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TRANSVERSE MODES IN A RESONANTLY COUPLED ACCELERATOR*

J. M. Potter
Los Alamos Scientific Laboratory
Los Alamos, New Mexico

Summary

In predicting the possibility of transverse mode beam blow-up in a linear accelerator it is useful to have some idea of the transverse mode shunt impedance of a typical accelerating cell. This paper discusses the techniques for identifying the modes associated with beam blow-up and for measuring their shunt impedance. A family of six $\beta = 0.65$ cavities of similar geometry and varying diameters, typical of the accelerating cells in the LASL $\pi/2$ mode structures, and with the TM_{010} mode near 805 MHz were studied. The results of these measurements are presented.

Introduction

The cavities used in this study are similar to the accelerating cells of Model K, a 60 cell, $\beta = 0.65$, side-coupled structure now being assembled. Figure 1 shows a typical cross-section and lists the dimensions of the six cavities. The half-gap was varied with the diameter to keep the resonant frequency of the accelerating mode (TM_{010}) for each cavity near 805 MHz. All other dimensions were kept constant, as indicated in the figure.

Estimates of the half-gap were made by interpolation and extrapolation of results from the L.A.L.A. field configuration calculation program. All cells were made with a reasonable margin of safety in the drift tube length to allow them to be individually tuned to 805 MHz before the halves were brazed together. Frequencies calculated by the L.A.L.A. program from the final dimensions are within a megahertz or so of the measured frequencies for each cell; the discrepancies are attributed to overall machining tolerances. The shunt impedances and Q's of the accelerating mode were then measured. Figure 2 shows a comparison of these measurements to the predictions of the L.A.L.A. program. The difference between the predicted Q's and the measured Q's is considered to be the result of the machined surface finish in reducing the effective conductivity at 800 MHz. The measured values of ZT^2/Q and T (not shown in the figure) are considered to be in good agreement with the computer predictions.

Theory

We define the effective shunt impedance for transverse modes in the normal manner:

$$Z_{d_{eff}} = \frac{\left[\frac{1}{\ell}\int_0^{\ell} E_{d_z}(\vec{r}(t),t)\, dz(t)\right]^2}{P/\ell} \qquad (1)$$

The modes associated with beam blow-up are the TM_{11X} modes characterized by a zero of E_z on the axis. For these modes we may write, (where x is the distance from the axis):

$$E_z(z,x) = 0 + \left[\frac{\partial E_z}{\partial x}\right]_{x=0} x + \cdots \qquad (2)$$

Substituting the expression for E_z into the definition for shunt impedance above and removing the x dependence from the equation results in a modified definition for shunt impedance:

$$Z_{d_{eff}} = \frac{\left[\frac{1}{\ell}\int_0^{\ell}\left[\frac{\partial E_{d_z}(\vec{r}(t),t)}{\partial x}\right]_{x=0} dz(t)\right]^2}{P/\ell} \qquad (3)$$

The shunt impedance at any distance x from the axis can then be obtained by multiplying the results obtained from our new definition by x^2. For an isolated single cell with particles of a constant velocity βc incident along the z axis we have:

$$Z_{d_{eff}} = \frac{\left[\frac{1}{\ell}\int_0^{\ell}\left[\frac{\partial E_{d_z}(z)}{\partial x}\right]_{x=0} \cos\left(\frac{2\pi z}{\beta\lambda_d} + \psi\right) dz\right]^2}{P/\ell} \qquad (4)$$

where λ_d is the free space wavelength of the deflecting mode and ψ is a phase constant chosen to maximize $Z_{d_{eff}}$. We may separate out the time factor in the usual way by defining:

$$T = \frac{\int_0^{\ell}\left[\frac{\partial E_{d_z}(z)}{\partial x}\right]_{x=0} \cos\left(\frac{2\pi z}{\beta\lambda_d} + \psi\right) dz}{\int_0^{\ell}\left[\frac{\partial E_{d_z}}{\partial x}\right]_{x=0} dz} \qquad (5)$$

Thus $Z_{d_{eff}} = Z_d T^2$ where

$$Z_d = \frac{\left[\frac{1}{\ell}\int_0^{\ell}\left[\frac{\partial E_{d_z}(z)}{\partial x}\right]_{x=0} dz\right]^2}{P/\ell} \qquad (6)$$

