ANALYTICAL AND SEMI-ANALYTICAL EXPRESSIONS FOR THE VOLTAGE IN A CAVITY UNDER DYNAMIC DETUNING

Marc Doleans and Sang-ho Kim
Spallation Neutron Source (SNS) Project
ORNL, Oak Ridge 37830, USA

Abstract

Elliptical superconducting radio frequency (SRF) cavities are sensitive to frequency detuning because they have a high Q value in comparison with normal conducting cavities and weak mechanical properties. Radiation pressure on the cavity walls, microphonics, and tuning system are possible sources of dynamic detuning during SRF cavity operation. The accelerating voltage evolution in a cavity under dynamic detuning is analytically expressed by a general integral formulation. In some cases, an explicit solution for the voltage can be obtained. For general cases, a semi-analytical calculation scheme is also derived. This scheme offers a fast and flexible computational tool to simulate the cavity voltage under various conditions. Examples of cavity voltage behavior under different detunings, including Lorentz dynamic detuning, are presented with the Spallation Neutron Source (SNS) SRF cavity parameters.

1 INTRODUCTION

In many designs or real constructions for pulsed mode linacs, elliptical SRF cavities have been chosen because of their capacity to operate at high accelerating gradient with high RF efficiency, large bore radius, etc. These elliptical cavities have, however, intrinsic weak mechanical properties. In pulsed mode operation the cavities will experience dynamic detuning, for example due to the time varying Lorentz forces acting on the cavity walls. The voltage behavior of a cavity under dynamic detuning is more complex than in the case of a static detuning. To model the behavior of the voltage in time, the usual parallel resonant circuit model representation [1] with fixed capacitance and inductance was generalized by considering these parameters as time varying [2]. In Sec. 2, the differential envelope equation for the voltage and its general solution will be presented. Since the dynamic detuning appears in the cavity due to the deformation of the cavity wall shape, a mechanical model for the cavity wall deformation is necessary. As proposed in [3], the mechanical vibrations will be addressed by a modal analysis. In the SNS, the main sources of the dynamic detuning are the Lorentz forces and the piezotuning system [4] that will be used for compensation. Because the action of these two sources on the cavity wall is different, the modal basis for each of them is clearly separated. In Sec. 3, the equations for the dynamic detuning from the mechanical model will be stated. The equation of the voltage envelope of Sec. 2 and the mechanical equations of Sec. 3 are combined in a semi-analytical scheme. In Sec. 4, preliminary comparison with measured data will be shown.

2 VOLTAGE FOR A CAVITY UNDER DYNAMIC DETUNING

In [2], the envelope equation for the voltage in a cavity under dynamic detuning was derived and based on a parallel resonant circuit model representation of the cavity system where the capacitance and the inductance are considered as time varying parameters. The necessary conditions for this modeling are, high Q system, slow variations of the driving current and of the cavity detuning compare to the resonance frequency, and small amplitude of the detuning compare to the resonance frequency. Under these considerations the equation for the voltage envelope can be written as a first order differential equation:

$$\dot{\tilde{V}} - j\tilde{\omega}\,\tilde{V} = R_L\omega_{1/2}\,\tilde{I} \qquad (1)$$

where $\tilde{V}$ is the voltage envelope, $\tilde{I}$ is the current envelope (linear sum of the RF source current and the beam source current), where the real part, $\Delta\omega$ of the complex frequency $\tilde{\omega} = \Delta\omega + j\omega_{1/2}$ is the time varying detuning of the cavity frequency with respect to the RF source. And where the loaded shunt impedance R_L and the cavity half-bandwidth $\omega_{1/2} = \dfrac{\omega}{2Q_L}$ are parameters constant in time. The general solution of Eq. (1) can be expressed under an integral formulation

$$\tilde{V} = \tilde{V}_0\, e^{j\int_0^t \tilde{\omega}\,dt'} + R_L\omega_{1/2}\int_0^t \tilde{I}\, e^{j\int_{t'}^t \tilde{\omega}\,dt''}\,dt' \qquad (2)$$

Where the first term on the right side corresponds to the solution of the source free equation which depends on the initial condition of the cavity voltage $\tilde{V}_0$, and where the second term corresponds to the driven part of Eq. (1). The Eq. (2) is the general integral formulation for the cavity voltage envelope in presence of dynamic detuning and time varying driving current. Obtaining a more explicit analytical expression for a given set of current and detuning functions $\{\tilde{I}, \tilde{\omega}\}$ is a matter of solving the integrals.

Some examples where the integrals can be solved are presented and illustrated in [2]. Since the dynamic detuning will be decomposed on a modal basis, the case of a sine dynamic detuning function and a fixed current source is of particular interest.

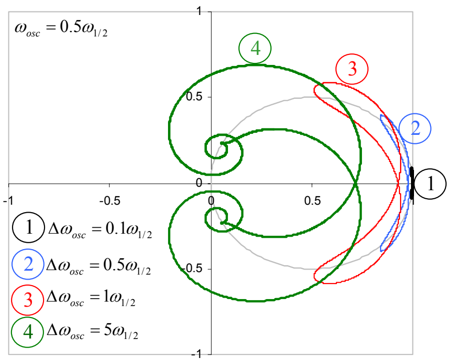

Figure 1: Parametric plot of the voltage for sinusoidal dynamic detuning function of different amplitude and constant frequency.

