LIMITING EFFECTS IN THE ROUND-TO-FLAT BEAM TRANSFORMATION

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Abstract

In the present paper we study the chromatic effects on transverse beam emittances in the transformation of an angular-momentum-dominated round beam into a flat beam. Analytical results are compared with numerical simulations and found in good agreement. We also attempt to study the effects caused by the asymmetries in the four dimensional transverse phase space distribution.

INTRODUCTION

The theory of generating a beam with high transverse emittance ratio, i.e., a flat beam, from an incoming angular-momentum-dominated beam is treated in several papers [1, 2, 3]. In this paper, we follow the theoretical treatment based on four dimensional beam matrix presented in [4], in which the round-to-flat beam (RTFB) transformation analysis was performed assuming that the beam and the transport channel upstream of the flat beam transformer are cylindrically symmetric and that the particle dynamics is symplectic. The experimental demonstration of such a round-to-flat transformation at Fermilab/NICADD Photoinjector Lab (FNPL) by using a RTFB transformer consists of three skew quadrupole channel is reported in [5].

CHROMATIC EFFECTS

The strength of a quadrupole is related to the particle's momentum. Consider an electron with a small fractional momentum deviation $\delta = \frac{p-p_0}{p_0}$ around the average beam momentum p_0. In practical units, the quadrupole strength q for an electron with momentum p is given by:

$$q[\text{1/m}] = \frac{300g[\text{T/m}]l_{eff}[\text{m}]}{pc[\text{MeV}]} = q_0(1-\delta+\delta^2+\mathcal{O}(\delta^3)),$$

where g the transverse magnetostatic field gradient, l_{eff} is the effective length of the quadrupole and c the speed of light, $q_0[\text{1/m}] \doteq \frac{300g[\text{T/m}]l_{eff}[\text{m}]}{p_0c[\text{MeV}]}$. Correspondingly, in thin lens approximation, the 2×2 transfer matrix M_Q of a normal quadrupole may be written as:

$$M_Q(q,\delta) \approx \begin{bmatrix} 1 & 0 \\ q_0 & 1 \end{bmatrix} + \delta \begin{bmatrix} 0 & 0 \\ -q_0 & 0 \end{bmatrix} + \delta^2 \begin{bmatrix} 0 & 0 \\ q_0 & 0 \end{bmatrix}.$$

Consider a RTFB transformer composed of three skew quadrupoles of strengths q_1, q_2, q_3 and separated dy drift space of lengths d_2 and d_3. The 4×4 transfer matrix of such a transformer takes the form of:

$$M(q_1,q_2,q_3,d_2,d_3) \approx M_0 + \delta\Delta_1 + \delta^2\Delta_2, \qquad (1)$$

where M_0 is the transfer matrix for reference particle with momentum p_0, Δ_1 and Δ_2 are the corrections to the transfer matrix to the first and second order of δ.

The general form of a cylindrically symmetric beam matrix [4] at the entrance of the RTFB transformer is:

$$\Sigma_0 = \begin{bmatrix} \sigma^2 & 0 & 0 & \kappa\sigma^2 \\ 0 & \kappa^2\sigma^2+\sigma'^2 & -\kappa\sigma^2 & 0 \\ 0 & -\kappa\sigma^2 & \sigma^2 & 0 \\ \kappa\sigma^2 & 0 & 0 & \kappa^2\sigma^2+\sigma'^2 \end{bmatrix}, \qquad (2)$$

where $\sigma^2 = \langle x^2\rangle = \langle y^2\rangle, \sigma'^2 = \langle x'^2\rangle = \langle y'^2\rangle$, $\kappa = \frac{eB_z}{2p}$, B_z is the longitudinal magnetic field on the photocathode and e the electron charge. The beam matrix at the exit of the RTFB transformer is:

$$\Sigma = M\Sigma_0\widetilde{M}, \qquad (3)$$

where $\widetilde{M}$ stands for the transpose of M. Since $\langle\delta\rangle$ vanishes, keeping only the first order modification to the beam matrix, from Eq. 1 and Eq. 3, we have:

$$\Sigma \approx M_0\Sigma_0\widetilde{M_0} + \langle\delta^2\rangle(M_0\Sigma_0\widetilde{\Delta_2} + \Delta_1\Sigma_0\widetilde{\Delta_1} + \Delta_2\Sigma_0\widetilde{M_0}). \qquad (4)$$

