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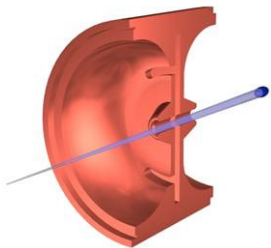


Low-Alpha Storage Ring Design for Steady-State Microbunching to Generate EUV radiation

On behalf of Tsinghua SSMB team

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Accelerator Laboratory of Tsinghua University



Content

- Introduction
- Linear lattice design
- Nonlinear study
- Summary





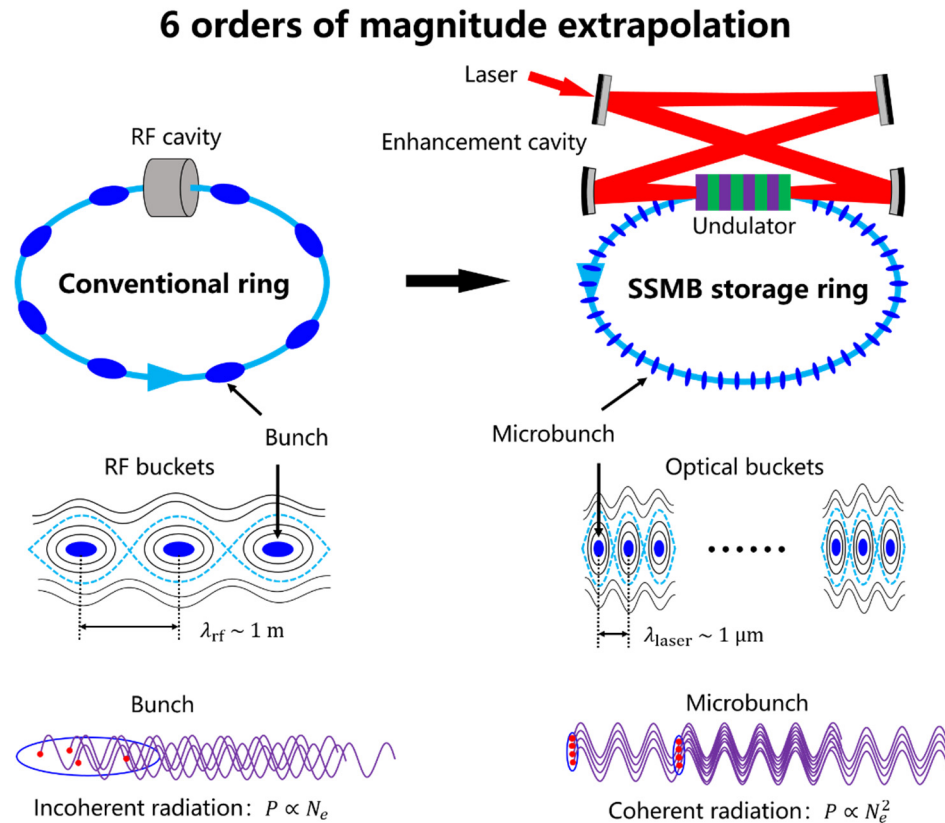
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Steady-state microbunching (SSMB)⁴*

- Electron storage ring-based, longitudinal dynamics study needed
- Bunching system laser modulator, instead of RF cavity
- Two key points
 - Microbunching for strong coherent radiation
 - turn-by-turn steady state for high repetition rate
- High average power, high repetition rate coherent radiation

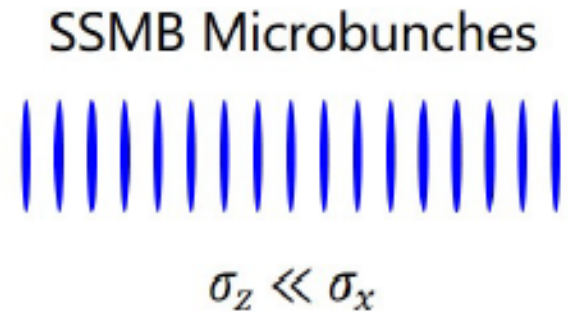
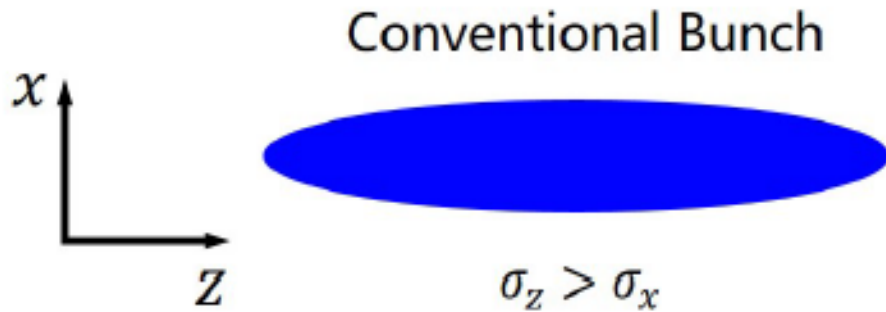
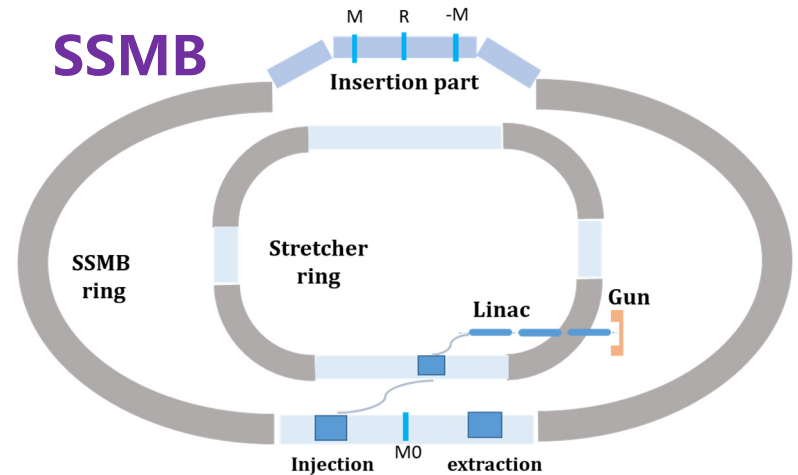


D. F. Ratner and A. W. Chao, Phys. Rev. Lett. 105, 154801 (2010).



Courtesy of Xiujie Deng





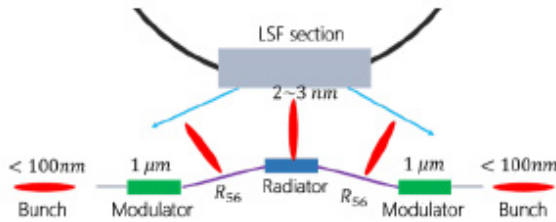
- ❑ DLSR: minimize transverse size to diffraction limitation
- ❑ SSMB: minimize longitudinal size for coherent radiation



