

## 3D BEAM DYNAMICS SIMULATION IN UNDULATOR LINAC\*

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### Abstract

The ion beam can be bunched and accelerated in linear undulator accelerator (UNDULAC). The acceleration and focusing of beam can be realised without using a synchronous wave. In this paper the computer simulation of high intensity ion beam dynamics in UNDULAC-RF was carried out by means of the "particle-in-cell" method.

### INTRODUCTION

The beam focusing and acceleration can be realized without using a synchronous wave of RF field, as it discussed in Ref. [1-2]. In this case accelerating force is to be driven by a combination of two non-synchronous waves (two undulators). Such linac was called linear undulator accelerator (UNDULAC). The ribbon ion beams can be accelerated in two types of undulator linac UNDULAC-E [1] and UNDULAC-RF [2]. In first case beam bunching, acceleration and transverse focusing are realised in combined wave field which is produced by one spatial RF field harmonic in a periodical resonator and field of electrostatic undulator. In second case the accelerator force is driven by two fundamental RF field harmonics. It should be noted that the ion beams can be accelerated in UNDULAC in low energy range using transverse or longitudinal fields [1].

The beam dynamics in UNDULAC can not be investigated by means of traditional analytical methods because it is no synchronous wave in this linac. The computer simulation and optimization of ion dynamics consist of two steps. At the first the equations of particles motion in polyharmonic fields is devised by means of smooth approximation. Hamiltonian analysis of this equation allows to find a velocity of reference particle in polyharmonic field and to formulate the conditions of effective longitudinal bunching and transverse beam focusing [3]. These conditions define the structure period and the field amplitudes distribution. At the second, using above founded characteristics, the 3D ion beam dynamics numerical simulation in an UNDULAC is provided. The space charge influence on the beam dynamics is investigated also by this simulation

The results of beam dynamics analytical study and numerical simulation for UNDULAC-E are discussed in Ref. [4-5]. The analytical investigation for UNDULAC-RF is provided in [3]. Let us consider briefly the results of this investigation. It helps to realize the numerical simulation.

### THE RESULTS OF BEAM DYNAMICS ANALYTICAL STUDY

The analytical investigation was provided for UNDULAC-RF using RF field with  $\mu = 0$  and  $\mu = \pi$  modes in Ref. [3]. The main results of ribbon ion beam dynamics analytical study in UNDULAC-RF:

1. The two sub-sections are necessary for providing of beam bunching in UNDULAC-RF. The reference particle phase must be chosen linearly decreased and fields amplitudes increased as sine function in first bunching sub-section. The synchronous phase and amplitudes are constant in second accelerating sub-section. These dependencies are providing the minimal particle losses.
2. It was shown that the optimal bunching conditions and maximal current transmission coefficient could to be driven with the optimal RF field harmonics ratio  $\chi = E_1 / E_0$ . The optimal value is equal  $\chi = 0.3-0.4$  in UNDULAC-RF using  $\mu = \pi$  mode of RF field and  $\chi > 1$  for  $\mu = 0$  mode.
3. In the smooth approximation the current transmission coefficient  $K_T$  is equal 90-95 % for UNDULAC-RF using  $\mu = \pi$  mode of RF field (for both transverse and longitudinal fields) and 85-90 % for  $\mu = 0$  mode.
4. The two bunches per one period of RF field will be configured.