Given proper transfer matrix M, the first term of Eq. 4 can be block diagonalized and the two transverse emittances are given by (see, for example, Ref. [4]):

$$\varepsilon^0_{x,y} = \varepsilon_{eff} \mp \mathcal{L}, \qquad (5)$$

where $\varepsilon_{eff} = \sigma\sqrt{\sigma'^2+\kappa^2\sigma^2}$, $\mathcal{L} = \kappa\sigma^2$.

When there is a relative momentum spread in the beam, the beam matrix varies as a function of it. The two transverse emittances can be calculated as the square roots of the determinants of the top left and bottom right 2×2 sub-matrices of the beam matrix expressed in Eq. 4. Let's rewrite the second term of Eq.4 as:

$$\langle\delta^2\rangle \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix} \doteq \langle\delta^2\rangle(M_0\Sigma_0\widetilde{\Delta_2} + \Delta_1\Sigma_0\widetilde{\Delta_1} + \Delta_2\Sigma_0\widetilde{M_0}). \qquad (6)$$

By using the convenient relation for the determinant of the sum of two 2×2 matrices P and Q,

$$|P+Q| = |P| + |Q| + Tr(P^\dagger Q), \qquad (7)$$

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where "| |" stands for the determinant, "Tr" for the trace of a matrix, and $P^\dagger$ is the symplectic conjugate of P, $P^\dagger = J^{-1}\widetilde{P}J$, where J is the 2×2 unit symplectic matrix:

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

we can write an expression for the transverse emittances in the presence of the aforementioned chromatic effect:

$$\begin{aligned} \varepsilon_x &= \sqrt{(\varepsilon_x^0)^2 + \langle\delta^2\rangle^2[|\Delta_{11}| + (\varepsilon_x^0)^2\mathrm{Tr}(T\Delta_{11}^\dagger)]}, \\ \varepsilon_y &= \sqrt{(\varepsilon_y^0)^2 + \langle\delta^2\rangle^2[|\Delta_{22}| + (\varepsilon_y^0)^2\mathrm{Tr}(T\Delta_{22}^\dagger)]}. \end{aligned} \tag{8}$$

As a numerical application we consider parameters (see Table 1) close to those achieved for the FNPL flat beam experiment. Note the normalized beam emittance upstream of the RTFB transformer is taken to be equal to the thermal emittance at the photocathode ($\gamma\sigma\sigma' = 1$ mm mrad, γ is the Lorenz factor). This underestimation of the emittance leads to higher transverse emittance ratio. In turn this means the calculations presented hereafter are more sensitive to chromatic effects.

Table 1: Parameters used as a numerical example for chromatic effects in flat beam generation

parameter	value	units
γ	30	
σ	1.00	mm
κ	0.78	m^{-1}
σ'	0.033	mrad
d_2	0.35	m
d_3	0.85	m

Using the thin lens approximation, and including the thermal emittance, the skew quadrupole strengths are calculated from Ref.[6] to be:

$$q_1 = 1.729\ \mathrm{m}^{-1}, \quad q_2 = -1.339\ \mathrm{m}^{-1}, \quad q_1 = 0.628\ \mathrm{m}^{-1}.$$

The normalized flat beam emittances (from Eq. 5) are:

$$\varepsilon_x^n = 0.021\ \text{mm mrad}, \quad \varepsilon_y^n = 46.82\ \text{mm mrad}.$$

The analytical calculations of the two transverse emittances and their ratio, as a function of relative momentum spread, are first compared with simulation results from ASTRA[7], ELEGANT[8] and SYNERGIA[9]. These results agree well in general, see Fig. 1.