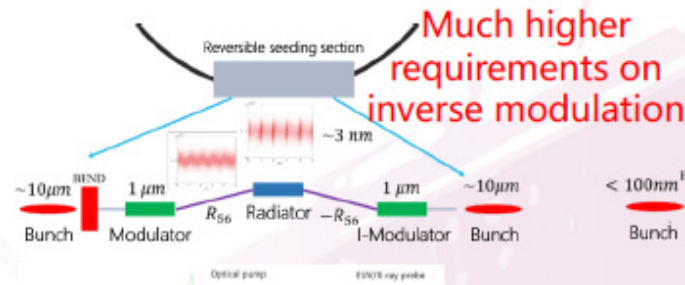


SSMB schemes

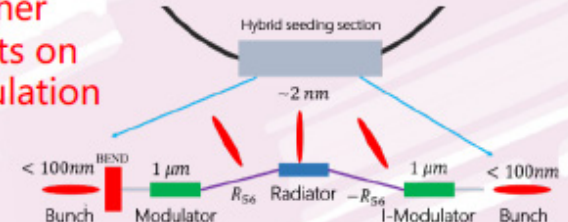
Longitudinal strong focusing



Reversible seeding



Hybrid



- | | | |
|--|---|---|
| <ul style="list-style-type: none"> ❑ Low-alpha ring (~100 nm bunch) + LSF(~3nm) ❑ Required laser power: hundreds MW, pulsed, Duty rate: 1% ❑ Pulse power : several kW, average power : several tens W | <ul style="list-style-type: none"> ❑ normal ring + ADM compress (~3nm) ❑ Required laser power: ~1 MW ❑ Low bunching factor, coasting beam (@10A) ❑ Average power : ~ kW | <ul style="list-style-type: none"> ❑ Low-alpha ring (~100 nm bunch) + ADM compress (~3nm) ❑ Required laser power: ~1 MW ❑ high bunching factor ❑ Average power : ~ kW (@1A) |
|--|---|---|





So, what should the SSMB ring looks like? Traditional low-alpha ring or MBA ring? We will find the answer is partially yes but not exactly.





Existing low-alpha mode ring

□ Diamond storage ring:

$$\alpha_1 = -10^{-5} \text{ or } -3 \times 10^{-6}$$

□ SLS storage ring:

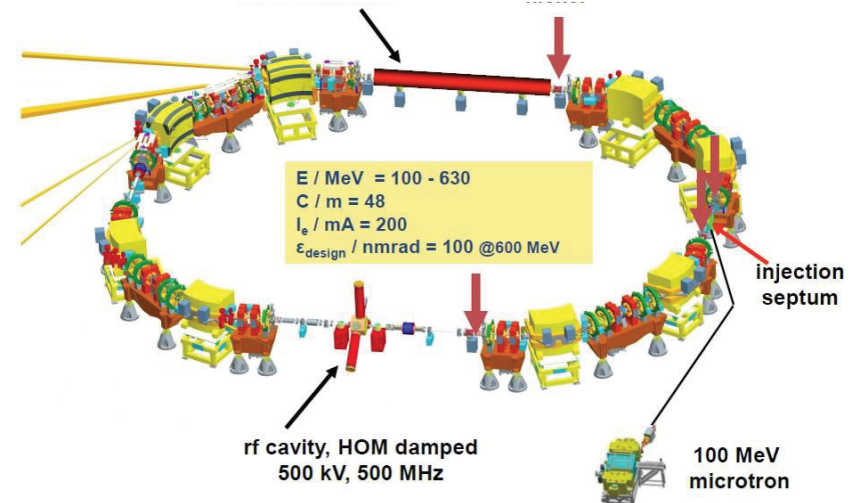
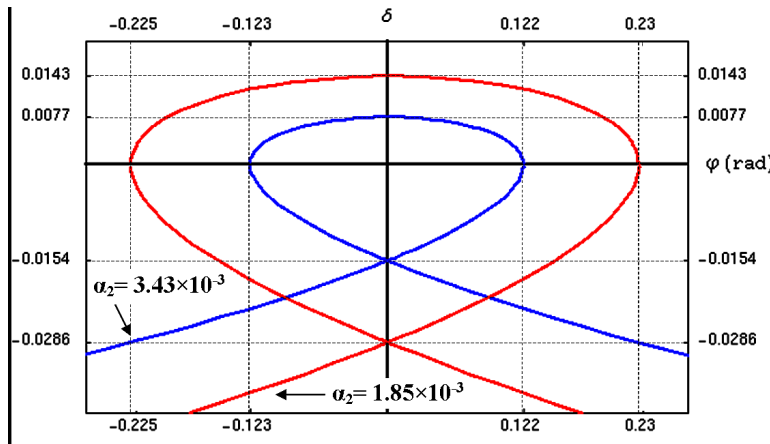
$$\alpha_1 = 3.6 \times 10^{-5}$$

□ MLS storage ring:

$$\alpha_1 = 1.3 \times 10^{-4}$$

Table 1: Main Parameters of the Two Low Alpha Lattices

Parameter	High ϵ	Low ϵ
Emittance	35.2 nm rad	4.4 nm rad
α_1	-3×10^{-5}	-1×10^{-5}
α_2 (no sext.)	0.0116	0.0050
α_3	-0.0426	0.0040
Q_x / Q_y	21.150/12.397	29.390/8.284
Nat. chrom. (ξ_x / ξ_y)	-37 / -26	-66 / -43
$\beta_{xID} / \beta_{yID}$	8.2 m / 2.4 m	1.1 m / 5.7 m
Nat. bun. len (3MV)	1.3 ps	2.4 ps
Synch. freq. (3MV)	346 Hz	629 Hz