### BEAMDULAC CODE

The beam dynamics can not be studied completely using analytical methods only. The time-averaged motion equation and effective potential function are obtained using smooth approximation. The influence of particle phase and velocities oscillations on beam dynamics does not take into account in this approximation. The numerical simulation in polyharmonic field is also necessary. The new code BEAMDULAC has been developed specially for beam dynamics simulations in undulator linear accelerator. This code is carried out by means of Cloud-in-Cell (CIC) method for accurate treat of space charge effects that is especially important in the case of high intensity beam. The motion equation for each particle is being solved including forces due to external fields and inter-particle Coulomb field. The Poisson equation is solving on the grid with periodic boundary conditions in order to find the potential of Coulomb. The Dirichlet boundary conditions are applied at transverse boundaries of the simulation domain. The interaction of the bunch space charge with the accelerating channel is taken into account. The Fast Fourier Transform algorithm is used to solve the Poisson equation on 3D grid with given charge density distribution. The external potential is

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represented as a series of space harmonics. The BEAMDULAC code was used for ribbon ion beam dynamics study in UNDULAC-RF and UNDULAC-E. Let us discussed the results of beam dynamics in UNDULAC-RF using  $\mu=0$  and  $\mu=\pi$  modes of transverse and longitudinal RF field.

### LONGITUDINAL RF FIELD ( $\pi$ MODE)

The numerical simulation was provided for ribbon beam of deuterium ions. The basic parameters of UNDULAC and the results of beam dynamics investigation are presented in Table 1. The simulation was started with next parameters: the initial energy of deuterium ions  $W_{in}=100$  keV; the full length of accelerator channel 2.5 m; the accelerator channel cross-section size  $2a \times 2b=0.8 \times 20$  cm. The output beam energy in UNDULAC-RF using  $\mu=\pi$  mode is equal 1.3-1.5 MeV with effective amplitude equal to  $E_{eff} = e\lambda E_0 E_1 / 2\pi W_0 \beta = 40$  kV/cm. Here  $e$  is the charge of electron,  $\lambda$  is the wave length ( $\lambda=1.5$  m in this case),  $W_0 = mc^2$  and  $\beta$  is the initial beam velocity. The energy gain in accelerating part is equal 700-800 keV/m. The optimal bunching sub-section length  $L_b$  must be equal to accelerating sub-section length  $L$  approximately. It was shown that the optimal value of RF field harmonics amplitude is equal  $\chi=0.3-0.4$  (see Fig. 1). This result coincides with previously analytically founded value and this ratio can be easily realised.

The current transmission coefficient is close to 100 % in smooth approximated field as it was shown above [3]. It is appreciably reduced if the beam dynamics simulation is done in polyharmonic RF field. The current transmission coefficient is equal 75–80 % (see Fig. 1, curve 1) for paraxial injected beam (when the beam cross-section size is equal to  $2l \times 2t=1 \times 0.04$  cm<sup>2</sup>). One has obtained that  $K_T$  mightily decreased if the beam size is larger of a critical value ( $2l \times 2t=5 \times 0.3$  cm<sup>2</sup>). The size of accelerator channel can be reduced to  $2a \times 2b=0.7 \times 10$  cm in this case. The particle loses are due to the fast oscillations of particle phases and longitudinal velocities for UNDULAC-RF using longitudinal RF field.

Numerical simulation of beam dynamics when a space charge field is taken into account shows that limit current in UNDULAC-RF is lower analytically predicted and its value is equal  $I_{max}=200-250$  mA when the cross-section is equal  $2l \times 2t = 5 \times 0.3$  cm<sup>2</sup> (see Fig. 1, curve 2). It was shown that  $K_T$  is not exceeding 60 % in this case.

The input and output beam cross-section (a), normalized transverse emittance  $E_y$  (b), phase (c) and energy (d) spectra are plotted on Fig. 2. This figure illustrates the formation of two bunches per one RF field period. The output normalized emittance  $E_y$  is twice as an initial one.

### TRANSVERSE RF FIELD ( $\pi$ MODE)

The parameters of system for simulation are equal for UNDULAC-RF using transverse and longitudinal RF field for  $\mu=\pi$  mode. The results of simulation are also close for both linac types (see Table 1). The optimal ratio of RF field harmonics amplitudes is equal  $\chi=0.35$  for UNDULAC-RF using transverse RF field. The length of bunching sub-section must be also close to the accelerator length.

