

THE COMMON DISPERSION EQUATION FOR HIGH-ENERGY COMPENSATED ACCELERATING STRUCTURES

V.V. Paramonov, Institute for Nuclear Research of the RAS, 117312 Moscow, Russia

Abstract

The general method for the field determinations in periodical structures has been developed. The field distribution of the travelling wave or standing wave mode for arbitrary phase shift in arbitrary passband is expanded over all 0- and π - modes field distributions. The resulting eigen value problem is the general dispersion equation of the structure for any passband. Considering such well known accelerating structures for high energy range as Coupled Cells Structures (CCS), such as SCS, ACS, APS, CCDTL, and DAW structure, it is founded the common equation for operating dispersion curve. The equation parameters are driven from the field distributions of four modes, limiting the operating passband. As compared to classical Knapp's equation for coupled cavities, the equation proposed is more complicated, but simultaneously describes both narrow (CCS) and wide (DAW) passband structures. The differences in equation parameters for different structures are presented and particularities of the structure analysis are discussed.

1 INTRODUCTION

The coupled cavities model [1] is widely used for description of CCS structures. This model is easy for treatment and describes the structure well if one can distinguish in the period of the CCS structure the clearly defined cells. Such structures are the Side Coupled Structure (SCS) [2], the Annular Coupled Structure (ACS) [3], Alternating Periodic Structure (APS) [4] or Coupled Cell DTL (CCDTL) [5] one with enough narrow operating passband. For the wide passband structures, like Disk and Washer (DAW) one [3], or Cut Disk Structure (CDS) [6], the coupled cavities model is at least not evident. Nevertheless, all structures, mentioned above, are compensated structures. To generalize the description of such quite different structures, we have to consider it from general point of view as periodical structures.

The method for the field description and frequency determination in periodical structures has been developed before [7]. Some results of applications are considered below.

2 THE METHOD FOR THE FIELD DESCRIPTION

Let simulate and store for the structure under investigation a set of field distributions for 0- and π -type modes. In the structure with symmetry plane 0-type modes can be calculated considering one half of the period in conditions **(ee)** (**e**) at $z = 0$ and **(e)** at $z = d/2$ and **(mm)**. The π -type modes should be calculated in conditions **(em)** and **(me)**. For the DAW structure an example of such set is

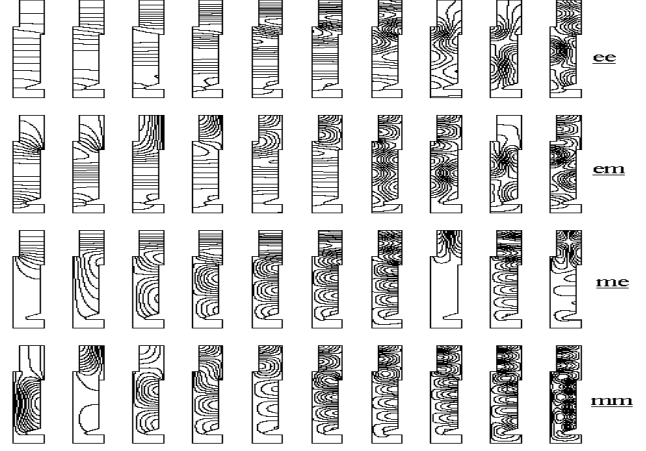


Figure 1. The basis of trial functions for the DAW structure. 0 modes - **(ee)**, **(mm)** and π modes **(em)**, **(me)**.

shown in Fig. 1. Considering these four families of field distributions (functions), **(ee)**, **(em)**, **(me)** and **(mm)**, as a basis of trial functions for variational approach, let represent the complex field amplitude $\vec{E} = \text{Re}\vec{E} - i\text{Im}\vec{E}$ of the travelling wave with the phase shift per period $\theta \neq 0, \pi$ in an arbitrary passband as:

$$\begin{aligned} \text{Re}\vec{E} &= \sum_n^{Nee} c_n^{ee} \vec{E}_n^{ee} + \sum_n^{Nem} c_n^{em} \vec{E}_n^{em}, \\ \text{Im}\vec{E} &= \sum_n^{Nme} c_n^{me} \vec{E}_n^{me} + \sum_n^{Nmm} c_n^{mm} \vec{E}_n^{mm}. \end{aligned} \quad (1)$$