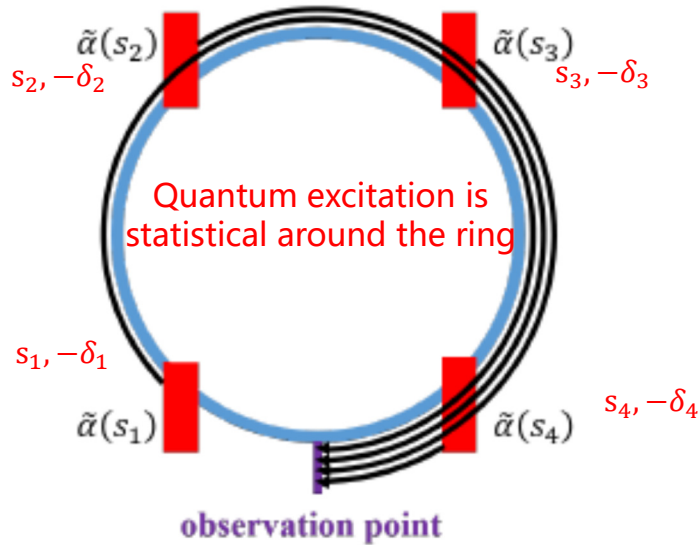
The bunch length in existing low-alpha mode ring: ~1 ps



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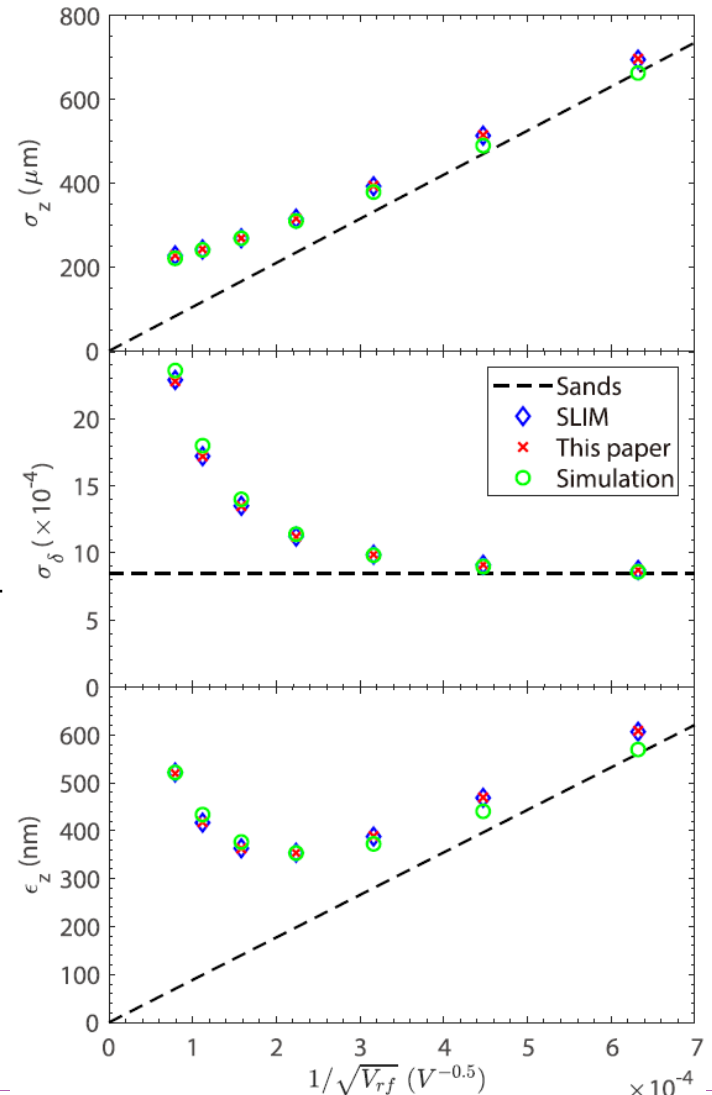




$$\tilde{\alpha}(s_j) = \frac{1}{C_0} \int_{s_l}^{\text{observation point}} \frac{\eta(s)}{\rho(s)} ds \quad \sigma_\tau = \sigma_\delta \sqrt{(\alpha_c/\omega_s)^2 + T_0^2 I_{\tilde{\alpha}}}$$

$I_{\tilde{\alpha}}$ the variance of $\tilde{\alpha}(s_j)$

- The scaling law $\sigma_z \propto \sqrt{|\eta|}$ breakdown when $\eta \rightarrow 0$
- The key is to control $I_{\tilde{\alpha}}$ in low alpha ring





$$\square I_{\bar{\alpha}} = \left\langle \left(\widetilde{\alpha}_{s_j} - \langle \widetilde{\alpha}_{s_j} \rangle \right)^2 \right\rangle = \langle \widetilde{\alpha}_{s_j}^2 \rangle - \langle 2\widetilde{\alpha}_{s_j} \rangle \langle \widetilde{\alpha}_{s_j} \rangle + \langle \widetilde{\alpha}_{s_j} \rangle^2 = \langle \widetilde{\alpha}_{s_j}^2 \rangle - \langle \widetilde{\alpha}_{s_j} \rangle^2$$

$$\square \tilde{\alpha}(s_j) = \frac{1}{c_0} \int_{s_j}^{OP} \frac{D_x(s)}{\rho(s)} ds$$

$\square$ **Control the max of dispersion**

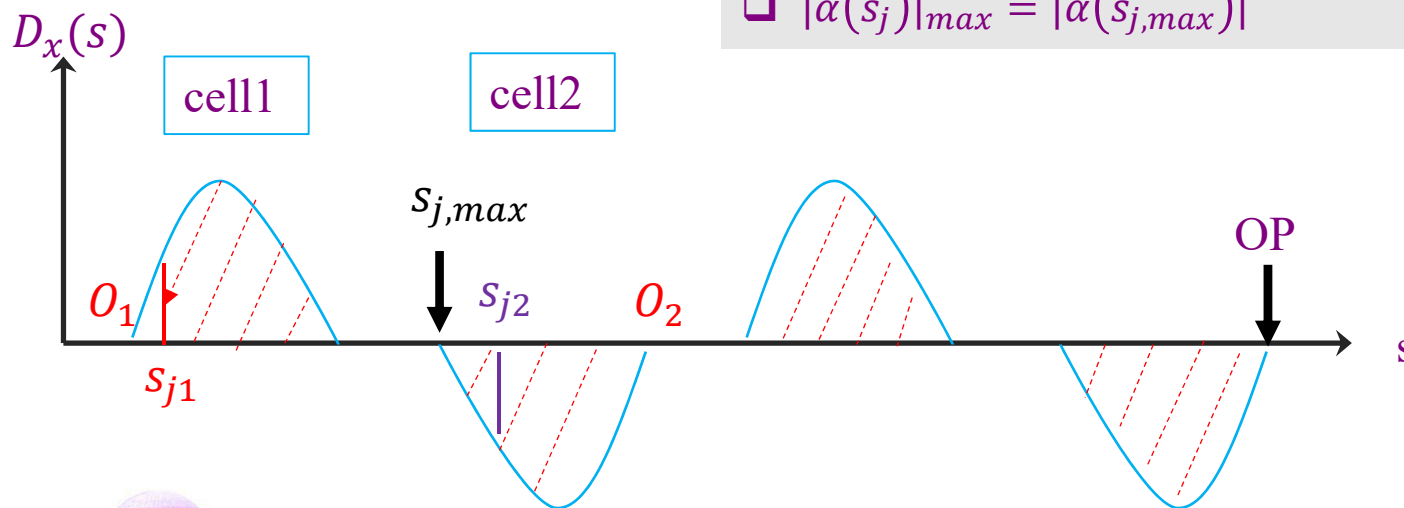
$\square$ **The nature idea: make $\alpha_c \sim 0$ in each dipole**

