NEW REAL-TIME SIMULATION TECHNIQUE FOR SYNCHROTRON AND UNDULATOR RADIATIONS

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Abstract

New mathematical method has been developed to compute radiation field from a moving charge in free space. It is not based on the retarded potential and its derivation. It uses the following two facts: (1) once a wave is emitted from a particle, it propagates as a spherical wave. It's wavelet (a part of the wavefront) runs with speed of the light, and does not change its direction, (2) the initial direction of the wavelet is determined by the Lorentz transformation from electron-rest-frame to the laboratory frame by taking into account the light aberration. 2D radiation simulator has been developed based on this method, which simulates synchrotron, undulator and dipole radiation in time domain.

1 INTRODUCTIONS

In various experimental applications of radiation, such as, the synchrotron, undulator and FEL radiations, discussions are usually made in terms of the angular and frequency spectrum of these radiations. These field properties are historically analysed by solving retarded potential for specified trajectory. Usually only the far-field radiation, whose field intensity is proportional to r^{-1}, is considered, and the Coulomb field is omitted since it decays quickly as r^{-2}. The results from this approximation have been widely used to evaluate the experimental data and its validity has been well confirmed.

However, understanding the realistic spactial distribution of radiation field and its time evolution becomes more important for studying the beam physics in electron accelerators. For example, the electron bunch-compressor for the e+e- Linear Collider, or the X-ray FEL, uses very short and intense bunch, and the CSR: Coherence Synchrotron Radiation breaks the transverse emittance of the beam. To cure this effect, the understanding the radiation field and beam dynamics is key issues of these accelerators.

Historically, the realistic field profile of the radiation field was firstly made by R. Y. Tsien[1] in 1972. He numerically integrated parametric equations for the field lines based on the retarded potential. By using IBM 360/65 computer, he visualized the electric field lines by California Computer. It took so long CPU time, because he needed to integrate every field line with fine step with computing fairly complicated vector algebra.

The method reported in this paper solves the field lines based on a kind of "con-formal mapping" technique. It was originally made by the author in 1974[2]. The algorizm is simple, and the code runs very fast, so that it can illustrate the animation of the radiation field in "real-time".

2 MATHMATICAL MODEL

2.1 Basic Equation

The Maxwell equation with field source is given by

$$\begin{aligned} \nabla\times\boldsymbol{H} &= \boldsymbol{J}+\frac{\partial\boldsymbol{D}}{\partial t} \\ \nabla\times\boldsymbol{E} &= -\frac{\partial\boldsymbol{B}}{\partial t} \\ \nabla\cdot\boldsymbol{B} &= 0 \\ \nabla\cdot\boldsymbol{D} &= \rho \end{aligned} \qquad (1)$$

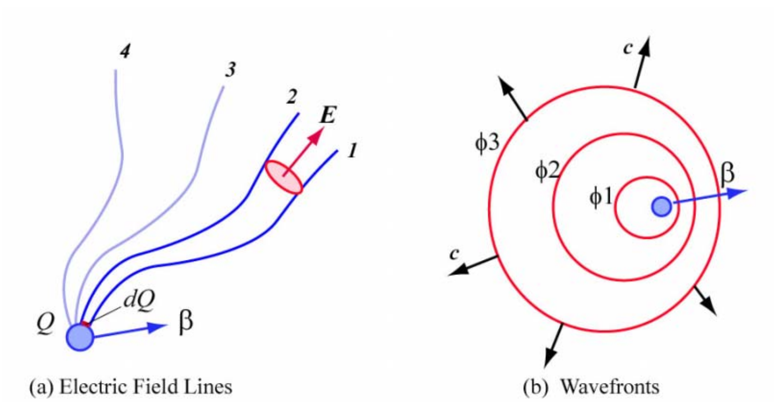

Fig. 1 Electric field lines of moving charge, and spherical wavefronts.

Here we treat radiation field from a single charge in free space. In the Maxwell equation, there are two driving terms, ρ and $\boldsymbol{J}$, which are related by the following continues equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{J} = 0 \tag{2}$$

Therefore, we do not need to treat $\boldsymbol{J}$ as explicit manner, which is automatically included when we correctly treat ρ on a moving charge. The magnetic field is derived by the first equation, so that it is enough to calculate only the $\boldsymbol{E}$ with ρ in the case of the single charge field.

Since we have Gauss's theorem, which is always satisfied for a moving charge, the flux enclosed area $d\boldsymbol{S}$ is kept constant when we follow the "flux pipe" as shown in Fig.1a. Thus

$$dQ = \varepsilon_0 \int \boldsymbol{E} \cdot d\boldsymbol{S} \tag{3}$$

where dQ is a part of total charge Q of moving particle. If we know the cross-sectional area $d\boldsymbol{S}$, we can determine the field strength E from eq. (3), then the associated magnetic field from the Maxwell equation (1a).