On the other hand, we can see that the agreement between the analytical results and simulations is better for lower relative momentum spread values. To further explore the difference, each particle used in the simulation is tracked through the transfer matrix for both the cases when the quadrupole are thick and thin lenses, using the transfer matrix as shown in Eq. 1. We found that in the thick lens case, the tracking results almost overlap with the simulation results, while the thin lens approximation tracking results agree quite well with the analytical predictions. Hence the difference between analytical and numerical results can be attributed to the fact that the thin lens approximation used in analytical model does not hold as the energy spread is introduced.

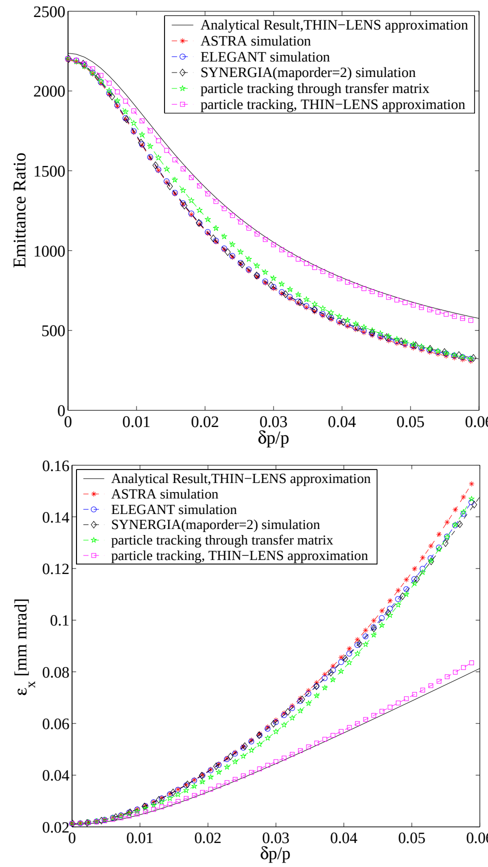

Figure 1: Chromatic effects on emittance ratio (top) and horizontal emittance (bottom). Solid line is obtained from Eq.8. Dashed lines with markers are computed using numerical methods.

ASYMMETRIES

Some elements in the beam line, such as the RF gun coupler, could cause asymmetry in the beam before it enters the RTFB transformer. In this case, the emittances of the flat beam is effected and the flat beam ratio is lowered comparing to the cylindrically symmetric beam case.

Suppose the y, y' deviate from the symmetric beam by the amount of ξ and η, respectively:

$$\begin{aligned} y &\to y + \xi, \\ y' &\to y' + \eta, \end{aligned}$$

where η and ξ could be functions of y. The beam matrix at the entrance of the RTFB transformer is:

$$\Sigma = \Sigma_0^* + \Delta, \tag{9}$$

where Σ_0^* is of the form of Eq. 2 with $\mathcal{L}$ replaced by $\mathcal{L}^* \doteq \kappa(\sigma^2 + \mu)$ and it can be block diagonalized; Δ is given by:

$$\Delta = \begin{bmatrix} 0 & 0 & 0 & -\kappa\mu \\ 0 & 0 & -\kappa\mu & 2\kappa\nu \\ 0 & -\kappa\mu & 4\mu + \langle\xi^2\rangle & 2(\mu+\nu) \\ -\kappa\mu & 2\kappa\nu & 2(\mu+\nu) & \langle\eta^2\rangle \end{bmatrix},$$

with $\mu \doteq \frac{1}{2}\langle y\xi\rangle$, $\nu \doteq \frac{1}{2}\langle y\eta\rangle$.