$$\square R_{56,c} = R_{56,1} + R_{56,2} \sim 0$$

$\square$ So $\tilde{\alpha}(s_j) < 0$ for all radiation points

$$\square |\tilde{\alpha}(s_j)|_{min} = 0$$

$$\square |\tilde{\alpha}(s_j)|_{max} = |\tilde{\alpha}(s_{j,max})|$$





□ In a low alpha cell, $\alpha_c = \frac{1}{c_0} \int_0^L \frac{\eta_x(s)}{\rho(s)} ds \sim 0$

$$\eta(\theta) = \eta_0 \cos\theta + \eta_0' \rho \sin\theta + \rho(1 - \cos\theta)$$

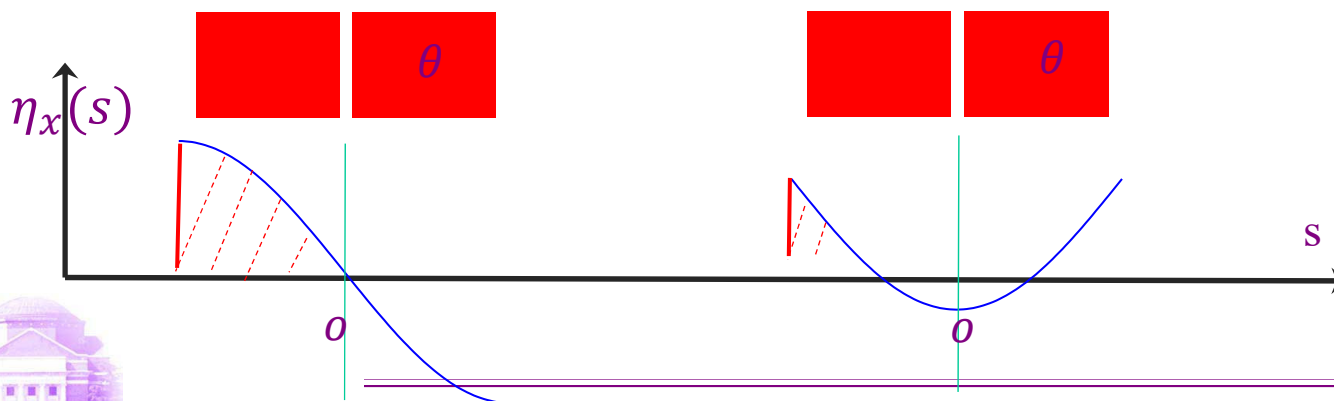
□ Define the dispersion functions at point o as (η_0, η_0')

□ θ is small, the increment of dispersion in the dipole is

$$\Delta\eta = |\eta(\theta) - \eta_0| = |\eta_0' \rho \sin\theta + \rho(1 - \cos\theta)|$$

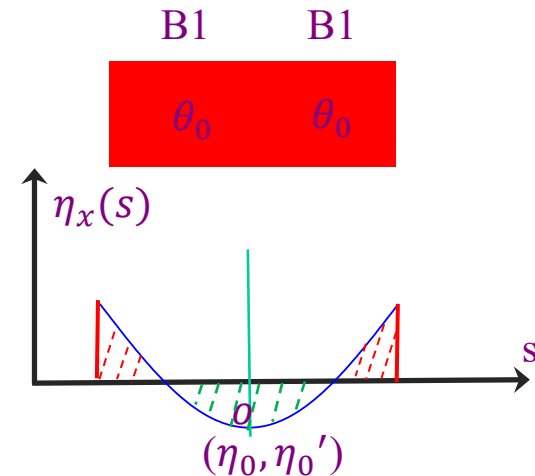
□ bigger the $\Delta\eta$ is, bigger the partial alpha

□ The best condition is $\eta_0' = 0$





- Make the α_c in each dipole be 0
- At the middle of the dipole $\eta_0' = 0$
- $\eta(\theta) = \eta_0 \cos\theta + \eta_0' \rho \sin\theta + \rho(1 - \cos\theta)$
- $\int_0^{\theta_0} \eta(\theta) d\theta = 0$



□ We get $\eta_0' = \frac{\sin(\theta_0) - \theta_0 - \frac{\eta_0}{\rho} \sin(\theta_0)}{1 - \cos(\theta_0)}$

□ And $\eta_0' = 0$, so $\eta_0 = \rho \frac{(\sin(\theta_0) - \theta_0)}{\sin(\theta_0)} \approx -\frac{1}{6} \rho \theta_0^2$, and $\eta(\theta) = \frac{1}{3} \rho \theta_0^2$

$$\tilde{\alpha}(\theta) = \frac{1}{C_0} \rho \left(\theta - \frac{\theta_0}{\sin \theta_0} \sin \theta \right) \quad I_{\tilde{\alpha}(s_j)} = \langle \tilde{\alpha}(\theta)^2 \rangle - \langle \tilde{\alpha}(\theta) \rangle^2 \approx \frac{\sqrt{2415}}{2520 C_0} \rho \theta_0^3$$