To measure $\frac{\partial E_{d_z}(z)}{\partial x}$ we return to the original approximation:

$$\frac{\partial E_z(z,x)}{\partial x} \simeq \frac{E_z(z,x)}{x} \qquad (7)$$

We see from Slater's perturbation theorem that we may determine $E_z(z)$ by moving a suitable perturbing object along the desired path and measuring the corresponding change in the resonant frequency of the cavity.

$$\frac{\Delta f(z)}{f_o} = -\frac{3}{4}\,\frac{\epsilon_o E^2(z)}{U}\,\eta\, v_b \qquad (8)$$

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where η is the effectiveness of the perturbing object relative to a conducting sphere of volume V_b. For a dielectric bead

$$\eta = \frac{\epsilon-1}{\epsilon+2} \tag{9}$$

For a wire of diameter to length ratio γ

$$\eta = \frac{(1-\gamma^2)^{3/2}}{\frac{1}{2}\ln\frac{1+(1-\gamma^2)^{\frac{1}{2}}}{1-(1-\gamma^2)^{\frac{1}{2}}} - (1-\gamma^2)^{\frac{1}{2}}} \tag{10}$$

(relative to a sphere of diameter equal to the length of the wire) where it is understood that only the component of E parallel to the wire is affected. Solving (8) for E and substituting the approximation (7) into equation (6) gives:

$$Z_d = \frac{4}{3}\frac{U}{\epsilon_o \eta V_b P \ell f_o x^2}\left[\int_o^{\ell}\sqrt{-\Delta f(z)}\,dz\right]^2 \tag{11}$$

and the transit time factor T is

$$T = \frac{\int_o^{\ell}\sqrt{-\Delta f(z)}\cos\left(\frac{2\pi z}{\beta\lambda_d}+\psi\right)dz}{\int_o^{\ell}\sqrt{-\Delta f(z)}\,dz} \tag{12}$$

Equation (11) is not of much use until we note that

$$Q = \frac{2\pi f_o U}{P}\,. \tag{13}$$

This results in our operational definition of transverse shunt impedance.

$$\frac{Z_d}{Q} = \frac{4}{3}\frac{1}{\epsilon_o \eta V_b \ell 2\pi f_o{}^2 x^2}\left[\int_o^{\ell}\sqrt{-\Delta f(z)}\,dz\right]^2 \tag{14}$$

Identification of Modes

Identification of cavity resonant modes is at best a tricky business. It is probably impossible to identify all the modes one might find in a typical 805 MHz linac accelerating cavity in the spectrum below 2 GHz with only three probe holes and the beam hole for access to the cavity interior. To avoid ruining our set of brazed copper cavities an aluminum duplicate of one of them was made with enough holes in it to permit positive identification of the various cavity resonances.

Once one has a cavity with enough holes in it, he is faced with two possible approaches to the problem of identifying its resonant modes. Perhaps the most obvious technique is to drive a given mode with a suitable signal source and use magnetic and electric probes in conjunction with a detector to map out the field magnitude and direction. This technique has many limitations. Observations are limited to the fields near the cavity surface. Field magnitude measurements are difficult to reproduce. Shielded magnetic probes must be used if one is to eliminate their coupling to the electric field. Degenerate sets of modes result in confusion because of the difficulty of driving only one of the modes with a finite sized probe.

These problems may be circumvented by utilizing the predictions of the Slater perturbation theorem. Using a sweep generator in conjunction with an oscilloscope display to observe the resonances one can easily measure the effects of various perturbations on the frequency of the mode being investigated.

The Slater perturbation theorem predicts a frequency change proportional to E^2 for a dielectric sphere and proportional to $E^2 - \frac{1}{2}H^2$ for a conducting sphere. While it is possible to get large enough spheres through the beam hole to get an observable perturbation, holes large enough to pass these beads would themselves seriously perturb the cavity resonances. Also these beads give no indication of field direction, a serious handicap in distinguishing between some modes. A conducting needle, on the other hand, can have a large effect from the component of E parallel to its axis, while having a quite smaller effect from the perpendicular component of E or either component of H.

This then serves as the basis for our investigation. One can observe directly on the oscilloscope the great effect of the needle when placed in a region of maximum E and the negligible effect when placed in a region of minimum E. Identification is made by comparison of the response obtained to the response expected as shown on the circular waveguide charts (see Fig. 3) in the various handbooks with possible recourse to the field-describing equations in order to interpret the diagrams. Since the waveguide diagrams do not apply directly to resonances in right circular cylindrical cavities it is necessary to remember the boundary conditions for E on the cavity end walls, i.e., the parallel component of E must be zero on a conducting surface. Assignment of the third integer to the mode follows directly from the number of reversals of E and H along the axis.