Along with the motion of charge, information of each event propagates outward with speed of the light as a spherical wave. Because of the causality, we can apply numbering on each wave-front as shown in Fig. 1b. The shape of the wavefront is always perfect sphere in free space, and continuously expands with speed of the light, only the origin (start point) differs for each event.

The discussion above is basically 3D problem, so that it is possible to develop 3D computer code to solve radiation field based on this method. However, to make the discussion simple, and ease code development, firstly the 2D code: Radiation2D was developed. The field is treated in 3D manner, but the electric field distribution on 2D plane is computed and plotted. We tread only the 2D trajectory of moving charge, such as, circular, sinusoidal, or dipole oscillation. In this condition, the electric field lines and wave-fronts are treated as a 2D grid space as seen in Fig. 2, and motion of the node points (crossing point) is tracked in real time manner. It should be noted that, this grid space is not always orthogonal. For a resting particle, the grid space becomes orthogonal, but for a moving charge the electric field line and wave-front becomes non orthogonal. This is due to the light aberration effect, which is discussed in the next section.

2.2 Initial Conditions

When a wave-front is emitted from a moving charge, direction of the wavelet (a part of the wave-front) is tilted toward the velocity of motion. This is due to the light-aberration effect. When a wavelet is emitted in the direction of unit wave-vector **k**' on the electron rest frame,

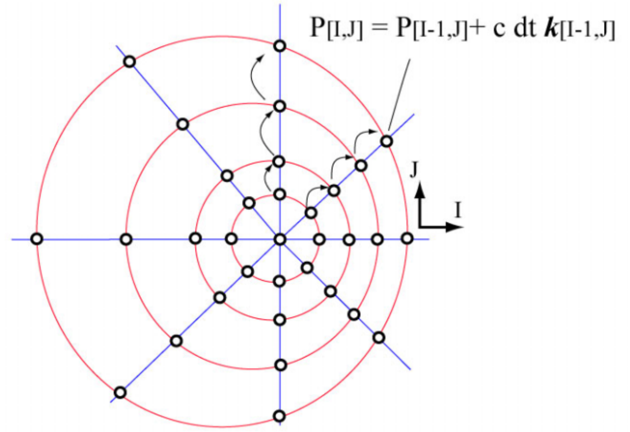

Fig. 2 Series of wavefronts and field lines form 2D grid space. The positions of the node points P are computed in time step.

the observed wavelet on the laboratory frame propagates along unit vector **k**,

$$\mathbf{k} = \begin{bmatrix} k_{//} \\ k_{\perp} \end{bmatrix} = k \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \frac{1}{1+\beta \cdot k_{//}'} \begin{bmatrix} k_{//}' + \beta \\ \dfrac{k_{\perp}'}{\gamma} \end{bmatrix} \qquad \text{.........(4)}$$

When a particle is running at relativistic speed, the direction of the wavelet is focused in the direction of motion. This is the physical origin of the radiation power of the synchrotron or undulator radiation being focused in the cone-angle of 1/γ along the direction of particle velocity.

3 THE 2D RADIATION SIMULATOR

The windows application has been created, which simulates 2D radiation field. It shows electric field line motion in real-time, and wavefront propagation. The code is available from our Web-site http://www-xfel.spring8.or.jp, which runs on Windows 98, 2000, XP. No Linux, nor Machintosh version is supported, at moment.

3.1 Numerical Model

To visualize radiation field in real time, 2D radiation simulator was developed. In this simulation, particle runs along 2D trajectory, such as, circle, sinuous wiggle or line trajectory, whose radiation becomes the synchrotron, the undulator and dipole radiation, respectively.

In the 2D radiation simulator, the positions of the node points of the electric field lines and wave-fronts are recorded in 2D matrix. One step of the computation is

(1) move the particle in one step: $c\beta dt$.
(2) compute the direction of wavelet by eq. (4).
(3) move node point one step with speed of light.
(4) shift the address one step along electric field line.

In each time steps, new wavefront is generated from the particle, and propagates outward by the following equations:

$$\begin{aligned} &\boldsymbol{P}[I,J] = \boldsymbol{P}[I-1,J] + cdt \cdot \boldsymbol{k}[I-1,J] \\ &\boldsymbol{k}[I,J] = \boldsymbol{k}[I-1,J] \end{aligned} \tag{5}$$

where P is the coordinate of the grid point.
Followings are snapshots from the simulator.