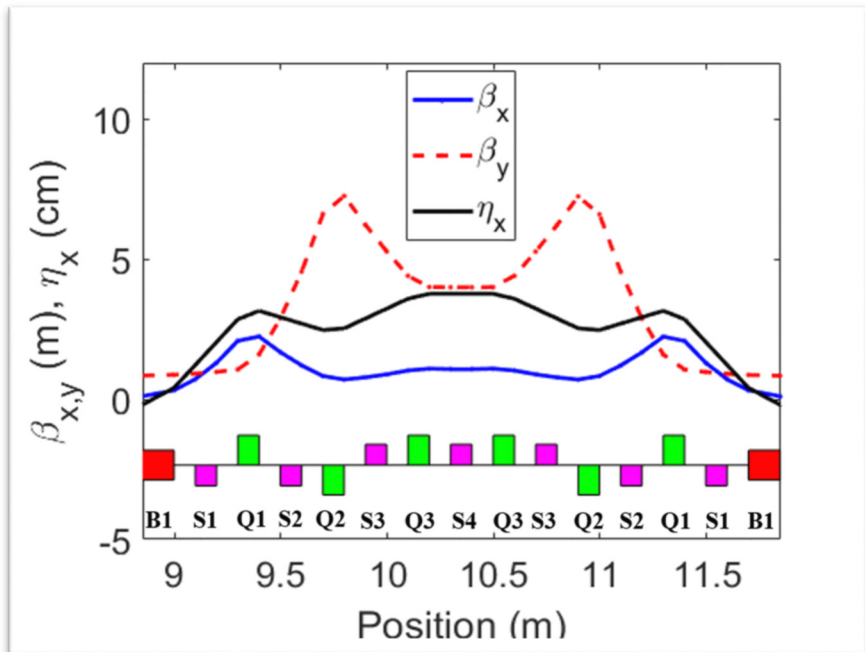
If make $\sigma_{s, \text{partial alpha}} = C_0 \sigma_\delta \sqrt{I_{\tilde{\alpha}(s_j)}} \sim 10 \text{ nm}$ $\rho = 2.0 \text{ m}, \quad \sigma_\delta = 2.4 \times 10^{-4}$

Then $\theta_0 < 5.86 \text{ deg}$





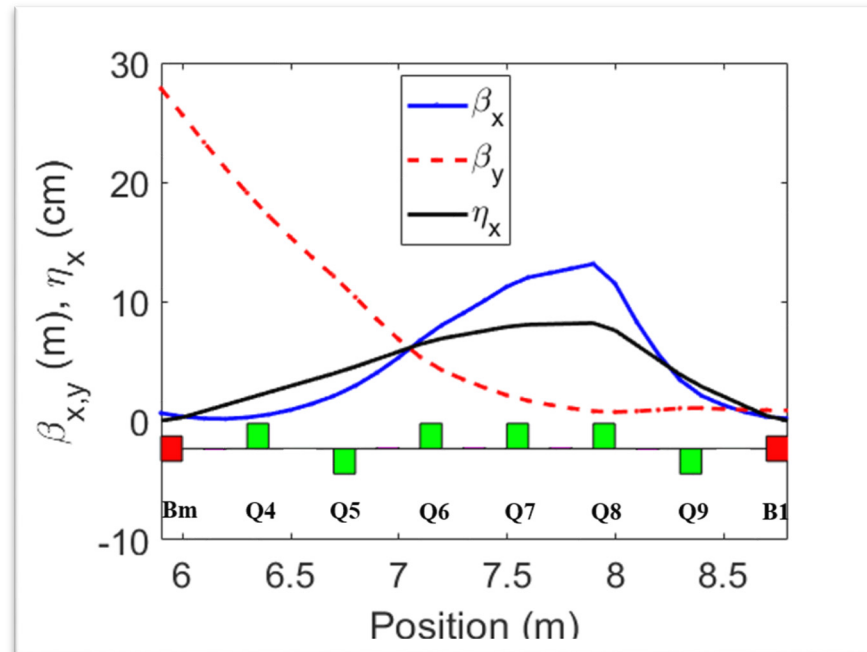
Adjust quadrupole strengths to control ν_x, ν_y, η_0



Main Cell

Tune: 0.75, 0.25

$$\eta_0 = -\frac{1}{6}\rho\theta^2$$



Match Cell

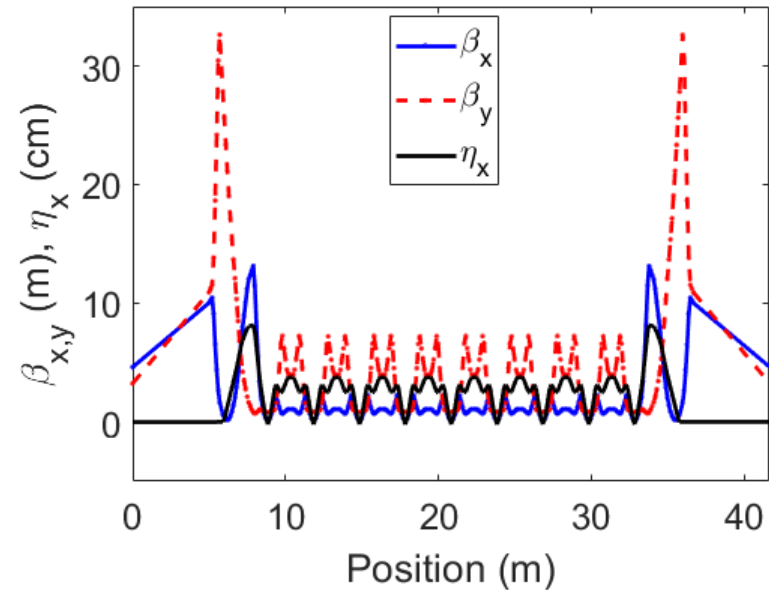
Match the dispersion
and alpha for straight





$$E_0 = 400 \text{ MeV}$$

Parameters	Value
Circumference[m]	166.78
Tunes(x/y)	30.677/11.391
Chromaticity(x/y)	0.006/-0.0008
alphac	2.11e-6
alphac2	6.39e-5
alphac3	7.13e-3
partial alpha	3.07e-7
Sdelta0 σ_δ	2.52e-4
U0 [keV]	1.23
Natural emittance [pm]	54.4
Hor./Ver damping time [ms]	361.4/362.2
Long. Damping time [ms]	181.3



Twiss for super-period

$$\tilde{\alpha}(s_j) = \frac{1}{C_0} \int_{s_j}^{obser} \frac{D_x(s)}{\rho(s)} ds - \frac{1}{\gamma^2} \frac{C_0 - s_j}{C_0}$$

$$\sigma_{s,partial\ alpha} = C_0 \sigma_\delta \sqrt{I_{\tilde{\alpha}(s_j)}} = 12.83 \text{ nm}$$





- ❑ 500 macro particles
- ❑ 1.5 million turns (~ 4.5 damping time)
- ❑ All particles survived
- ❑ Rms bunch length: 40 nm (simulation)

$\lambda = 1\mu m$	
Voltage 250 kV	Half bucket height δ_{rf} 2.22×10^{-3}
Synchrotron tune ν_s 0.089	Bunch length $\sigma_{s,sands}$ 36.07 nm

$$\sigma_s = \sqrt{\sigma_{s,sands}^2 + \sigma_{s,partial\ alpha}^2} = \sqrt{12.83^2 + 36.07^2} = 38.29\ nm,$$

