

# TUNING OF EXTERNAL Q AND PHASE FOR THE CAVITIES OF A SUPERCONDUCTING LINEAR ACCELERATOR

V.Katalev, S.Choroba, DESY, Germany

## Abstract

The RF power required for a certain gradient of a superconducting cavity depends on the beam current and coupling between the cavity and waveguide. The coupling with the cavity may be changed by variation of  $Q_{ext}$ . Different devices can be used to adjust  $Q_{ext}$  or phase. In this paper three stub and E-H tuners are compared and their usability for the RF power distribution system for the superconducting accelerator of the European X-Ray Laser and the TESLA linear collider is considered. The tuners were analyzed by using the scattering matrix. Advantages and limitations of the devices are presented.

## INTRODUCTION

The linear accelerators of the European XFEL and TESLA linear collider make use of the same type of superconducting cavities [1],[2]. The cavities are operated at a frequency of 1.3 GHz and an unloaded quality factor of  $Q_0 > 10^9$ . The accelerating gradient is 23.4 MV/m with a beam current of 9.5 mA for the TESLA linear collider and 22.9 MV/m with 5 mA of a beam current for the XFEL.

The power required by the cavity is 122 kW for the XFEL and 231 kW for TESLA with a pulse duration of 1.5 ms at a repetition rate of 10 Hz and 5 Hz respectively. To accelerate the beam with optimum consumption of RF power it is necessary to change the loaded quality factor of the cavity within several units of  $10^6$ . With constant coupling between the cavity and the waveguide this can be achieved by changing the external losses namely by changing of  $Q_{ext}$ .

The cavities are connected to the RF power source, a klystron, by a waveguide distributing system with a circulator and a fixed dummy load in front of each cavity. In order to change the external losses of the cavity it is necessary to use an additional impedance transformer between the cavity and the circulator like a three stub tuner or an E-H tuner.

## RF POWER FOR BEAM ACCELERATION

Let us consider the case of cavity operation at resonance and beam acceleration on-crest. It is possible to neglect power dissipation in the walls of the cavity in comparison with losses in the external load because  $Q_0 \gg Q_{ext}$ . The required RF power [3] for filling of the superconducting cavity with energy up to the required accelerating voltage  $V_{c0}$  is

$$P_{g0} = \frac{V_{c0}^2}{4 \frac{R}{Q_0} Q_{ext}} \frac{1}{(1 - e^{-\frac{t_{inj}}{\tau}})^2}$$

With  $R/Q_0$  – normalized shunt impedance for a linac  
 $\tau$  =  $Q_{ext} / \pi f$  time constant of the cavity  
 $t_{inj}$  – time of beam injection

In order to accelerate the DC beam current  $I_b$  the RF power

$$P_{gb} = \frac{V_{c0}^2}{4 \frac{R}{Q_0} Q_{ext}} \left(1 + \frac{I_b \frac{R}{Q_0} Q_{ext}}{V_{c0}}\right)^2$$

for flattop operation is required. It is the well-known formula for the power of a generator connected to a cavity with heavy beam loading [4].

Fig. 1 shows the RF power for the XFEL and TESLA cavity  $P_g$  as a function of the external quality factor  $Q_{ext}$ . The minimum RF power for cavity filling or for beam acceleration has a minimum for different conditions. For cavity filling the condition is

$$\frac{t_{inj} \pi f}{Q_{ext}^0} = 1.256,$$

whereas for beam accelerating it is

$$Q_{ext}^b = \frac{V_{c0}}{I_b \frac{R}{Q_0}}$$

Typically  $Q_{ext}$  is chosen between these two minima.

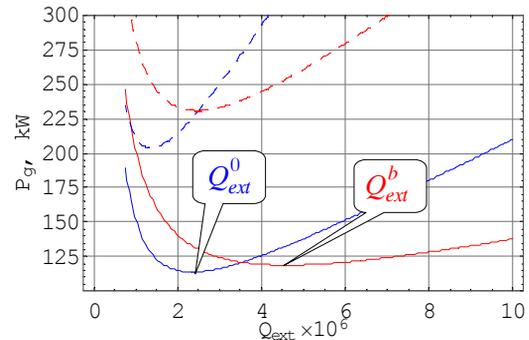


Figure 1: Cavity forward power  $P_{g0}$  (blue) and  $P_{gb}$  (red) for a XFEL cavity with gradient  $E=22.9$  MV/m,  $I_b=5$  mA and  $t_{inj}=720$   $\mu$ s. TESLA cavity –  $E=23.4$  MV/m,  $I_b=9.5$  mA  $t_{inj}=420$   $\mu$ s (dashed).

The conditions of minimum RF power for cavity filling and beam acceleration are fulfilled with  $Q_{ext}^0 = 2.36 \times 10^6$  and  $Q_{ext}^b = 4.59 \times 10^6$  in the XFEL case and with  $Q_{ext}^0 = 1.36 \times 10^6$  and  $Q_{ext}^b = 2.47 \times 10^6$  in the TESLA case, respectively. For the case of operation at a reduced cavity gradient one has to reduce the external quality factor in order to operate at minimum of RF power consumption. In the other case of a reduced beam current one might consider increasing  $Q_{ext}$  to meet the minimum condition for  $Q_{ext}^b$ . But one has to observe that by increasing  $Q_{ext}$  the RF power for cavity filling would increase dramatically (see Fig. 1). Therefore the recommended  $Q_{ext}$  value are in the range from 2.3 up to

$4.6 \times 10^6$  for the XFEL and from  $1.3$  up to  $2.5 \times 10^6$  for TESLA.

