THE RADIATION FIELD BY THE TRANSFORM SPECTRAL METHOD IN FEL

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Abstract

Three dimensional radiation field of the free-electron laser is numerically studied. The radiation field is expanded by using the Fourier expansion coefficients to satisfy the free space wave equation so that the field could be solved semi-analytically. The final form of the semi-analytical solution is then integrated to get the physical properties of radiation field. Instead of using Lagrangian variables, the Eulerian variables, for the electron beam density, is used and numerically integrated. The shape of three-dimensional radiation field according to the wiggler distance and to the size of transverse dimensions are obtained. Magnitudes of the radiation field according to the transverse domain, the wiggler distance, the number of terms of Fourier expansion and the various radiation wavelength are investigated.

1 INTRODUCTION

For the one-dimensional free-electron laser (FEL) study, a number of articles have been published until now[1, 2, 3, 4, 5, 6, 7, 8, 9]. But since the FEL experiments are complex, simple one-dimensional theory is not often adequate. Therefore, in the field of experimental FEL, the importance of three-dimensional numerical simulations is emphasized in design, optimization and interpretation. For the FEL based on the self-amplified spontaneous emission (SASE), understanding 3D effects and 3D simulation are becoming to be more important in constructing a FEL[10].

In studying the transverse variation of the wave equation of the FEL, many different numerical and analytical methods are employed such as the transverse mode spectral method[11, 12, 13], the transform spectral method[14, 15], the finite difference method[16], and the Lienard-Wiechert potential method[17]. Recently, in Ref. [13], the three-dimensional time dependent transverse mode scheme was rather expanded to include longitudinal mode competition.

The transform spectral method evaluates the transverse second order differentiate terms of vector potential in the wave equation. And the method involves representing the solution of the radiation field as a truncated series of known functions of the independent variables. The method has a great advantage to reduce the wave equation to a first order differential equation, where the current can be described in terms of the Lagrangian or Eulerian variables, and the current terms can be evaluated analytically or numerically. The advantages of the transverse spectral method such as analytical handling of the free space wave propagation, easily including the transverse particle motion exactly, lending to analytical and semi-analytical solutions, and automatical including the transverse boundary conditions in the waveguide mode expansion can be used in the transverse transform spectral method.

2 WAVE EQUATION

The vector potential representing the linearly polarized wiggler and radiation field are expressed as follow: The linearly polarized wiggler is the most common wiggler field.

$$\mathbf{A}_w(y,z) = A_w(z)\cosh(k_w y)\cos[\bar{k}_w(z)] \qquad (1)$$

where $\bar{k}_w(z) = \int_0^z k_w(z')dz'$, $A_w(z)$ and $k_w(z)$ are the slow varying amplitude and wave number of the wiggler.

For $k_w y \ll 1$, the amplitude of wiggler can be approximated as $A_w(z)\cosh(k_w y) \sim A_w(z)$.

The radiation field can be expressed as

$$\mathbf{A}_R(x,y,z,t) = -\frac{1}{2}\mathbf{A}(x,y,z)e^{i(kz-\omega t)} + \text{c.c}, \qquad (2)$$

where

$$\mathbf{A}(x,y,z) = -\frac{1}{D_x D_y}\sum_{l,m=0}^{L,M} A_{l,m}(z)e^{i(k_l x + k_m y)}\hat{e}_x \qquad (3)$$

is the slowly varying amplitude of the radiation field expressed by the Fourier series, $A_{l,m}$ is the complex amplitude of the normal modes. The domain for the radiation field is $-D_x/2 \le x \le D_x/2$ and $-D_y/2 \le y \le D_y/2$ and the boundary conditions are periodic, and $k_l = 2\pi l/D_x$ and $k_m = 2\pi m/D_y$.