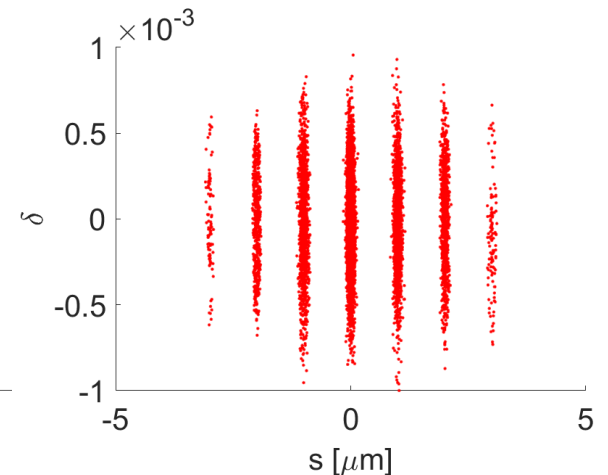
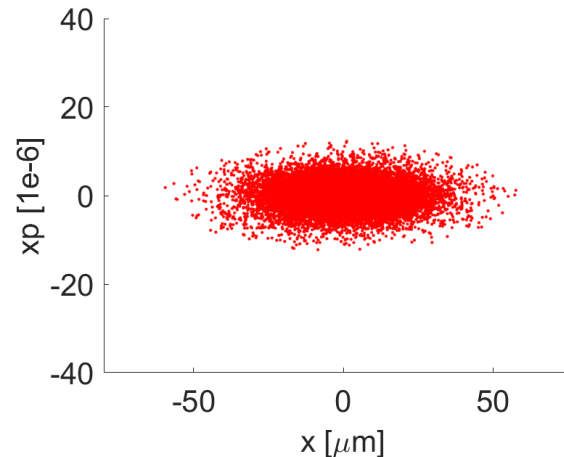
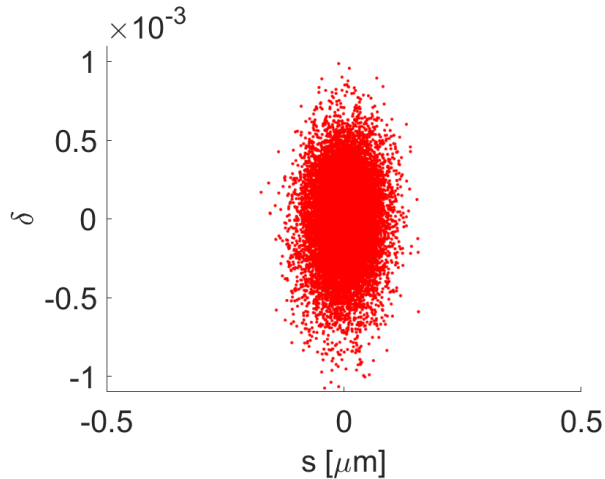
$$V_0[V] = \sqrt{2}E_0R_zJk_u\omega_0 \int_0^{\frac{L_u}{2R_z}} \frac{A}{\sqrt{1+z^2}} e^{-\frac{A^2}{1+z^2}} dz$$

$$P[W] = \frac{\pi\omega_0^2}{4}E_0^2\sqrt{\frac{\epsilon_0}{\mu_0}},$$

$$L_u = 2.5\ m, \quad \omega_0 = 450\ \mu m$$

$$\lambda = 1\ \mu m, \quad N=500, \quad K=6.86$$

$$V_0: 250\ kV \rightarrow P: 0.56\ MW$$





The difference between SSMB ring and MBA ring

- ❑ SSMB rings focus on longitudinal dynamics, minimizing longitudinal emittance will naturally require the MBA structure.
- ❑ A MBA structure will not naturally be the SSMB ring, there is precise dispersion control in dipole for SSMB rings.
- ❑ SSMB rings do not require very low transverse emittance, instead prefer relatively larger transverse emittance considering IBS effects





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□ Second order alpha

- Destroy RF bucket if too large

$$\alpha = \alpha_c + \alpha_2 \delta + \dots$$

$$\alpha_{c2} = \frac{1}{C} \int_0^C \left(\frac{\eta_2}{\rho} + \frac{\eta_1'^2}{2} \right) ds$$

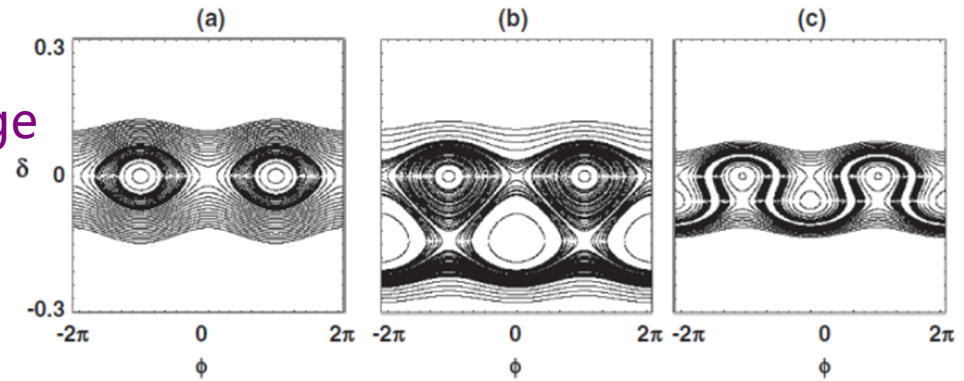


Fig. 8. Effect of second-order momentum compaction factor on longitudinal phase space in ring. The first-order momentum compaction factor in (a), (b), and (c) is 2.2×10^{-3} . The second-order momentum compaction factors in (a), (b), and (c) are 0.006, 0.02311, and 0.06, respectively.

when $|\alpha_2| > \alpha_{2Cr}$, the RF bucket will transmit to α -bucket, bucket area will shrink

$$\alpha_{2Cr} = \sqrt{\frac{E_0 h |\alpha_1|^3}{12 e V_{rf} [-\cos \varphi_s + (\frac{\pi}{2} - \varphi_s) \sin(\varphi_s)]}}$$

$$\eta_2(s) = -\eta_1(s) + \frac{1}{2 \sin \pi Q_x} * \int_s^{s+C} \sqrt{\beta_x(s) \beta_x(\sigma)} \cos(\pi Q_x - \mu_{\sigma s}) * \left(K_1(\sigma) \eta_1(\sigma) - \frac{1}{2} K_2(\sigma) \eta_1^2(\sigma) \right) d\sigma$$

The sextupoles should be located at dispersive location to correct chromaticities for three directions