Δ generates x-y phase space coupling terms in the beam matrix at the exit of the RTFB transformer. It also modifies the two transverse emittances of the flat beam. As an example, we use the same initial beam matrix as used in previous section, plus Δ induced by the RF gun coupler kick [10]:

$$\Delta = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_1\sigma_-^2 \\ 0 & 0 & a_1\sigma_-^2 & a_1^2\sigma_-^2 + (ka_2\sigma_y\sigma_z)^2 \end{bmatrix},$$

where $\sigma_-^2 = \sigma_y^2 - h^2$, σ_y is the vertical rms bunch size, h the difference in vertical direction between the geometry and the electro-magnetic axes due to the RF coupler kick, k the rf wave number, σ_z the rms bunch length, $a_1 \doteq \alpha k \sin(kl)\sin(2kz_m)$, $a_2 \doteq \alpha k \sin(kl)\cos(2kz_m)$, where $l = z_f - z_i$, $z_m = \frac{l = z_f + z_i}{2}$, z_i and z_f be the start and end of the coupler region, $\alpha \doteq \frac{eE_0}{2mc^2k}$ [11], where E_0 is the gun peak accelerating field, m the electron mass. Take the following typical values at FNPL:

$f = 1.3$ GHz $\rightarrow$ k $= 27$ m^{-1},

$E_0 = 35$ MV/m $\rightarrow \alpha = 1.27$,

$\sigma_z = \sigma_y = 1$ mm,

$h = 1$ mm [12],

$z_i = 0.11$ m, $z_f = 0.19$ m $\rightarrow z_m = 0.15$ m, $l = 0.08$ m,

$\varepsilon_y^0 = 1$ mm mrad.

All the elements of Δ matrix vanish apart from $\Delta_{44} = 7.65 \times 10^{-10}$. For a beam without energy spread, this increases the smaller one of the two transverse emittances of the flat beam by 30%. The emittance ratio drops from 2237 to 1715, see Fig. 2 for the emittance ratio and horizontal emittance as a function of fractional momentum spread.

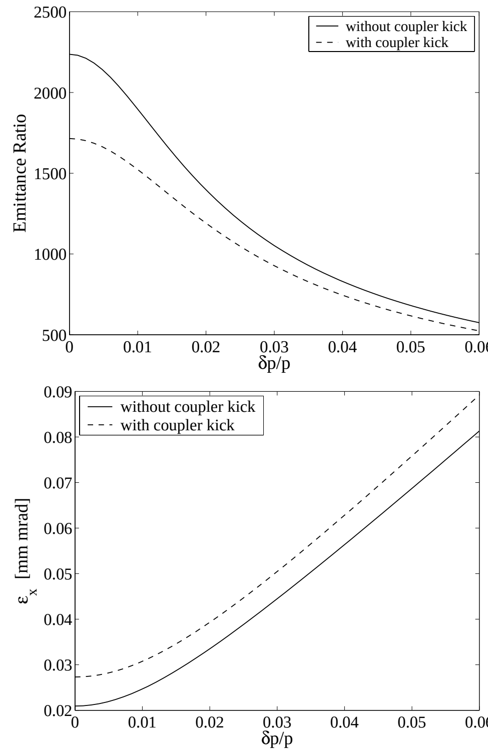

Figure 2: Asymmetry in the initial beam matrix caused by gun RF coupler kick effects emittance ratio (top) and horizontal emittance (bottom).

SUMMARY

We have shown that both the chromatic and asymmetric effects induce some residual x-y coupling downstream of the RTFB transformer, comparing to the symmetric beam without energy spread case. However the transverse emittances calculated when these two latter effects are considered remain constant in the downstream transport line, as long as no element in the beam line introduces x-y coupling.

As far as chromatic effects are concerned, we find our simple analytical treatment provide some insight regarding the dependencies of the transverse emittances on chromatic effect. Our predictions are in decent agreement with simulations taking into account that our model assumes the quadrupoles to be thin lenses. Regarding the impact of RF asymmetries, we have estimated the effects on the flat beam emittances but not yet compared with simulations.

The next step will be trying to find a possible cure for the aforementioned limiting effects on the round-to-flat beam transformation.

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