THE TECHNIQUE FOR THE NUMERICAL TOLERANCES ESTIMATIONS IN THE CONSTRUCTION OF COMPENSATED ACCELERATING STRUCTURES

V. V. Paramonov, A.K. Skassyrskaya, INR, 117312 Moscow, Russia

Abstract

The requirements to the cells manufacturing precision and tuning during multi-cells accelerating structures construction came from the required accelerating field uniformity, based on the beam dynamics demands. The standard deviation of the field distribution can be expressed in terms of accelerating and coupling cells frequencies deviations, stop-band width and coupling coefficient deviations. The deviations of the cells frequencies and coupling coefficient can be expressed from 3D fields distributions for accelerating and coupling modes and the cells surface displacements. With the modern software possibilities it can be done separately for every specified part of the cell surface. Finally, the surface displacements are expressed through characteristic cells dimensions deviations. This technique allows both qualitatively to define the critical regions and quantitative to optimize the tolerances definition.

INTRODUCTION

In the multi-cell accelerating structures all time exists the problem of cells identity - to realize the structure design parameters, the cells should be identical within the defined tolerances. The tolerances for cells parameters are directly related to tolerances for geometrical parameters. And the tolerances for cell geometry are directly related to the price (and possibility) of the cavity construction. The cavity mast realize design parameters, so, tolerances can not be too soft. But extremely rigid tolerances lead, at least, to the extra costs in the construction. The tolerances definition is all time the important part in the cavity development.

COUPLED CIRCUIT ANALYSIS

The original demands originate from the beam dynamics, which provides the requirements to the accelerating field distribution - the standard (rms) field deviations in accelerating gaps σ_E or field tilt along the cavity. The relation between σ_E value and deviations in cells rf parameters can be obtained by using well known coupled circuit model [1]. The rms value of the accelerating electric field deviation σ_E is [2]:

$$\sigma_E^2 = \sigma_{E_f}^2 + \sigma_{E_k}^2, \quad \sigma_{E_k}^2 = \sigma_{k_c}^2 \frac{N_p+2}{3}, \tag{1}$$

and

$$\sigma_{E_f}^2 \approx \frac{16\sigma_{f_a}^2}{k_c^4}(\sigma_{f_c}^2 \frac{N_p^2+3N_p}{12} + (\frac{\delta f}{f_a})^2 \frac{N_p^3+4N_p^2+6N_p}{3}), \tag{2}$$

where σ_{f_a} and σ_{f_c} are the rms frequency deviations for the accelerating (f_a) and coupling (f_c) cells, $\delta_f = f_c - f_a$ is the stop-band width, N_p - number of structure periods in the cavity, σ_{k_c} is the rms deviation of coupling coefficient k_c. As it usually is supposed in coupled circuit approach, the influences of δk_c and $\delta f_{a,c}$ on the operating field distribution are independent.

Suppose the cell geometry (the shape and dimensions) are defined with the set of geometrical parameters x_i. Assuming the deviations of different parameters as independent, for relative rms deviations $\sigma_{f_{a,c}}, \sigma_{k_c}$ it is valid:

$$\sigma_{f_{a,c}} = \frac{\sqrt{\sum_i (\frac{\partial f_{a,c}}{\partial x_i})^2 \sigma_{x_i}^2}}{f_{a,c}}, \sigma_{k_c} = \frac{\sqrt{\sum_i (\frac{\partial k_c}{\partial x_i})^2 \sigma_{x_i}^2}}{k_c}, \tag{3}$$

where $\frac{\partial f_a}{\partial x_i}, \frac{\partial f_c}{\partial x_i}, \frac{\partial k_c}{\partial x_i}$ are the sensitivities of the cell parameters f_a, f_c, k_c with respect the small deviation in the geometrical cell parameter x_i, σ_{x_i} is the rms x_i deviation. The value σ_{x_i} corresponds to the tolerance $\pm 3\sigma_{x_i}$ for the parameter x_i.

To estimate values in numbers, we need in the sensitivities values.

Coupling Coefficient Deviations

Considering k_c deviations, we have to distinguish symmetrical and nonsymmetrical k_c distortions with respect coupling cells. Suppose k_c value of the j-th coupling cell with adjacent $j-1$-th and $j+1$-th accelerating cells has the same deviation δk_c^s. The total coupling between the coupling cell and adjacent accelerating cells remains equal. As it is easy to see from coupled equations, [1], the fields in $j-1$-th and $j+1$-th accelerating cells will be equal.

Estimating k_c deviations, we have to exclude from consideration the symmetrical (with respect coupling cells) parts.

3D NUMERICAL MODELS

The sensitivity values $\frac{\partial f_a}{\partial x_i}, \frac{\partial f_c}{\partial x_i}, \frac{\partial k_c}{\partial x_i}$ depend on the field distributions of both accelerating and coupling cells (modes).