In retrospect, the experience gained in working with the aluminum model, with its many holes, has enabled us to identify the modes which can cause beam blow-up, the TM_{110} and TM_{111} modes with only two probe holes, for source and detector, and with the only access for perturbing objects being through the beam hole. In fact probably all of the modes listed in Table I which we found in the 10.5 inch model could be positively identified with needles and access only through the beam hole with the possible exception of the TM_{020} mode would, with these tests, appear identical to the TM_{010} mode. The confusion is immediately eliminated, however, since we hope the mode at 805 is the TM_{010}. The TM_{11x} series of modes, in particular, are quite readily identified by these techniques. Considering the mode diagrams in Fig. 3, one expects the mode to orient itself so that the regions of E maximum lie in plane joining the source and detection loops (180° apart) and containing the longitudinal axis of the cavity. Inserting a needle near the periphery of the beam hole, parallel to the axis, and in this reference plane perturbs this family of modes greatly. The perturbation is enhanced by the presence of the drift tubes which serve to bring the electric field maxima nearer the axis. Placing the needle anywhere in the plane perpendicular to our reference plane, including directly along the cavity axis, results in no noticeable perturbation. Placing the needle at intermediate positions on the periphery of the beam hole, splits the degeneracy as one would expect.

This behaviour is identical for the entire family of TM_{11x} modes. Thus it is necessary to distinguish between them by studying the longitudinal variations in an electric field. If we were dealing with right circular cylinders distinguishing the TM_{110} mode from the TM_{111} could be done with a dielectric bead since one expects E(z) (slightly off the axis) to be constant in the former case and exhibit a maximum in the latter. The presence of the drift tubes eliminates this means of distinction since it distorts the fields and results in the TM_{110} exhibiting a maximum in E(z) also.

A sensitive test to distinguish the modes is to insert a conducting disc perpendicular to the axis, through the beam hole. In the absence of an H_z (since we are dealing with transverse magnetic modes) we expect a perturbation only if there is a radial component of E present. For the TM_{110} E is parallel to the axis (except near the drift tubes) and there will be no perturbation from the disc. However the TM_{111} mode has a maximum in the radial component at the center of the cavity and is therefore quite readily distinguished from the TM_{110}. Figure 4 shows the frequencies of the TM_{110} and TM_{111} modes versus diameter for the six cavities tested.

Shunt Impedance Measurement

Once these modes have been singled out one may proceed with the task of measuring this shunt impedance. The technique for measuring the transverse mode shunt impedance is only slightly modified from the procedure used on the accelerating mode. We cannot get away with using a conducting bead as a perturber as we could for the TM_{010} mode since the TM_{11x} modes have a large magnetic field near the axis.

For the TM_{110} mode, which has no significant radial electric field component near the axis, one may use a dielectric bead. The relative efficiency of the bead may be calculated if the dielectric constant is known for the frequencies at which it is to be used or it may be calibrated quite simply by comparing the perturbation it produces in a magnetic field free region of a cavity compared to the perturbation from a conducting bead of the same diameter at the same position in the cavity. A suitable test cavity would be one whose TM_{010} mode is near the frequency of the higher order modes. We used a half scale model of accelerator cavities which resonated at 1600 mc.

The efficiency factor we measured for the sapphire bead used in these tests was 75% nearly the same as we measured at 800 MHz in earlier tests. This corresponds to a dielectric constant of 10. The dielectric constant of most common materials is not a very strong function of frequency in the region of interest and it would suffice to calibrate the bead at the TM_{010} mode frequency of the cavity being studied. Teflon for instance has a dielectric constant of 2.1 (η = 27%) almost independent of frequency up to 10 GHz. However, sapphire and ruby beads are more useful because of their high dielectric constant.

For the TM_{111} mode it was necessary to use a short needle, parallel to the axis, as the perturber to single out the axial component of electric field. The effectiveness coefficient for the needle may be calculated as described above. For a typical needle (say of aspect ratio 0.025) the effectiveness for the parallel component of E is about 10% compared to a conducting sphere of diameter equal to the length of the needle. The effect on H and the perpendicular component of E is down another factor of 10 or so.

To make these measurements properly it is necessary to first remove the degeneracy with a suitable perturbation. This can be done with a thin wire through the cavity parallel to the axis on the periphery of the beam hole, 90° from the reference plane. In each case the perturber is pulled along on a nylon string parallel to the axis but some distance x from it and the resonant frequency of the perturbed cavity versus position of the perturber is observed. From this data the shunt impedance may be calculated as described earlier.