The expression:

$$\omega^2 = \frac{\int_V \frac{1}{\mu_0} \text{rot}\vec{E}^* \text{rot}\vec{E} dV + \frac{I_S}{2}}{\int_V \epsilon_0 \vec{E}^* \vec{E} dV}, \quad (2)$$

where the boundary intergral I_S is required for Floquet boundary conditions at $S_1, z = -d/2$ and $S_2, z = d/2$

$$\begin{aligned} I_S &= \int_{S_1} \vec{v}[(\vec{E}e^{-i\theta/2} + \vec{E}^*e^{i\theta/2})\frac{1}{\mu_0}(\text{rot}\vec{E}^*e^{i\theta/2} + \\ &+ \text{rot}\vec{E}e^{-i\theta/2})]dS + \int_{S_2} \vec{v}[(\vec{E}e^{i\theta/2} + \vec{E}^*e^{-i\theta/2}) \\ &\frac{1}{\mu_0}(\text{rot}\vec{E}e^{i\theta/2} + \text{rot}\vec{E}^*e^{-i\theta/2})]dS, \end{aligned}$$

is variational one for the problem of the travelling wave propagation in symmetrical periodic structure without losses. Substituting (1) in (2) and following to a standard

variational technique, one come to the generalized symmetrical eigen-value problem:

$$AC - k^2 BC = 0, \quad (3)$$

with square block-type matrixes A and B :

$$\mathbf{A} = \begin{pmatrix} A_{ee}^{ee} & A_{ee}^{em} & 0 & A_{ee}^{mm} \\ A_{ee}^{em} & A_{em}^{em} & A_{em}^{me} & 0 \\ 0 & A_{em}^{me} & A_{me}^{me} & A_{me}^{mm} \\ A_{ee}^{mm} & 0 & A_{me}^{mm} & A_{mm}^{mm} \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} B_{ee}^{ee} & B_{ee}^{em} & 0 & 0 \\ B_{ee}^{em} & B_{em}^{em} & 0 & 0 \\ 0 & 0 & B_{me}^{me} & B_{me}^{mm} \\ 0 & 0 & B_{me}^{mm} & B_{mm}^{mm} \end{pmatrix}$$

Coefficients of these matrixes are for blocks

$$\begin{aligned} (A_{ee}^{ee}, A_{em}^{em}, A_{me}^{me}, A_{mm}^{mm}), a_{ij} &= \delta_{ij} k_i k_j, \\ (B_{ee}^{ee}, B_{em}^{em}, B_{me}^{me}, B_{mm}^{mm}), b_{ij} &= \delta_{ij}, \\ (B_{ee}^{em}), b_{ij} &= \frac{\epsilon_0}{W_0} \int_{V_1} \vec{E}_i^{ee} \vec{E}_j^{em} dV, \\ (B_{mm}^{me}), b_{ij} &= \frac{\epsilon_0}{W_0} \int_{V_1} \vec{E}_i^{mm} \vec{E}_j^{me} dV, \\ (A_{ee}^{em}), a_{ij} &= \frac{\epsilon_0}{W_0} \int_{V_1} \text{rot} \vec{E}_i^{em} \text{rot} \vec{E}_j^{ee} dV - \\ &\quad - \frac{\epsilon_0(1 + \cos \theta)}{2W_0} \int_{S_2} \vec{\nu} [\vec{E}_i^{em}, \text{rot} \vec{E}_j^{ee}] dS, \\ (A_{mm}^{me}), a_{ij} &= \frac{\epsilon_0}{W_0} \int_{V_1} \text{rot} \vec{E}_i^{mm} \text{rot} \vec{E}_j^{me} dV - \\ &\quad - \frac{\epsilon_0(1 - \cos \theta)}{2W_0} \int_{S_2} \vec{\nu} [\vec{E}_i^{mm}, \text{rot} \vec{E}_j^{me}] dS, \\ (A_{em}^{me}), a_{ij} &= \frac{\epsilon_0 \sin \theta}{2W_0} \int_{S_2} \vec{\nu} [\vec{E}_i^{em}, \text{rot} \vec{E}_j^{me}] dS, \\ (A_{ee}^{mm}), a_{ij} &= \frac{\epsilon_0 \sin \theta}{2W_0} \int_{S_2} \vec{\nu} [\vec{E}_i^{mm}, \text{rot} \vec{E}_j^{ee}] dS. \end{aligned}$$