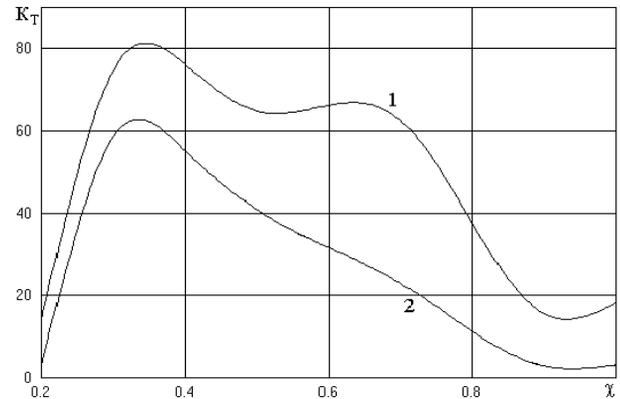


Figure 1: Current transmission coefficient versus ratio of RF field space harmonics amplitudes  $\chi$ .

Table 1: The parameters of UNDULAC-RF ( $\mu=\pi$  mode)

	lg.	tr.
Length of accelerator $L$ , m	1.3	
Injection energy $W_{in}$ , keV ( $\beta_{in}$ )	100 (0.01)	
Amplitudes of RF field harmonics,		
base $E_0$ , kV/cm	200	210
first $E_1$ , kV/cm	80	70
Buncher length, m	1.2	
Accelerator channel size $2a \times 2b$ , cm	10 $\times$ 0.7	
Input beam size $2l \times 2t$ , cm <sup>2</sup>	5 $\times$ 0.3	7 $\times$ 0.3
Input emittance		
$E_x$ , mm-mrad	$30\pi$	$30\pi$
$E_y$ , mm-mrad	$0.7\pi$	$0.06\pi$
$E_\phi$ , keV-mrad	25	40
Acceptance of accelerator channel		
$A_x$ , mm-mrad	$60\pi$	$60\pi$
$A_y$ , mm-mrad	$2\pi$	$2.5\pi$
$A_\phi$ , keV-mrad	40	40
Limit beam current $I_{max}$ , mA	200– 250	300– 350
Output beam energy $W_{max}$ , MeV ( $\beta_{max}$ )	1.2–1.5 (0.034–0.04)	

The limit input beam size is  $2l \times 2t = 7 \times 0.3$  cm<sup>2</sup>. It is higher than for UNDULAC-RF using longitudinal field. The current transmission coefficient is equal 65 % for this input beam size. The limit current is equal 300–350 mA for this type of undulator linac. The particle losses are caused by longitudinal motion in the bunching sub-section and all losses in accelerating sub-section are caused by transverse motion and Coulomb field influence.

The transverse emittance threefold. The transverse particles velocity and beam size both. The halo is not forming. The limit initial emittance  $E_y$  is lower than for UNDULAC-RF using longitudinal field (see Table 1).