Suppose the structure period has a length $d = 2l, -l \le z \le l$, z is the structure axis, and at least the accelerating cells has a plate of mirror symmetry. It means, at $z = \pm l$, in the middles of accelerating cells, one can apply boundary conditions of electric walls (to simulate accelerating mode) or magnetic ones (to simulate coupling mode).

3D fields distributions and frequencies can be calculates for accelerating ($\vec{E}_a, \vec{H}_a, f_a$) and coupling ($\vec{E}_c, \vec{H}_c, f_c$) modes at one structure period, with appropriate boundary conditions at $z = \pm l$, can be calculated by using such software, as MAFIA and ANSYS.

Calculating and combining energy densities for electric $(\vec{E}_a, \vec{E}_c)$ and magnetic $(\vec{H}_a, \vec{H}_c)$ fields, one gets at the structure surface:

$$\frac{(\epsilon_0 E^2_{a,c} - \mu_0 H^2_{a,c})}{W_{a,c}} \sim \frac{\delta f^{(d)}_{a,c}}{f_{a,c}}, \qquad (4)$$

where W_a, W_c are the stored energies for accelerating and coupling modes.

At the case of confluence ($f_a = f_c$) in compensated structures exists the traveling wave $\vec{E}_{tr} = \vec{E}_a - i\vec{E}_c$ [3] with normalization conditions $W_a = W_c$ [4]. The relative group velocity of the traveling wave β_g, by definition, is:

$$\beta_g = \frac{dP_t}{cW_t} = \frac{\beta\lambda Re \int_{S_{z=l}} [\vec{E}_{tr}\vec{H}^*_{tr}]d\vec{S}}{2c(W_a + W_c)}, \qquad (5)$$

or,

$$|\frac{\beta_g}{\beta}| = |\frac{\lambda \int_{S_{z=0}} ([\vec{E}_a\vec{H}_c] - [\vec{E}_c\vec{H}_a])d\vec{S}}{4c\sqrt{W_a W_c}}|, \qquad (6)$$

where β is the particles relative velocity. The operating π mode is assumed. The ratio $\frac{\beta_g}{\beta}$ is related with coupling coefficient k_c as:

$$\frac{\beta_g}{\beta} = \frac{\pi k_c}{4}. \qquad (7)$$

The β_g reference value should be calculated according (6) - it is general expression and do not requires coupling cell mirror symmetry. The surface integral $I_{(z=0)} = \int_{S_{z=0}} ([\vec{E}_a\vec{H}_c] - [\vec{E}_c\vec{H}_a])\vec{z}_0 d\vec{S}$ in (6) can be calculated numerically, by using 3D software output. But β_g value doesn't depends on the reference plane coordinate and, remembering accelerating cell mirror symmetry, one has:

$$|\frac{\beta_g}{\beta}| = |\frac{\lambda \int_{S_{z=l}} [\vec{E}_c\vec{H}_a]d\vec{S}}{4c\sqrt{W_a W_c}}|, \qquad (8)$$

Let transform the integral $I_{(z=l)} = \int_{S_{z=l}} [\vec{E}_c\vec{H}_a]d\vec{S} = 2\pi f_0 \int_{V_0} (\epsilon_0 \vec{E}_a\vec{E}_c - \mu_0 \vec{H}_a\vec{H}_c) + \int_{S_{z=0}} [\vec{E}_c\vec{H}_a]d\vec{S}$. For β_g deviations one can neglect variations of the surface integral $\int_{S_{z=0}} [\vec{E}_c\vec{H}_a]d\vec{S}$, and, similar to (4):

$$\frac{(\epsilon_0 \vec{E}_a\vec{E}_c - \mu_0 \vec{H}_a\vec{H}_c)}{\sqrt{W_a W_c}} \sim \frac{\delta\beta^{(d)}_g}{\beta_g} = \frac{\delta k^{(d)}_c}{k_c}, \qquad (9)$$

The normalized scalar fields $(\epsilon_0 E^2_{a,c} - \mu_0 H^2_{a,c})$, $(\epsilon_0 \vec{E}_a\vec{E}_c - \mu_0 \vec{H}_a\vec{H}_c)$ (4), (9) represent at the structure surface the densities of sensitivity values $\frac{\partial f_a}{\partial x_i}, \frac{\partial f_c}{\partial x_i}, \frac{\partial k_c}{\partial x_i}$. The shift values are proportional to these densities and the volume of perturbation. We can estimate visually the relative influence of deviations in different structure elements on the cells parameters.