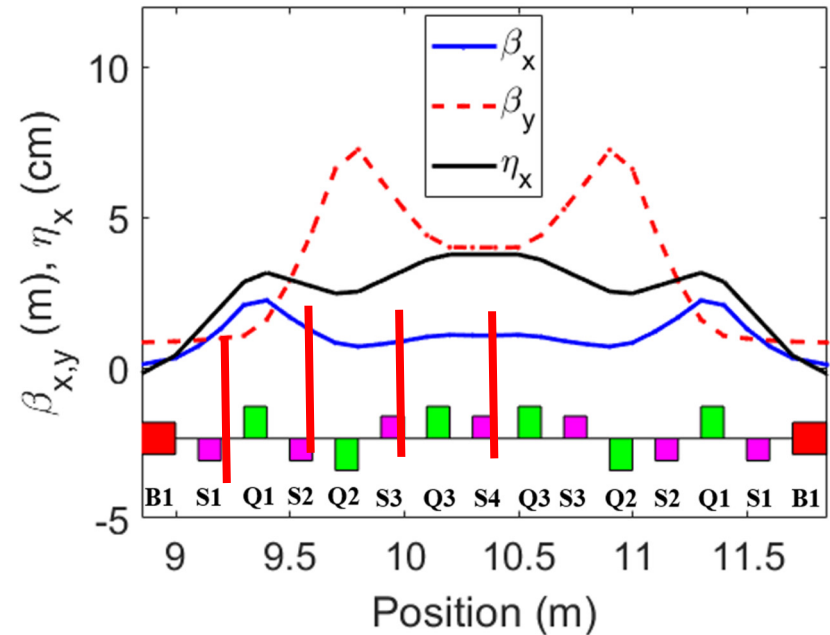
DA optimization

High order achromat scheme; add some geometrical sextupoles when needed

First order driving terms: Second order driving terms:

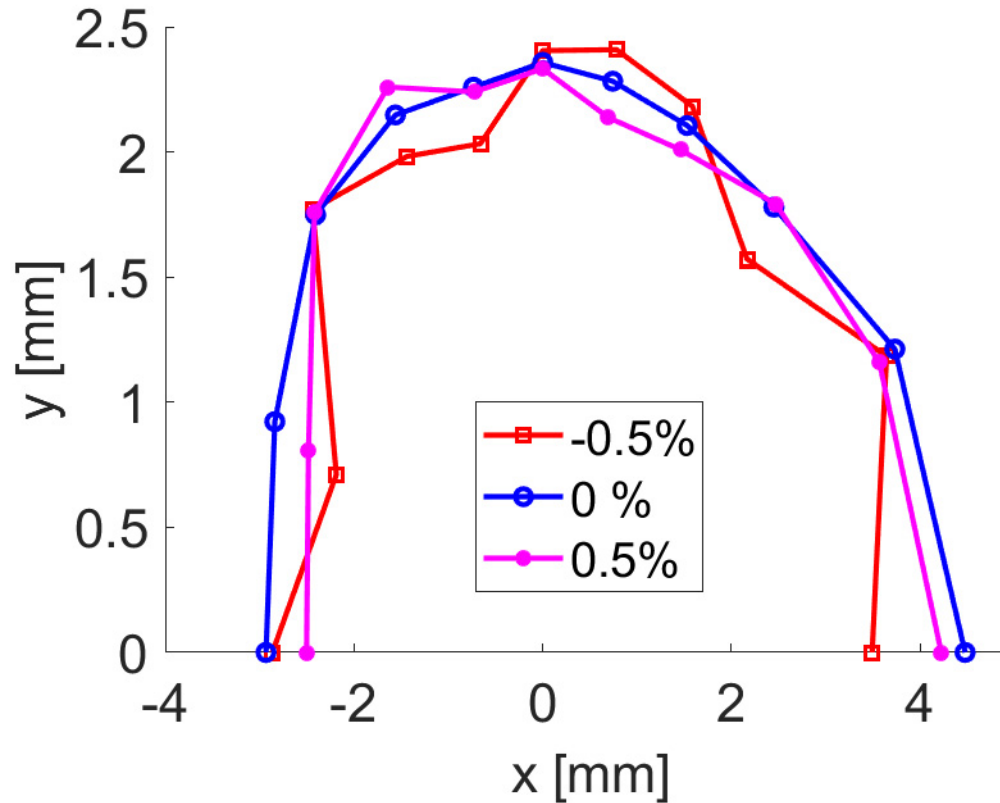
h10110	2.21e-2
h21000	6.32e-3
h30000	7.28e-3
h10020	1.08e-2
h10200	3.19e-2
h20001	6.53
h00201	5.80
h10002	3.25e-2

h22000	7.76e4
h00220	2.87e5
h11110	1.10e5
h40000	8.16e3
h00400	1.85e5
h31000	1.97e1
h20110	1.04e2
h11200	5.23e2
h00310	2.26e3
h20020	4.35e4
h20200	1.52e4





DA optimization



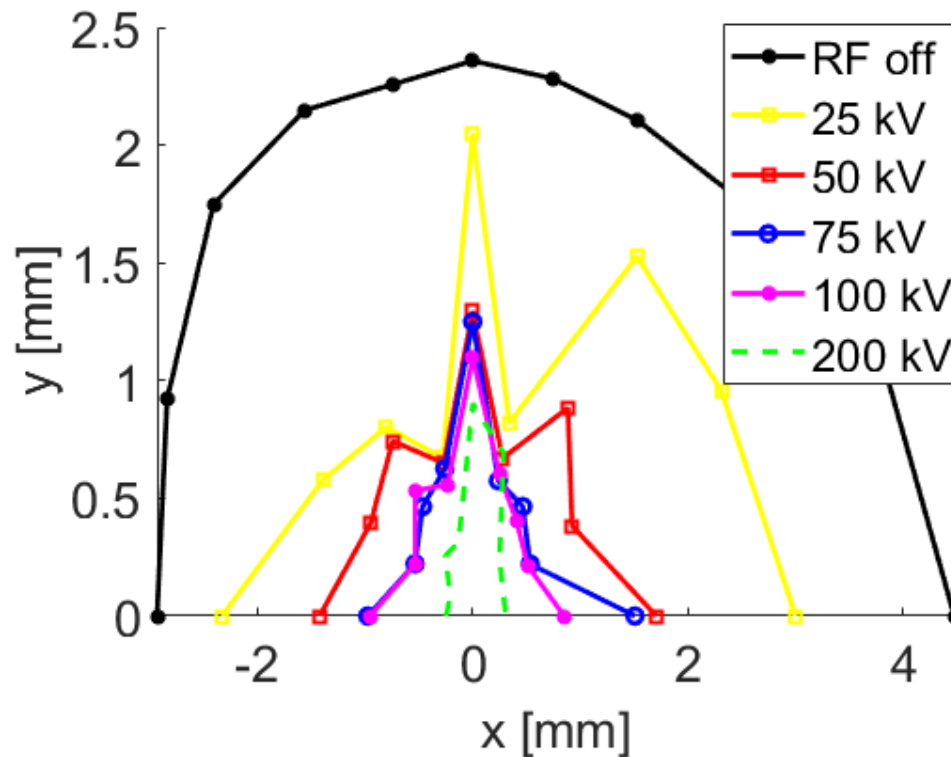
4-D DA, $x: \sim 3-4$ mm, $y: \sim 2.5$ mm