3.2 Static Field

One important example is the static field of a rest particle. Even when the particle rests, wavefronts are generated from the particle, and propagate outward. Since time derivation of E is zero, thus the magnetic field is zero, as a result, the pointing vector becomes zero. Therefore, there is no energy loss, and only the information is transferred.

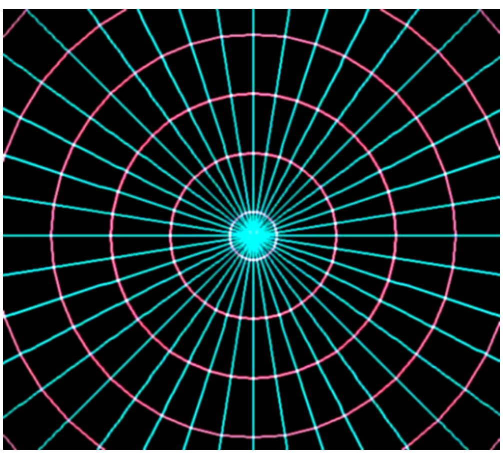

Fig. 3 Static field. Snapshot from the Radiation 2D.

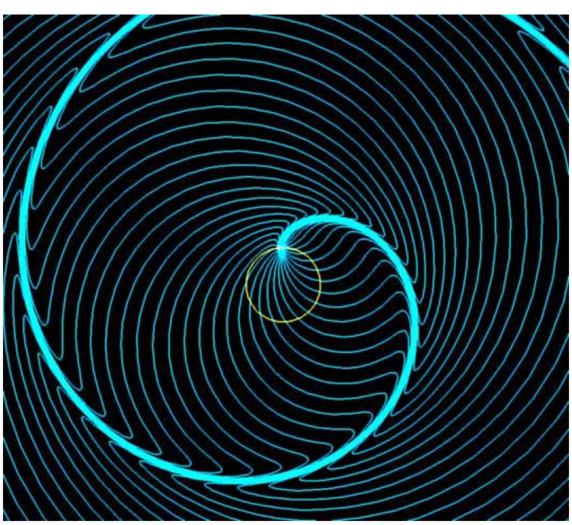

Fig. 4 Synchrotron radiation at v = 0.9c. Snapshot from the Radiation 2D.

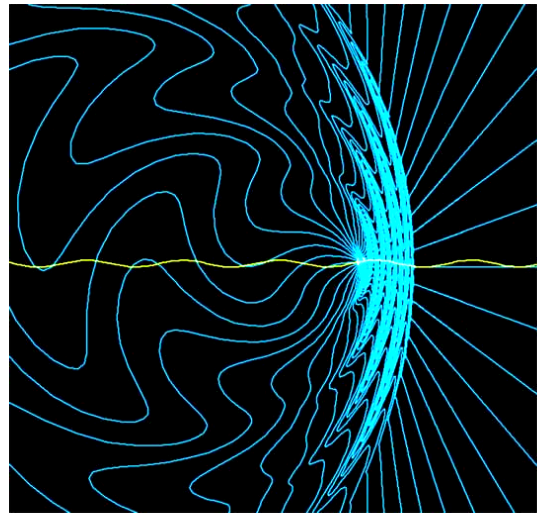

Fig. 5 Undulator radiation, v = 0.9c, K = 1. Snapshot from the Radiation 2D.

3.3 Synchrotron Radiation

When a charge particle runs along a circular trajectory, it generates spiral shape electric field as shown in Fig. 4. The field lines are condensed in bright spiral zone, where the electric field is very high. Increasing particle velocity, the bright zone becomes narrower, which corresponds to short impulse field, which has wide frequency spectrum. This is the synchrotron radiation.

3.4 Undulator Radiation

When a charged particle runs through an undulator, it is periodically deflected due to series of transverse magnetic field. In each bending, particle generates radiation in the direction of motion. Since the particle velocity is slightly lower than the speed of the light, wavelength of the accumulated periodic radiation becomes very short due to Doppler effects. This is clearly shown in Fig. 5.

4 DISCUSSION

The extension to 3D is straight forward. Problem will be extension to multi-particle problem. However, as seen in the snapshot of the undulator radiation, at the location where the radiation power is, we have much data point (node point), which provide enough spatial resolution. This is a kind of auto-zooming function. This will be suitable to particle tracking of short bunch and high frequency field problem, like CSR.

5 REFERENCES

[1] R. Y. TSIEN, “Picture of Dynamic Electric Fields”, AJP Vol. 40, January 1972

[2] T. Shintake, “Simulation of field lines generated by a moving charge”, private note 1984 March 19 at KEK, not published.