3 DISPERSION EQUATION

The equation (3) represents the dispersion equation of the periodical structure in terms of coupled modes for arbitrary passband. If we take into account $N = N_{ee} + N_{em} + N_{me} + N_{mm}$ modes, we can consider $\min(N_{ee} + N_{mm}, N_{em} + N_{me})$ passbands simultaneously [7]. For compensated structures the operating dispersion curve consists from two branches and at least four modes, which limit these branches, are required. All high-energy accelerating structures have operating π -mode, so operating (accelerating) mode \vec{E}_a with wavevalue k_a belongs to **(em)** group (to have the strong electric field in the accelerating gap). The coupling π -mode \vec{E}_c, k_c has the conjugated parity of the field distribution and belongs to the **(me)** group. Both of two 0-modes (\vec{E}_1^0 with k_{01} and \vec{E}_2^0 with k_{02}), in all high-energy CCSs satisfy the boundary conditions of electric wall in the middle of accelerating gap, so, belong to the **(ee)** group. Restricting the field description (1) with these

modes - $\vec{E}_1^0, \vec{E}_2^0, \vec{E}_a, \vec{E}_c$, one get the dispersion equation for the description of the operating passband of the high-energy CCSs as the fourth order determinant:

$$\begin{vmatrix} k_{01}^2 - k^2 & 0 & I_1^h - I_2 C - I_1^e k^2 & 0 \\ 0 & k_{02}^2 - k^2 & I_3^h - I_4 C - I_3^e k^2 & 0 \\ a_{13} & a_{23} & k_a^2 - k^2 & I_5 \sin \theta \\ 0 & 0 & a_{34} & k_c^2 - k^2 \end{vmatrix}$$

where $C = (1 + \cos \theta)$, a_{13}, a_{23}, a_{34} - the same terms as in the upper triangle of the determinant, and, in accordance with previous section,

$$I_1^e = V_e(\vec{E}_1^0, \vec{E}_a), I_1^h = V_h(\vec{E}_1^0, \vec{E}_a), I_2 = S_h^e(\vec{E}_a, \vec{E}_1^0),$$

$$I_3^e = V_e(\vec{E}_2^0, \vec{E}_a), I_3^h = V_h(\vec{E}_2^0, \vec{E}_a), I_4 = S_h^e(\vec{E}_a, \vec{E}_2^0),$$

$$I_5 = S_h^e(\vec{E}_a, \vec{E}_c), V_e(\vec{E}_1, \vec{E}_2) = \frac{\epsilon_0}{W_0} \int_{V_1} \vec{E}_1 \vec{E}_2 dV,$$

$$V_h(\vec{E}_1, \vec{E}_2) = \frac{\epsilon_0}{W_0} \int_{V_1} \text{rot} \vec{E}_1 \text{rot} \vec{E}_2 dV,$$

$$S_h^e(\vec{E}_1, \vec{E}_2) = \frac{2\epsilon_0}{W_0} \int_{S_2} \vec{\nu} [\vec{E}_1, \text{rot} \vec{E}_2] dS.$$

Deriving this common dispersion equation, we don't use any particular information about structures - just consider the field parity of 0- and π -modes. It are common for all high-energy CCSs and reflect the usage of the TM_0 -like modes.