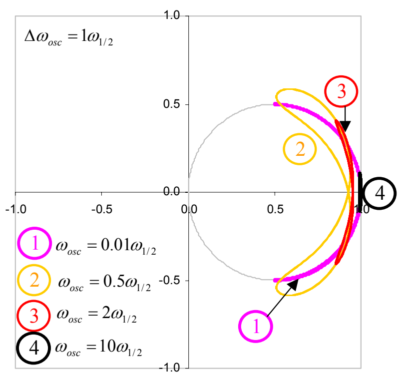

Figure 2: Closed orbits of the Voltage for sinusoidal dynamic detuning function of different frequency and constant amplitude.

$$\tilde{\omega} = \tilde{\omega}_0 + \Delta\omega_{osc}\sin(\omega_{osc}t) \qquad (3)$$

where $\tilde{\omega}_0 = \Delta\omega_0 + j\omega_{1/2}$ contains some possible value for the initial detuning $\Delta\omega_0$, where $\Delta\omega_{osc}$ is the amplitude of the dynamic detuning oscillations, and ω_{osc} the frequency of these oscillations.

The cavity voltage in steady state is periodic of period ω_{osc}. In a parametric representation, this periodicity means that the voltage moves on a closed orbit. Examples of such closed-orbits are presented on Fig. 1 for a fixed value of the frequency of the detuning and for different values of the amplitude of the detuning. The circle shape corresponding to all the possible value of the voltage in steady state for static detuning cases is drawn in gray color. If the voltage of a cavity under dynamic detuning was behaving as in the static case, the motion of the voltage would remain on the circle, but as shown in Fig. 1, the motion in a dynamically detuned cavity is more complex. In fig. 2 some closed orbits of the voltage are presented in the case of a constant amplitude of the detuning and different value of the frequency of the detuning.

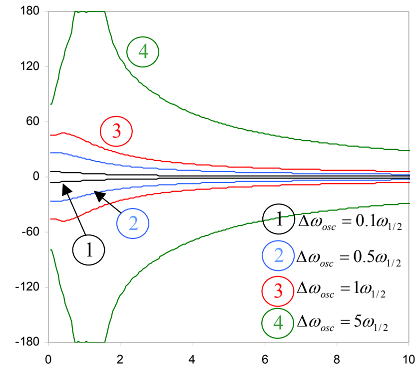

Figure 3: Minima and maxima of the voltage phase (deg) swing in function of the frequency of the sinusoidal detuning function (bandwidth unit).

When the frequency of the detuning is very small compare to the cavity half-bandwidth, the closed orbits are essentially following the gray circle. When the frequency of the detuning becomes large compare to the cavity half-bandwidth, the extensions of the closed orbits tend to shrink. This is understandable because the half-bandwidth is a measure of how fast the field builds in the cavity. When the detuning is very fast, the field in the cavity can not follow the detuning and an averaging effect occur. The extrema of the voltage orbits for the amplitude and the phase of the voltage are functions of the amplitude and frequency of the detuning function. These dependencies are presented in Fig. 3 and Fig. 4. The saturation in Fig. 3 is due to the fact that the voltage phase reaches pi and –pi.

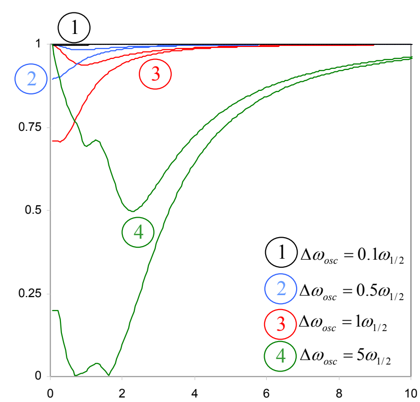

Figure 4: Minima and maxima of the voltage amplitude swing in function of the frequency of the sinusoidal detuning function (bandwidth unit).

3 MECHANICAL MODEL FOR THE DYNAMIC DETUNING

A mechanical model is necessary to calculate the dynamic detuning. In the SNS, the detuning is mainly created by the Lorentz forces and by the piezoelectric tuners and a modal basis can be applied to each of these vibration sources. It is assumed that the relation between the amplitude of the wall vibrations and the amplitude of the induced detuning is linear for every mode and that each mode can be represented by a damped oscillator.