### PHASE VARIATION

The phase difference caused by the accuracy of assembling the waveguide distributing system is smaller  $\pm 20^\circ$ . The maximum phase shift due to thermal expansion of the waveguides is about  $12^\circ$ . Therefore a tuner should have the capability to adjust the phase within a range of about  $\pm 40^\circ$ .

### EQUIVALENT CIRCUIT

The superconducting cavity is connected to the RF power waveguide distributing system through a circulator with matched dummy load [1]. A tuner described by a scattering matrix [S] is inserted between the cavity and the circulator. The port 1 of the tuner is connected to a cavity coupler through a waveguide without losses with impedance  $Z_0$  and fixed length  $l$  (see Fig. 2). The port 2 of the tuner is connected to the matched load  $Z_0$  of the circulator.

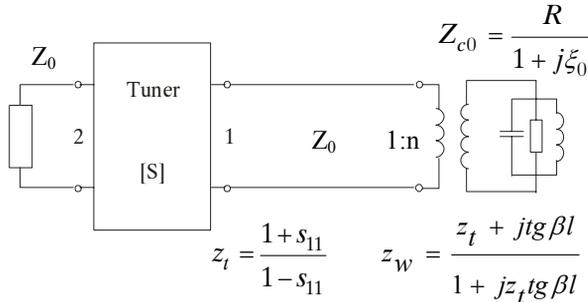


Figure 2: The equivalent circuit of a superconducting cavity with impedance  $Z_{c0}$  connected to a RF power waveguide distributing system through circulator and tuner.

A normalized tuner impedance  $z_t$  is transformed into  $z_w$  by the waveguide. This impedance is transformed by the power coupler to a cavity gap of  $z_w Z_0 n^2$  in parallel to the cavity impedance. Having in mind  $Q_0 \gg Q_{ext}$  we get a normalized cavity impedance of

$$z_c \approx \frac{z_w \frac{R}{Q_0} Q_{ext0}}{1 + j\xi z_w}$$

$$\left[ \begin{array}{c} \frac{j(-1 + x_1(-j + x_2 - x_3) + (j + x_2)x_3)}{j(-1 + (j + x_2)x_3) + x_1(-1 + jx_3 + x_2(j + 2x_3))} \\ \frac{2x_1x_2x_3}{j(-1 + (j + x_2)x_3) + x_1(-1 + jx_3 + x_2(j + 2x_3))} \end{array} \right] \left[ \begin{array}{c} \frac{2x_1x_2x_3}{j(-1 + (j + x_2)x_3) + x_1(-1 + jx_3 + x_2(j + 2x_3))} \\ \frac{j(-1 + x_1(-j + x_2 - x_3) + (j + x_2)x_3)}{j(-1 + (j + x_2)x_3) + x_1(-1 + jx_3 + x_2(j + 2x_3))} \end{array} \right]$$

Figure 3: Scattering matrix of three stub tuner for quarter wave length between the posts and post impedances  $x_i$ .

With  $Q_{ext0}$  – external quality factor without tuner

$\xi$  – detuning of the cavity equal to  $2Q_{ext0} \Delta f/f_0$

By changing elements of the tuner scattering matrix we can tune both the external quality factor of the cavity and the phase of the forward power. In order to phase the cavity it is necessary to fix  $s_{11}$  and vary  $s_{21}$ . For changing external losses it is necessary to keep the phase of  $s_{21}$  constant and vary the absolute value of  $s_{11}$ . The phase of  $s_{11}$  has to be fixed, otherwise the cavity resonance frequency will be changed.

### THREE STUB TUNER

Three stub tuner for TESLA are made of a straight waveguide with three movable posts located 80 mm from each other. The diameter of a post is 33 mm and the maximal depth of penetration is 42 mm. The relation between the penetration depth  $h$  of the post and his normalized impedance  $x$  is determined experimentally and given by  $h = 29.253 \times x^{-0.5153}$ . This means that the minimum impedance of a stub can not be less than 0.5 for the maximum penetration depth of 42 mm.

By knowing the impedance of the posts and distances between them it is possible to find a scattering matrix of the three stub tuner. Calculation of the scattering matrix was carried out by Mathematica 5.0 [5]. The resulting scattering matrix has a lengthy expression. To simplify the analysis of the matrix expression the distance between the posts has been taken equal to a quarter of wave length 80.66 mm. The simplified scattering matrix is shown on fig.3.

To understand the three stub tuner when it operates as a pure phaseshifter let us assume a special case:  $s_{11}=0$ . From this condition and  $x_3 = x_1$  we can get  $x_2 = x_1^2 / (1 + 2x_1)$  and correspondingly calculate the next element of the matrix  $s_{21} = (-j + x_1) / (j + x_1)$ . By changing the penetration depth of posts in a proper way it is possible to vary the phase by  $\pm 50^\circ$  without any reflection from the tuner.