The experimental set up is shown in Fig. 5. The oscilloscope display is set for an indication of zero phase shift for maximum transmission through the unperturbed cavity. As the position of the perturber is varied the frequency is adjusted for zero phase shift indication and the corresponding change in frequency is noted as a function of position. It is desirable to have a stable rf source which may easily be tuned over a small range. Varying the i.f. reference in a phase-locked system is a convenient way to achieve this. For accurate results it is desirable that the maximum perturbation to be measured exceeds the 3 dB bandwidth by a factor of 10 or so. Another method which we have used successfully at 800 mc is to include the cavity in the feed back loop of an amplifier, which can then be made to oscillate at the frequency of the mode being studied. The frequency of oscillation will then vary as the perturber is moved through the cavity. This technique is valuable when used with a frequency counter and digital-to-analog converter when a graphical display is desired. Some care must be taken with this method to insure that the phase shift change in the external circuit is small for the frequency changes to be observed.

Experimental Results

Figure 6 shows typical data from this experiment. One curve is a plot of $\sqrt{\Delta f(z)/\eta}$; the other is $\sqrt{\Delta f(z)/\eta}\cos 2\pi(z-z_0)/\beta\lambda_d$. Table II shows the results obtained thus far. The two measurements made in the reference plane on the 10.5 inch model agree reasonably well. The third measurement, made at 45° to the reference plane also agrees with the other two since $E(\vartheta)$ should vary as $\cos\vartheta$ and thus Z as $\cos^2\vartheta$. The measured value of shunt impedance at 45° is about one half ($\cos^2 45°$) of the value measured in the reference plane.

The fourth measurement reflects the decrease in transit time factor one expects from the increased g/ℓ.

The fifth measurement, on the TM_{111} mode, is only an estimate based on the maximum perturbation measured on each side of the center of the cavity. Because of the low effectiveness of the perturbing needle the maximum perturbation was only 25 kc compared to 250 kc or so for some other cases.

The Q's quoted in Fig. 7 were measured with a power meter by the transmission method.

Conclusion

The shunt impedances of the transverse modes measured are quite low compared to the 45 MΩ/m shunt impedance of the accelerating mode, even at two centimeters off the axis. In any case, it appears to be a simple matter to choose the accelerating cell dimensions to avoid exciting either the TM_{110} or TM_{111} modes strongly. Further work is indicated to more completely determine the transverse mode shunt impedance as a function of diameter for the family of six cavities which we tested. We also plan to determine the band width of these unwanted modes in a long structure of similar design and investigate means of reducing their band width and spoiling their Q.

Acknowledgments

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Table I

Mode Spectrum for the 10.5 inch $\beta = 0.65$ Cell

Frequency (MHz)	Mode
804.8	TM_{010}
1519.3	TM_{110}
1628.6	TM_{011}
1651.8	TM_{111}
1870.1	TM_{210}
1885.0	TM_{020}
1985.1	TM_{211}

DISCUSSION

J. POTTER, LASL

BLEWETT, BNL: Can you say anything about the effect of coupling slots on the transverse modes?

POTTER: Since the TM_{010} and TM_{011} modes have a degeneracy, another mode exists at a 90° orientation at the same frequency. It is necessary to remove this degeneracy to make the measurements of shunt impedance and Q. I presume the coupling slots will remove the degeneracy in a long accelerator, and only one of the modes would be propagated. Also the coupling slots will distort the field distribution. I have observed this in one model with two cells coupled together by single accelerating cell. However, the asymmetry is magnified because there is only one coupling cell and not an alternating set of coupling cells. The E maximum on the coupling slot side is shifted toward the coupling hole. The other E maximum remains close to the end of the drift tube.

LAPOSTOLLE, CERN: I would like to ask you, whether, from these impedance measurements, estimates have been made of the intensity threshold for transverse instabilities?

POTTER: I don't know. Perhaps Dr. Nagle would care to comment.

NAGLE, LASL: We have done very little, but we plan to make estimates.

POTTER: We do plan to complete these measurements for the whole family of six cavities. These are only preliminary data.

CITRON, Karlsruhe: I was a little confused by the units--megohms per meter per centimeter squared--that you used for shunt impedance. Now, how does the 'centimeter squared' slip in there?