In Fig. 1 the Disk And Washer (DAW) structure [5] is shown (Fig. 1a) - the option for INR proton linac,

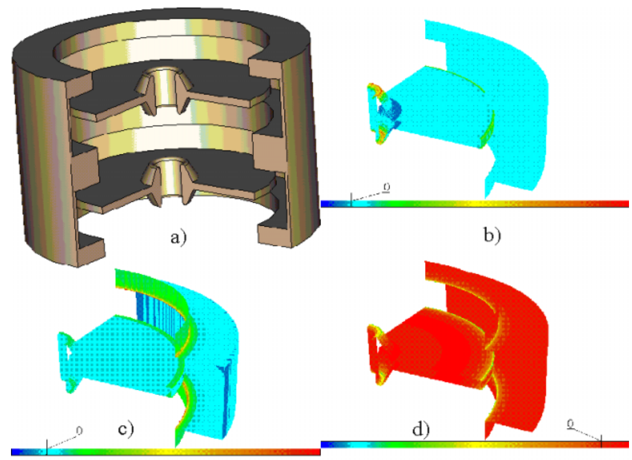

Figure 1: The DAW structure (a) and $\delta f^{(d)}_a$ (b), $\delta f^{(d)}_c$ (c), $\delta k^{(d)}_c$ (d) densities distribution at the structure surface.

$\frac{\beta_g}{\beta} = 0.512$. The scalar densities $\frac{\delta f^{(d)}_a}{f_a}, \frac{\delta f^{(d)}_c}{f_c}, \frac{\delta k^{(d)}_c}{k_c}$ are shown in Fig. 1 b,c,d, respectively. It is well known - the drift tube cone nose region has a strong influence on the frequency of accelerating mode f_a, Fig. 1b. As one can see from Fig. 1c, 1d, this region also has the influence on f_c and k_c values. In the DAW structure the field of coupling mode is distributed in the large part of the volume, providing strong $\vec{H}_a, \vec{H}_c$ overlapping and, as a sequence (see (9)), high β_g value . But for high $\beta \approx 1$ the fields $\vec{E}_c, \vec{H}_c$ penetrate to the drift tube region, leading to the influence on both k_c and f_c .

In Fig. 2 the CDS structure [6] is shown with two coupling windows, Fig. 2a, $\frac{\beta_g}{\beta} = 0.075$ and with four windows, Fig.2b, $\frac{\beta_g}{\beta} = 0.057$ [6]. The concept of k_c increasing without Z_e reduction in the CDS structure is described in [6] - for coupling mode $\vec{H}_c$ has no own space and trough coupling windows penetrate in accelerating cell. The windows opening angle has a large influence on f_a, f_c, k_c. Because the coupling windows are placed close to drift tubes, the drift tube nose surface has large $(\mu_0 \vec{H}_a\vec{H}_c - \epsilon_0 \vec{E}_a\vec{E}_c)$ density, mainly due to electric part in this combination.

The qualitative consideration shows - for the CDS structure the highest sensitivities for $\delta f_a, \delta f_c, \delta k_c$ deviations have the coupling windows and drift tubes dimensions.

Quantitative Estimations

For the values of $\delta f_a, \delta f_c, \delta k_c$ deviations both the sensitivity densities (4), (8) and surface areas are important. Basing on the FEM software (ANSYS) possibilities, the special procedure has been developed. The structure surface is described, (see Fig.3), in ANSYS as a set of numbered simple shapes. After rf problem solution, we have 3D distributions $\vec{E}_a, \vec{H}_a, \vec{E}_c, \vec{H}_c$ and can calculate deviations $\delta f_{ai}, \delta f_{ci}, \delta k_{ci}$, related to the fixed (say, $10 \mu km$) normal displacement of the $i - th$ surface.