When the tuner works like an impedance transformer the phase of  $s_{21}$  must be constant. This means, that

$$x_1 = \frac{-1 + x_2x_3 - x_3tg\varphi_{21}}{-x_2 - x_3 + tg\varphi_{21}(1 - 2x_2x_3)}$$

In order to keep the resonance frequency of the superconducting cavity constant the imaginary part of  $z_w$  has to be equal to 0 and therefore the following condition

should be satisfied

$$x_2 = 1/x_3 - ctg\varphi_{21}$$

From these two limitations and keeping in mind the minimum impedance of the stub of 0.5 we get the possible range of the external quality factor depending on the phase of the forward power without changing of cavity frequency (see fig.4).

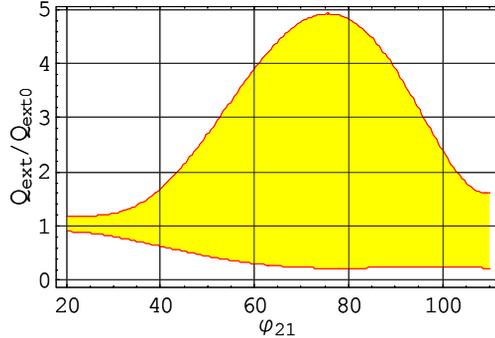


Figure 4: External quality factor  $Q_{ext}$  range relative to factor  $Q_{ext0}$  depending on phase shifts  $\varphi_{21}$  of forward power (yellow area).

### E-H TUNER

This type of a tuner is proposed for the waveguide system for the 8 GeV Fermilab linac [6] and is considered in [7]. An E-H tuner consists of a matched magic tee and two sliding short circuits. Taking the ideal scattering matrix of the tee junction and assuming full reflection by sliding short circuits it is straight forward to get the elements of the E-H tuner scattering matrix

$$s_{11} = \frac{1}{2} \left( e^{-j\frac{4\pi l_1}{\Lambda}} + e^{-j\frac{4\pi l_2}{\Lambda}} \right)$$

$$s_{21} = \frac{1}{2} \left( e^{-j\frac{4\pi l_1}{\Lambda}} - e^{-j\frac{4\pi l_2}{\Lambda}} \right)$$

With  $l_1, l_2$  – position of the short circuit,  
 $\Lambda$  – wave length in the waveguide

If  $l_1$  and  $l_2$  are changed in the same way in one direction the amplitudes of  $s_{11}$  and  $s_{21}$  are constant but their phases are varied. The E-H tuner works like a phasemifter. In the other case, when  $l_1$  and  $l_2$  are changed in the same way in opposite directions the E-H tuner is an impedance transformer. This means that the amplitudes of  $s_{11}$  and  $s_{21}$  are varied and phases do not change.

To avoid a cavity resonance frequency change the condition  $Im(z_w)=0$  must be fulfilled. It is straight forward to get the relation for the waveguide length  $l$  and the short circuit positions

$$l = \frac{1}{2}(l_1 + l_2) + n\frac{\Lambda}{2}$$

This requirement cannot be met easily if there is no additional phasemifter between cavity and tuner. Therefore the cavity frequency will change during the adjustment of phase or external quality factor by the E-H tuner.

This dependence is shown in fig.5 for the special case when  $l = n\frac{\Lambda}{2}$ . This can only be avoided if either an additional phasemifter is installed between the cavity and tuner or a frequency cavity tuner [8] is used in addition.

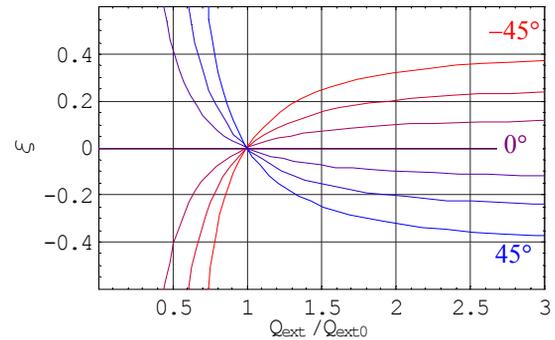


Figure 5: Detuning of cavity  $\xi = 2Q_{ext0} \Delta f/f_0$  due to variation of  $Q_{ext}$  by an E-H tuner for different phase shifts  $\varphi_{21}$  of forward power.

It is also possible to use the usual quadrature coupler instead of the magic tee junction. In this case the tuner has higher power capability and has also more compact dimensions.

### SUMMARY

Besides the restrictions mentioned above the power capabilities of the devices must be considered.

The measured power breakdown for three stub tuner is about 400 kW for full reflection. The power capability of E-H tuner is more than 1 MW

Three stub tuner or E-H tuner have enough dynamic range for  $Q_{ext}$  and phase settings and have acceptable power capability. Therefore they can be used for the power distributing system for the XFEL and the TESLA linac.

### REFERENCES

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