POTTER: Possibly to avoid confusion in having megohms per meter cubed. The centimeter-squared term refers to the position from the axis. At one cm from the axis, the shunt impedance which you will measure will be the number I quoted above in megohms per meter. If you are two cm from the axis, the shunt impedance will be four times, the square of two cm, the number quoted. In this way I have removed the x dependence from the shunt impedance definitions.

Table II

TABLE OF EXPERIMENTAL RESULTS

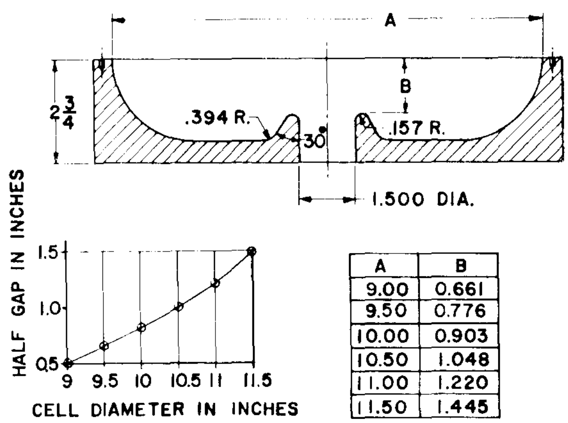
Cell Diameter (inches)	Mode	Freq. (Mhz)	Z_d/Q (Ω/m/cm^2)	T_{max} (β=0.65)	Z_dT^2/Q (Ω/m/cm^2)	Q ($\times 10^3$)	Z_d (MΩ/m/cm^2)	Z_dT^2 (MΩ/m/cm^2)
10.5[1]	TM_{110}	1519	161	0.58	54	25	4.0	1.4
10.5[2]	TM_{110}	1519	180	0.48	42	25	4.5	1.0
10.5[3]	TM_{110}	1519	95	0.52	26	25	2.4	0.6
11.5	TM_{110}	1366	105	0.27	7.7	28	2.9	0.21
10.5[4]	TM_{111}	1652	--	--	4	25	--	0.1

[1] measured 1.2 cm off axis in plane of E_{max}

[2] same, 0.7 cm off axis

[3] 1.0 cm off axis at 45° to plane of E_{max}

[4] estimated results



A	B
9.00	0.661
9.50	0.776
10.00	0.903
10.50	1.048
11.00	1.220
11.50	1.445

Fig. 1. Details of the family of $\beta = 0.65$ cell tested.

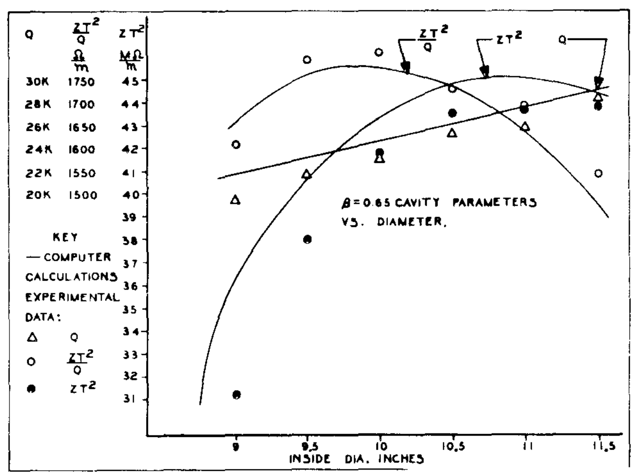


Fig. 2. Summary of TM_{010} mode data.

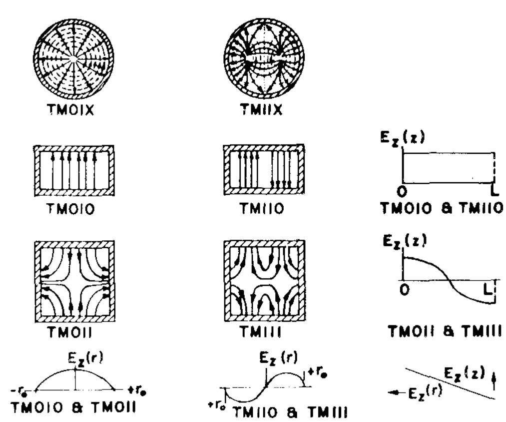


Fig. 3. Some TM modes in a right circular cylinder.