The equation proposed is match more complicated as compared to the Knapp's equation [1]:

$$(\gamma \cos \frac{\theta}{2})^2 = (1 - (\frac{k_a}{k})^2 + \gamma_a \cos \theta)(1 - (\frac{k_c}{k})^2 + \gamma_c \cos \theta), \quad (4)$$

where γ - is the coupling coefficient, γ_c, γ_a - the neighbor coupling coefficients for accelerating and coupling cells. But the proposed equation realized the multi-mode description of the field and describes well both narrow and wide passbands.

At the Fig. 2 the field distributions and dispersion curves for two options of the APS structure are shown. The dispersion curves were calculated directly at first, by using the high precision FEM (second order approximation) codes [9] with the Floquet boundary conditions, and after that with the decomposition (1) with four modes into account. For narrow band APS structure the frequency difference is of order $10kH_z$ for all θ and is inside the plot resolution. For the wide band DAW structure the equation proposed also well enough describes the operating dispersion curve. At Fig. 3 dispersion curves, obtained in the same way as for APS structure at Fig. 2, are shown for different cavity radius values R . The R is an additional free parameter in the DAW structure and for different R different shape of the curve can be obtained. For $R/\lambda_0 = 0.66, \partial f^t / \partial \theta < 0, 0 < \theta < \pi$, (DAW1 option at Fig. 3), for $R/\lambda_0 = 0.73, \partial f^t / \partial \theta = 0, 0 < \theta < \pi/2$, (DAW2 option) and for $R/\lambda_0 = 0.81, \partial f^t / \partial \theta > 0, 0 < \theta < 2\pi/3$, (DAW3 option). The frequency difference of

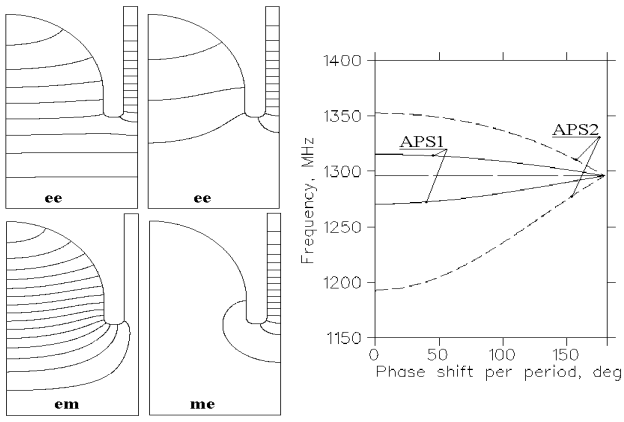


Figure 2. The field distributions (left) and dispersion curves for APS structure. APS1 - aperture radius $r_a = 3.0\text{cm}$, $\frac{r_a}{\lambda_0} = 0.13$, APS2 - $r_a = 6.0\text{cm}$, $\frac{r_a}{\lambda_0} = 0.26$.

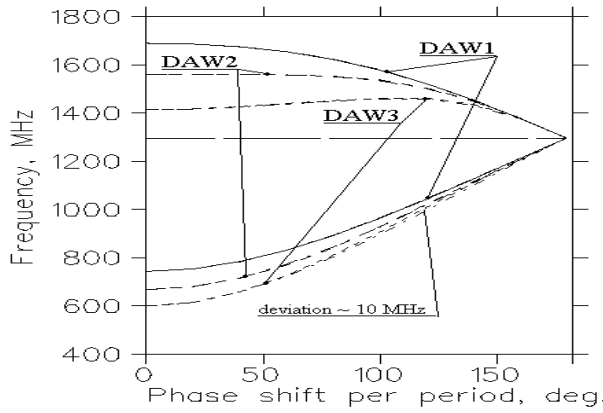


Figure 3. The dispersion curves for DAW structure. 1 - $\frac{R}{\lambda_0} = 0.66$, 2 - $\frac{R}{\lambda_0} = 0.73$, 3 - $\frac{R}{\lambda_0} = 0.81$.