The radiation field, Eq. [2] should satisfy the wave equation for FEL as

$$\left(\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}_R = -\frac{4\pi}{c}\mathbf{J}_\perp. \qquad (4)$$

The orthogonality condition over the radiation field domain is

$$\int_{-D_x/2}^{D_x/2} e^{i(k_l - k_{l'})x}dx \int_{-D_y/2}^{D_y/2} e^{i(k_m - k_{m'})y}dy = D_x D_y \delta_{ll'}\delta_{mm'}. \qquad (5)$$

Using the slow varying approximation and the orthogonality condition, and dotting both sides with $\hat{e}_x$ yield

$$\left[\frac{d}{dz} - \frac{k_l^2 + k_m^2}{2ik}\right] A_{l,m} = -\frac{i}{k}\frac{2\omega}{c}\int_0^{2\pi/\omega} dt \int_{-D_x/2}^{D_x/2} dx \int_{-D_y/2}^{D_y/2} dy \cdot J_x e^{-i(k_l x + k_m y)} e^{-i(kz-\omega t)}. \quad (6)$$

The current density J_x can be formulated in Eulerian variables as well as in Lagrangian variables[18]. For the former case in the transform spectral method, the right-hand side of Eq. [6] has to be evaluated numerically.

A general form of current density in FEL is given by[19]

$$J_x(x, y, z, t) = \frac{w_b^2}{4\pi c}\int_{-\infty}^{\infty} dt_0 \int_{-\infty}^{\infty} dx_0 \int_{-\infty}^{\infty} dy_0 e^{i\omega_w t} \cdot \theta(x_0, y_0)\delta(x-\tilde{x})\delta(y-\tilde{y})\delta(t-\tilde{t})\frac{A_w(\tilde{y}, z)}{\tilde{\gamma}}, \quad (7)$$

where $w_b = (4\pi e^2 n_0/m_0)^{1/2}$, n_0 is the beam density, $\theta(x_0, y_0)$ is a function describing the initial transverse electron beam profile and $\tilde{\gamma}$ is the total electron energy. Neglecting the gradients in the wiggler, i.e., $k_w y_0 \ll 1$, the zeroth order transverse electron coordinates $\tilde{x}$, $\tilde{y}$ are replaced by the Lagrangian independent variables x_0 and y_0. Choosing a Gaussian electron beam profile, i.e., $\theta(x_0, y_0) = \exp[-(x_0^2 + y_0^2)/r_b^2]$, and integrating the transverse coordinates and the time over the periods, the electron beam density, Eq. [7], was then numerically integrated. It should be noted that the Gaussian electron beam profile is independent of axial distance. Physical property, e.g., self-focusing of radiation field cannot then be studied with this beam profile. An alternative way to express the beam profile is the Gaussian-Hermite expansion[18], about which we are studying:Part of results will be shown in Fig. 2.

Using the Gaussian beam profile, the final form of the first order differential equation for the Fourier expansion coefficients, $A_{l,m}$, can be written as,

$$\frac{dA_{l,m}}{dz} = \alpha A_{l,m} + \beta\zeta\eta\cos(k_w z)e^{ikz} \quad (8)$$

where

$$\alpha = -\frac{i}{k}\frac{k_l^2 + k_m^2}{2}$$

$$\beta = \frac{i}{k}\frac{\omega\omega_w\omega_b^2}{4\pi^2(\omega+\omega_w)\gamma c^2} A_w\left(e^{i2\pi\frac{\omega_w}{\omega}} - 1\right)$$

$$\zeta = \int_{-D_x/2}^{D_x/2} dx e^{-ik_l x - \frac{x^2}{r_b^2}}$$

$$\eta = \int_{-D_y/2}^{D_y/2} dy e^{-ik_m y - \frac{y^2}{r_b^2}} \cosh(k_w y).$$

The Eulerian formulation of the current is known to be inferior to the Lagrangian formulation because the numerical transformations of Eq. [6] require the current to be evaluated at prespecified grids, but the problem can be reduced if the number of grids used across the electron beam is large, which in turn requires a larger number of terms in the expansion. The Eulerian formulation used in this study is somehow superior in the sense of the speed of computation, if the differential equation is solved analytically. By reducing the computational time, the number of grids used across the electron beam and then the number of expansion terms can be enlarged.

3 RESULTS AND DISCUSSION

The spectral transform method was applied to show the behavior of radiation field employing Fourier expansion, and choosing Gaussian beam profile with a radius of 2.25×10^{-2} cm. The domains for the radiation field, D_x and D_y, were not fixed but checked for several values: The main working value was 5.0×10^{-4} m. Predefined wavelengths were used to compute the correspondent field. Wiggler period of $\lambda_w = 2.8$ cm, axial electron velocity of $v_z = 0.99c$, electron beam density of $n_0 = 1.3 \times 10^{11}$ cm^{-3} and $A_w = 2.2 \times 10^3$ statvolts were used.