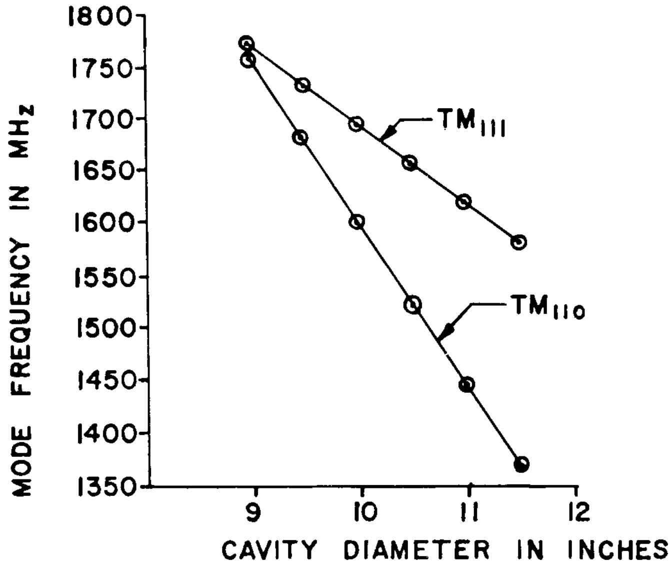


Fig. 4. TM_{110} and TM_{111} mode frequency vs diameter for the family of six = 0.65 cell tested.

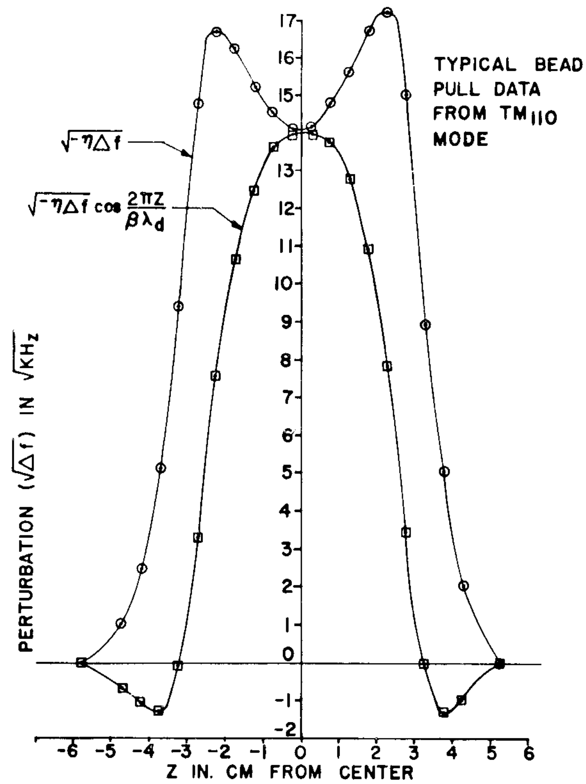


Fig. 6. Typical bead pull data from TM_{110} mode.

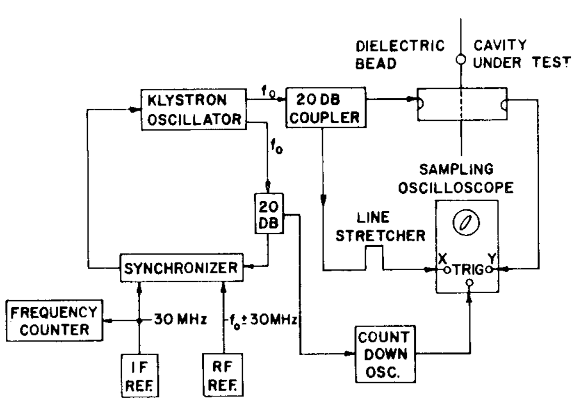


Fig. 5. Experimental apparatus for shunt impedance measurements.

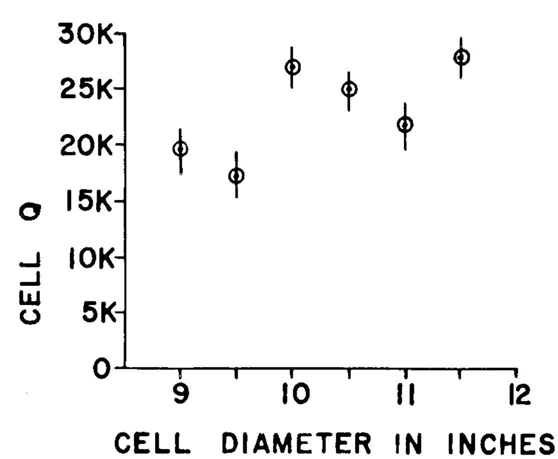


Fig. 7. TM_{110} mode Q versus diameter.