order $\leq 8\text{MHz}$ takes place for DAW3 option at the bottom branch f^b of the curve for $\theta \approx \frac{2\pi}{3}$ - for wide band DAW structure even for four modes field expansion the influence of the higher TM_0 -like passbands takes place.

Parameters of the dispersion equation for different structures are listed in the Table 1. The equation parameters are obtained in the single way from the fields distribution

Table 1. Parameters of the dispersion equation for different structures.

	APS1	APS2	DAW1	DAW2	DAW3
k_{01}/k_a	0.980	0.920	0.574	0.515	0.463
k_{02}/k_a	1.015	1.044	1.304	1.203	1.091
I_1^e	0.742	0.750	0.477	0.419	0.367
I_2/k_a^2	0.014	0.058	0.160	0.154	0.144
I_3^e	-0.663	0.592	0.684	0.638	0.580
I_4/k_a^2	0.010	-0.026	-0.240	-0.143	-0.055
I_5/k_a^2	0.017	0.066	0.371	0.388	0.398

$\vec{E}_1^0, \vec{E}_2^0, \vec{E}_a, \vec{E}_c$ - it is the big advantage of the equation proposed. But not all parameters are independent. For example, the relationship takes place:

$$I_2 = 2I_1^h + 2k_{01}^2 I_1^e, \quad I_4 = 2I_3^h + 2k_{02}^2 I_3^e.$$

The coupling integral value I_5/k_a^2 represents [8] the group velocity β_g value at the operating π mode:

$$\frac{\beta_g}{\beta} = \left| \frac{\pi \int_{V_1} (\epsilon_0 \vec{E}_a \vec{E}_c - \mu_0 \vec{H}_a \vec{H}_c) dV}{4W_0} \right|, \quad (5)$$

which is related with coupling coefficients as $\frac{\beta_g}{\beta} = \frac{\pi\gamma}{4\sqrt{(1-\gamma_a)(1-\gamma_c)}}$. Usually in the analysis of the field stability in the structure against manufacturing and tuning errors the influence of frequencies deviations $\delta f_a, \delta f_c$ and coupling coefficient deviation $\delta\gamma$ is considered as independent. If there exists the perturbation with the volume δV in the structure, one can derive with perturbation theory:

$$\frac{\delta f_a}{f_a} \sim \int_{\delta V} (H_a^2 - \frac{E_a^2}{\eta^2}) dV, \quad \frac{\delta f_c}{f_c} \sim \int_{\delta V} (H_c^2 - \frac{E_c^2}{\eta^2}) dV,$$

$$\frac{\delta\gamma}{\gamma} = \frac{\delta\beta_g}{\beta_g} \sim \left| \int_{\delta V} (H_a H_c - \frac{E_a E_c}{\eta^2}) dV \right|, \quad \eta = \sqrt{\frac{\mu_0}{\epsilon_0}}.$$

For the narrow band structures the fields distributions \vec{E}_a, \vec{E}_c overlap in the small part of the period volume (see, for example, Fig. 2). It results in relatively low $\beta_g \sim \gamma \approx 0.04$ value, but the assumption for independent $\delta f_a, \delta f_c$ and $\delta\gamma$ influence works well. For wide band structures (DAW, CDS) the distributions \vec{E}_a, \vec{E}_c overlap in the main part of the period (see Fig. 1), resulting in $\beta_g \sim \gamma \approx 0.4$, but during the stability analysis the correlation between $\delta f_a, \delta f_c$ and $\delta\gamma$ should be taken into account.

4 REFERENCES

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