The cells geometry is described with parameters, or di-

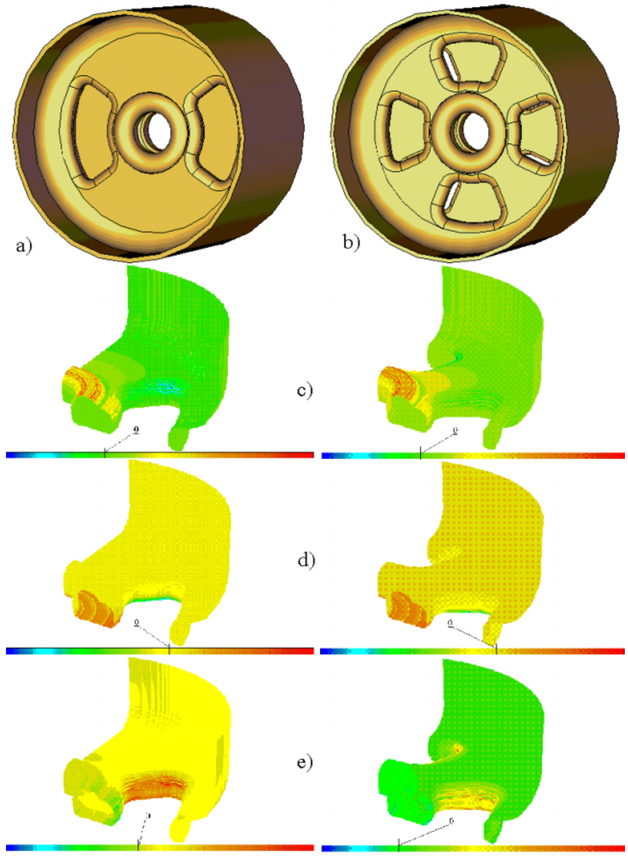

Figure 2: The CDS structure with two (a) and four (b) coupling windows and $\delta f_a^{(d)}$ (c),$\delta f_c^{(d)}$ (d), $\delta k_c^{(d)}$ (e) densities distribution at the structure surface for each CDS option (the left column - two windows, the right one - four windows).

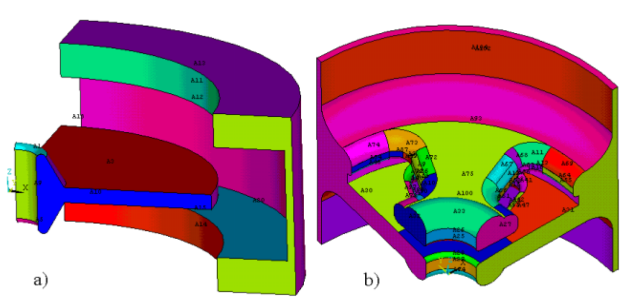

Figure 3: The surfaces of the DAW (a) and CDS (b) cells , divided in simple numbered shapes for deviations $\delta f_{ai}, \delta f_{ci}, \delta k_{ci}$ calculations.

mensions. The deviation in one parameter can lead to several simple shapes displacements. The relation between shape displacements and dimension deviations can be defined from geometry consideration.

EXAMPLES OF APPLICATIONS

In the DAW structure $10 \mu km$ each surface displacements (due to simple shapes, see Fig. 3a, it is equivalent to dimensions deviations) lead to $\sigma_{f_a} = 7.3 \cdot 10^{-3}, \sigma_{f_c} = 1.3 \cdot 10^{-2}, \sigma_{k_c} = 2.7 \cdot 10^{-2}$. The main influence to k_c deviations provide the disk and the washer thickness, due to large surface area of these elements. For these dimensions the tolerances should be more rigid than $\pm 60 \mu km$, which are considered in this example.

This technique was applied to determine the tolerances for the CDS booster cavity [7] construction. For the dimensions deviations, which correspond to $10 \mu km$ simple shapes displacements, see Fig. 3b, we have $\sigma_{f_a} = 1.2 \cdot 10^{-4}, \sigma_{f_c} = 7.3 \cdot 10^{-4}, \sigma_{k_c} = 1.2 \cdot 10^{-3}$. The booster cavity is not too long, $N_p = 14$. For the reasonable stopband width $\frac{\delta f}{f_a} = 3.0 \cdot 10^{-4}$ the first term (σ_{E_f} in (1), related to the $\delta f_{a,c}$ deviations, is two order less, as compared to the second one, related to the δk_c deviations influence. The coupling coefficient deviations determine σ_E value in the CDS booster cavity. Finally, reasonably comfortable tolerances for cells dimensions were chosen, taking into account technological reasons, to expect in the booster cavity $\sigma_E \leq 1\%$.

CONCLUSION

In this report the technique for numerical tolerances estimations in the construction of compensated accelerating is presented. The combination of different approaches provides the flexible, precise and visual method for problem understanding and well-grounded tolerances choice.

REFERENCES

[1] E.A. Knapp et al., Coupled resonator model for standing wave accelerators tanks. Rev. Sci. Instr., v.38, p. 22, 1967

[2] V.F. Vikulov, V.E. Kalyuzhny. The influence of manufacturing errors on parameters of standing wave accelerating structures, Soviet Journ. of techn. physics, V. 50, n4, p. 773, 1980.

[3] G. Dome. Review and survey of accelerating structures. in Linear Accelerators. /Ed. P. Lapostolle, A. Septier, North. Holland Pub. Co., 1970

[4] V. Paramonov. General relations for mode parameters of compensated structure in the vicinity pf operating point. Proc. of the 2000 Linac Conference, USA, p. 401, 2000

[5] V.G. Andreev et al., Study of high-energy proton linac structures. Proc. of the 1972 Linac Conference, p. 114

[6] V.V. Paramonov, The Cut Disk Structure for High Energy Linacs, Pros. 1997 PAC, v.3, p. 2962. 1998.

[7] V. Paramonov et al., Design parameters of the normal conducting booster cavity for the PITZ-2 test stand, MOP77, this Conference.