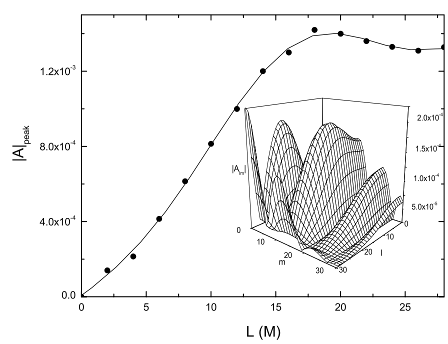

Figure 1: Variation of the radiation amplitude peaks according to the number of Fourier expansion terms. The peaks saturate as the number increases (Solid curve is for eye guide). (inset: Behavior of the Fourier expansion coefficients in transverse coordinates.)

Fig. 1 shows the peak values of slow varying amplitude, $|A(x, y, z)|$, corresponding to the number of terms in Fourier expansion. The peak values begin to saturate from around $L = 20$ (L and M were taken to have the same values for all computations in the study.) The Fourier expansion coefficients according to the l and m were shown in inset. The absolute values of the coefficients are gradually decreased with wiggling as l and m increased. Thus fifty terms of expansion are thought to be enough to demonstrate the physical behaviors of radiation field.

The peak values of slow varying amplitude as a function of axial distance obtained at $z = 1.5$, 3, 4.5 and 6 m are shown in Fig.2. Contrary to the case of the transverse spectral method, the peak seems to increase monotonically. The radiation is being amplified all the way of axial distance.

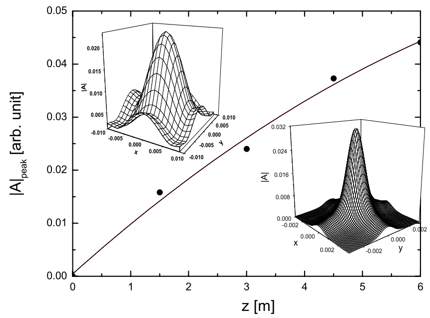

Figure 2: Variation of the radiation amplitude peaks according to the axial distance. The peaks monotonically increase along the axial distance (Solid curve is for eye guide). [Inset: Typical shape of radiation amplitude computed by using Gauss-Hermite (up) and Gauss (down) beam profiles.]

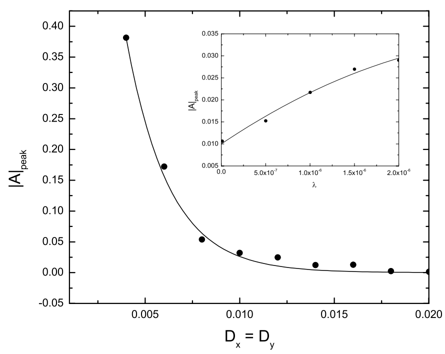

Figure 3: Variation of the radiation amplitude peaks according to the radiation domain. The peaks exponentially decrease as the domains increase. (inset: Amplitude peaks according to the radiation wavelength. The peaks show a monotonic increment as the wavelength increases.) (Solid curve is for eye guide).

In the inset, a typical figure of output is shown at $z = 3$ m. The transverse domain of radiation, in this case, is symmetric: We checked the asymmetric case, it turned out that the shape of radiation amplitude maintained the symmetry. One of the disadvantages of the Eulerian formulation is that, since the beam profile does not depend on the axial coordinate, the self-focusing property of the radiation field could not be observed. Fig. 3 shows the peak values, at $z = 3.0$ m, as a function of symmetric radiation domain. As was mentioned above, the amplitude was checked to be symmetric even if the asymmetry cases of domain, As shown in the figure, the values exponentially decreased as D_x (D_y) increased. It is believed that the large domain intensity generates large peak to keep the peak density constant. In the inset of Fig. 3, energy conservation of the formulation is shown at $z = 3.0$ m. As the wavelength of output increased the peak values also increased to conserve the energy.

In summary, this paper illustrated the physical properties of three-dimensional radiation field using the transform spectral method. Even if the method has disadvantages that the numerical transformations require the current to be evaluated at prespecified grids, the fast computation speed of this method can compensate for the disadvantage. Peak values of the radiation amplitude were mainly observed: 1) The values saturate as the number of expansion terms increased, 2) Those increased monotonically along the axial distance, 3) The domain intensity plays a role making those decrease exponentially, 4) The Energy conservation was conformed by calculating the peaks according to the wavelengths of radiation.

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