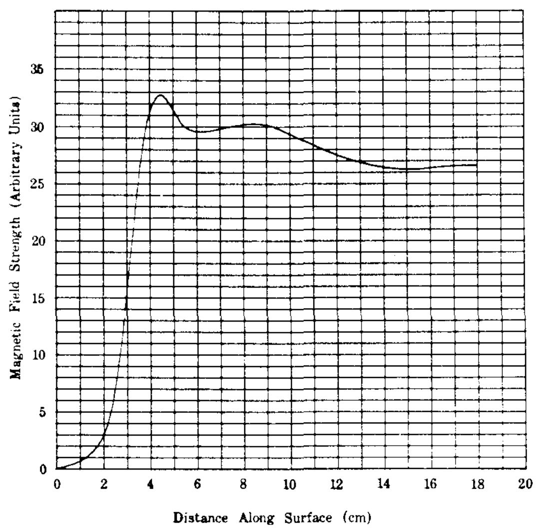


Fig.7. Magnetic field strength at conductor surface for Prob. 69.18.

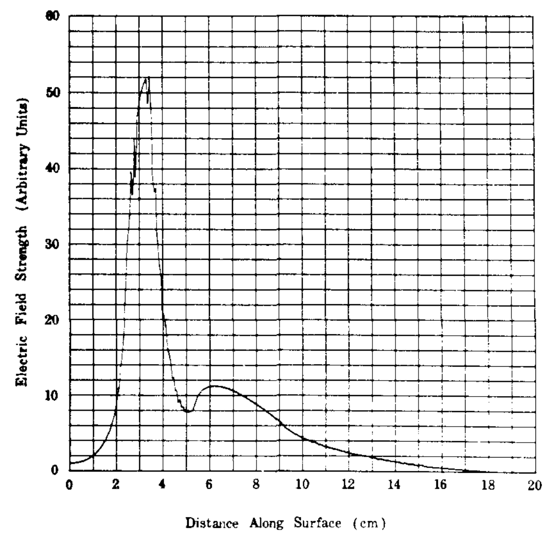


Fig.8. Electric field strength at conductor surface for Prob. 69.18

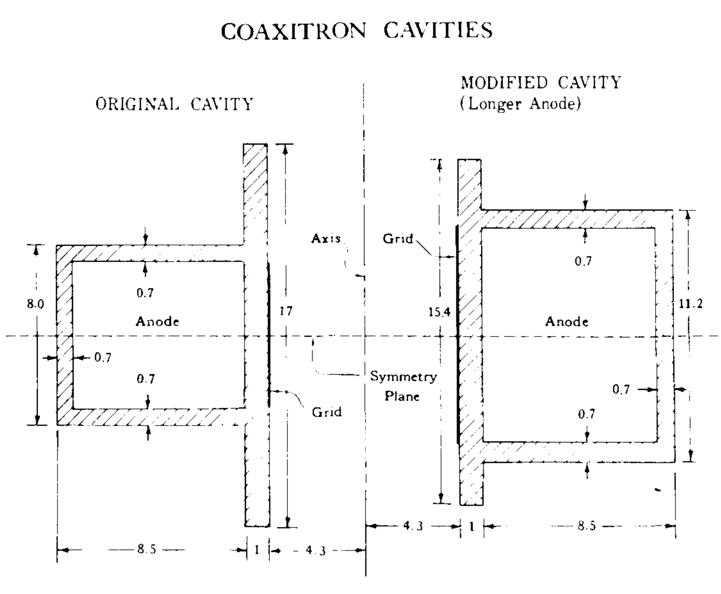


Fig.9. Calculational models of coaxitron cavities.

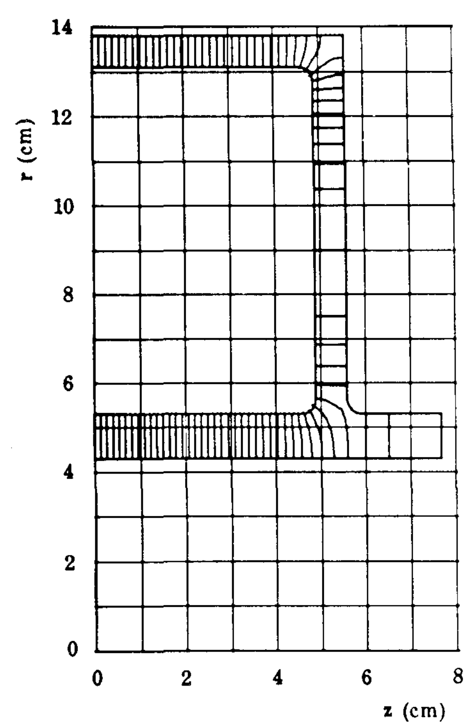


Fig.10. Computed electric field lines for coaxitron model with extended anode. $f_{calc} = 802.6$ MHz, $f_{exp} = 807.3$ MHz.