Indexing by a letter l, the quantities referring to the Lorentz forces action and by a letter p the quantities referring to the piezo tuner action, assuming respectively L and P modes and writing respectively $\{\Omega_l, Q_l, k_l\}$ and $\{\Omega_p, Q_p, k_p\}$ the parameters for the two modal basis, the detuning induced in each mechanical mode satisfies the differential equation

$$\Delta\ddot{\omega}_{l,p} + \frac{\Omega_{l,p}}{Q_{l,p}}\Delta\dot{\omega}_{l,p} + \Omega_{l,p}^2\Delta\omega_{l,p} = F_{l,p}k_{l,p}\Omega_{l,p}^2 \qquad (4)$$

where the forcing function for the Lorentz forces action can be written $F_l = -|\tilde{V}|^2$ and where the forcing function for the piezo action can be written $F_p = V_{Piezo}$, with V_{Piezo} the input voltage of the piezo tuner. The total induced dynamic detuning is given by the sum:

$$\Delta\tilde{\omega} = \Delta\tilde{\omega}_0 + \Delta\omega_{Lorentz} + \Delta\omega_{Piezo} \qquad (5)$$

where the total dynamic detuning generated by the Lorentz forces action or by the piezo action is the linear sum on all their mode contributions.

4 RESULTS FOR FULL MODELING

The two aspects of the modeling of a cavity under dynamic detuning, modeling of the voltage described in Sec. 2 and modeling of the dynamic detuning described in Sec. 3, should be combined. The two parts are not independent and a semi-analytical calculation scheme is derived and presented in Fig. 5. For each time interval, the solutions for the voltage and for the dynamic detuning are analytical and given by (Eq. (6))

$$\Delta\omega_{l,p}[t+\Delta t] = e^{-\eta_{l,p}\Omega_{l,p}\Delta t}\begin{pmatrix}\Delta\omega_{l,p}[t]\cos(\mu_{l,p}\Omega_{l,p}\Delta t) \\ +\mu_{l,p}^{-1}(\Omega_{l,p}\Delta\dot{\omega}_{l,p}[t]-\eta_{l,p}\Delta\omega_{l,p}[t])\sin(\mu_{l,p}\Omega_{l,p}\Delta t)\end{pmatrix} + F_{l,p}[t+\Delta t/2]k_{l,p}\left(1-\mu_{l,p}e^{-\eta_{l,p}\Omega_{l,p}\Delta t}\cos(\mu_{l,p}\Omega_{l,p}\Delta t-\Phi_{l,p})\right)$$

$$\tilde{V}[t+\Delta t] = \tilde{V}[t]e^{j\tilde{\omega}[t+\Delta t/2]\Delta t} + R_L\tilde{I}[t+\Delta t/2]\frac{j\omega_{1/2}}{\tilde{\omega}[t+\Delta t/2]}(1-e^{j\tilde{\omega}[t+\Delta t/2]\Delta t})$$

with $\eta_{l,p} = 1/(2Q_{l,p})$, $\mu_{l,p} = (1-\eta_{l,p}^2)^{1/2}$, $\tan\Phi_{l,p} = \eta_{l,p}\mu_{l,p}^{-1}$

For practical use, the parameters of the two mechanical modal basis should first be obtained.

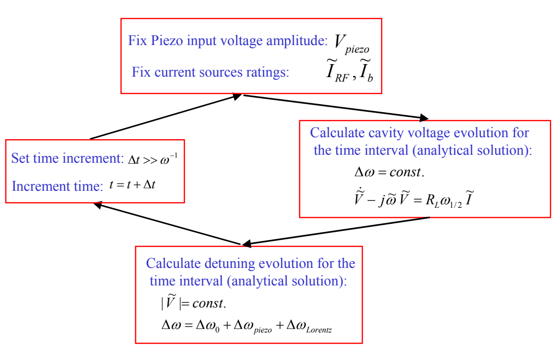

Figure 5: principle of the semi-analytical calculation scheme

The response mechanical spectrum to the piezo excitation was measured at JLAB on the SNS cryomodule prototype cavity number 2. A preliminary mechanical modal basis for the piezo action was built to fit the measured spectrum. If the full modeling is correct the voltage behavior under any piezo excitation should be reproducible in the simulation by use of this modal basis. Some measurements for the voltage behavior under square piezo excitation wave forms were performed and compared with the results from simulations as displayed in Fig. 6. The preliminary agreements between simulations and measurements are satisfying and validate the entire modeling approach.

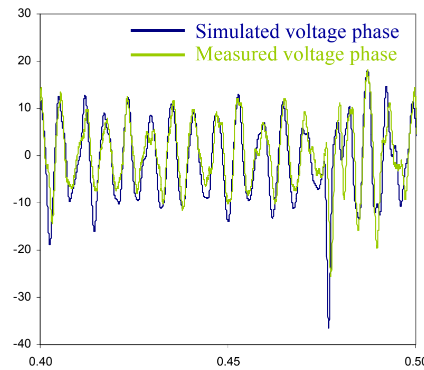

Figure 6: Voltage phase (deg) Vs time (s) for a 10Hz/1ms square piezo voltage excitation

5 CONCLUSION

A modeling combining the calculations of the voltage for a cavity under dynamic and the calculations of the dynamic detuning from a mechanical modal analysis was presented. The validity of this approach is confirmed by comparison with measured data. Simulations can for example be used to optimize the piezo tuning process.

6 AKNOWLEDGEMENT

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