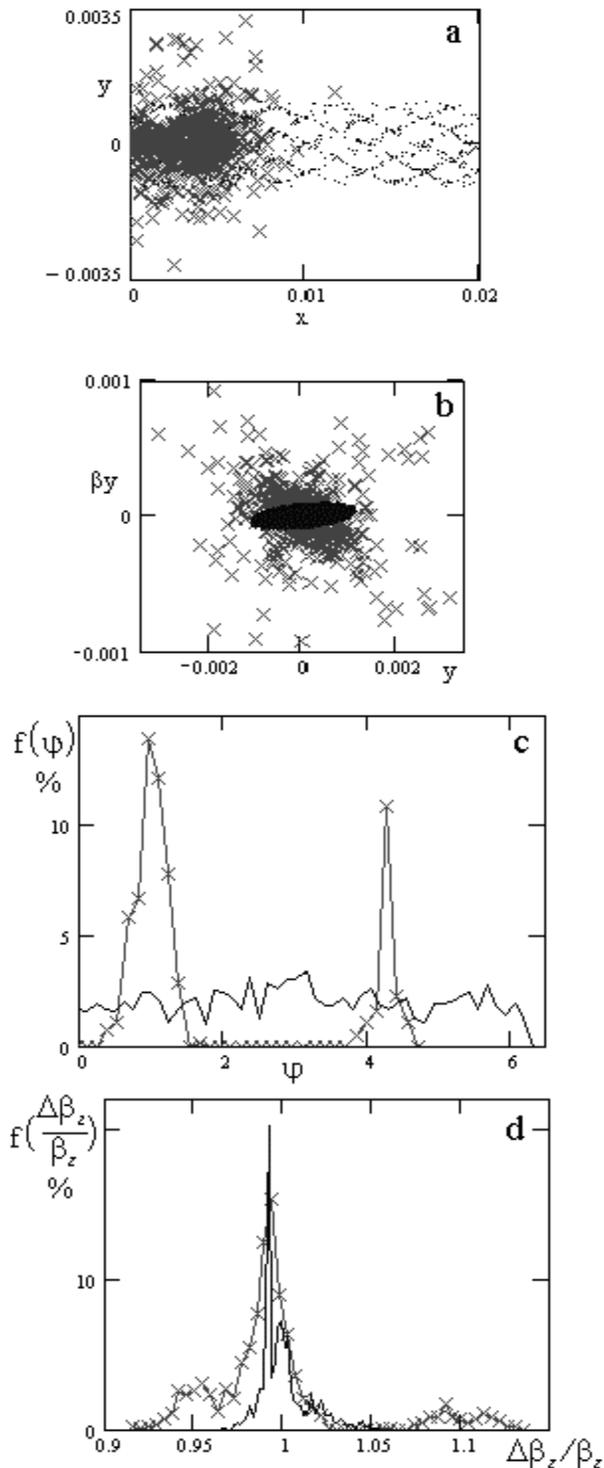


Figure 2: Input and output beam cross-section (a), normalized transverse emittance  $E_y$  (b), phase (c) and energy (d) spectra. ("points" – initial values, "x" – output).

## UNDULAC-RF USING LONGITUDINAL AND TRANSVERSE RF FIELD (0 MODE)

The numerical simulation of ribbon ion beam dynamics in UNDULAC-RF for  $\mu = 0$  mode of RF field shows that the current transmission coefficient is very small. The current transmission coefficient decreases to 55–60 % for paraxial injected beam if the simulation is provided in the polyharmonic field. For larger beam cross-sections current transmission coefficient reduces to 30–35 % (UNDULAC-RF using longitudinal field) and to 5–10 % (using transverse field). These results are confirmed by previous analytical investigation.

The optimal value of effective amplitude of combined wave is equal  $E_{eff}=30$  kV/cm and output energy  $W = 0.9$ –1.1 MeV. The limit current was not calculated for this type of undulator linac because the current transmission coefficient is small.

## CONCLUSION

The results of deuterium ion beam dynamics investigation in RF undulator linacs are presented. These results are obtained using the BEAMDULAC code. It was shown that UNDULAC-RF using  $\mu = \pi$  mode RF field is more preferable for further design. The ribbon beam of deuterium ions can be bunched and accelerated up to energy  $W = 1$ –1.5 MeV. The limit current is up to 350 mA and current transmission coefficient  $K_r=65$  % in this type of undulator linac. It should be noted that the limit beam current and current transmission coefficient are lowest than in electrostatic undulator linac [5]. However the energy gain in accelerating sub-section of UNDULAC-RF is twice as larger than in UNDULAC-E. The current transmission enlarging is possible by means of numerical optimization of function of synchronous phase and RF field harmonics amplitudes variation.

It was also shown that UNDULAC-RF using  $\mu = 0$  mode RF field is not perspective system for further design because the transverse beam focusing is not effective here.

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