DA optimization-6D issue

RF will be replaced by laser modulator in SSMB ring, the wavelength is $1\ \mu m$, the path length oscillation amplitude by transverse longitudinal coupling can be larger than $1\ \mu m$ easily. So the particles with large transverse emittance will transmit to different bucket turn by turn, which is not stable.





DA optimization-6D issue

- Typically, h_{20001} will be ~ 10 for high order achromat scheme.
- h_{22001} and h_{40001} will also be important in many cases.
- 1) Minimize some important terms by COSY & 2) using MOGA (diffusion rate)

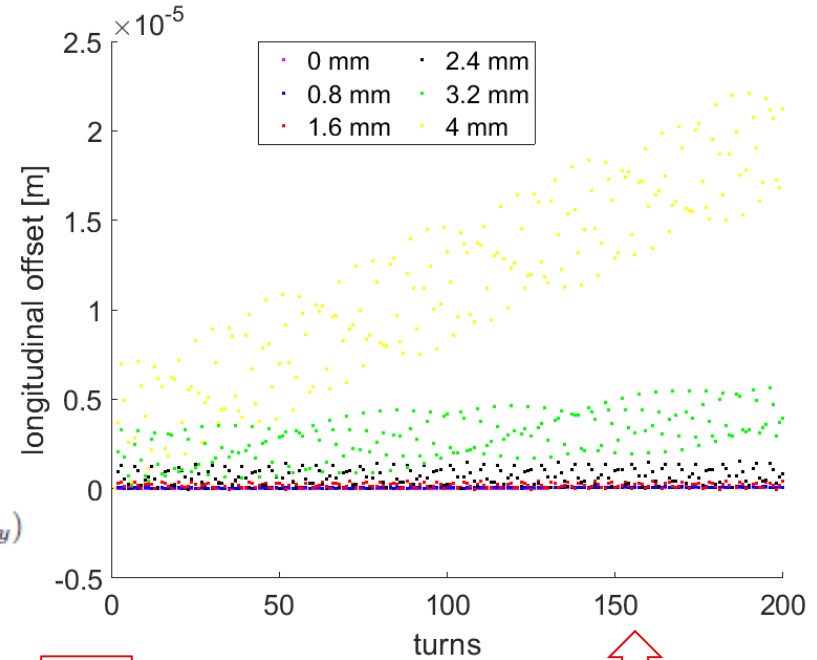
$$H^{(n)} = \sum_{a+b+c+d+e=n} h_{abcde} J_x^{+a} J_x^{-b} J_y^{+c} J_y^{-d} \delta^e \exp(i(a-b)\phi_x) \exp(i(c-d)\phi_y)$$

$n=3$

$$dz = \frac{\partial H^{(3)}}{\partial \delta} = 2 * h_{11001} * J_x + 2 * h_{00111} * J_y + 2 * h_{20001} * J_x * \exp(i2\phi_x) + 2 * h_{00201} * J_y * \exp(i2\phi_y).$$

$n=5$

$$\begin{aligned} dz = \frac{\partial H^{(5)}}{\partial \delta} = & 4 * h_{22001} * J_x^2 + 4 * h_{00221} * J_y^2 + 4 * h_{40001} * J_x^2 * \exp(i4\phi_x) + 4 * h_{00401} * J_y^2 * \exp(i4\phi_y) \\ & + 4h_{11111} * J_x * J_y + 4 * h_{31001} J_x^2 * \exp(i2\phi_x) + 4 * h_{13001} * J_x^2 * \exp(-i2\phi_x) \\ & + 4 * h_{00311} J_y^2 * \exp(i2\phi_y) + 4 * h_{00131} * J_y^2 * \exp(-i2\phi_y) \\ & + 4 * h_{20201} * J_x * J_y * \exp(-i2(\phi_y + \phi_x)) + 4 * h_{30101} * J_x^{3/2} J_y^{1/2} * \exp(i(3\phi_x + \phi_y)) \\ & + 4 * h_{10301} * J_y^{3/2} J_x^{1/2} * \exp(i(3\phi_y + \phi_x)) + c.c.. \end{aligned}$$



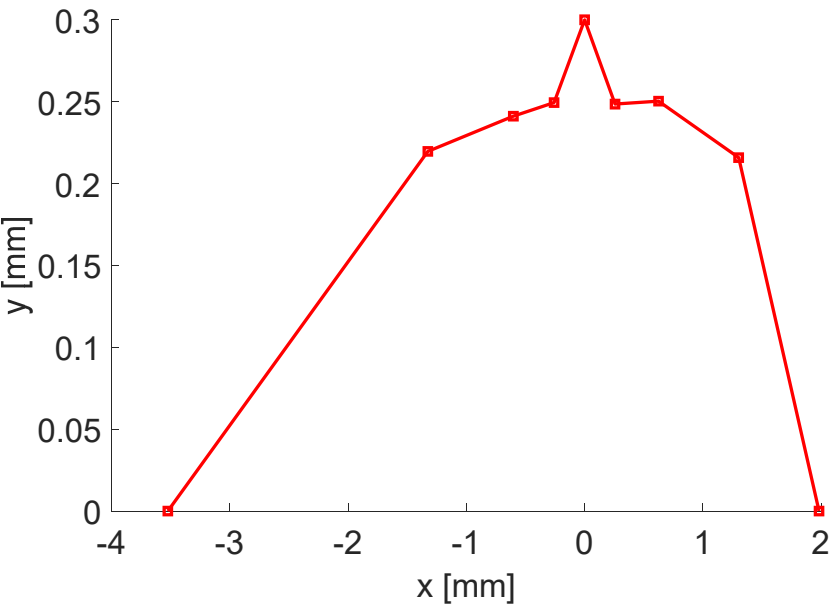
RF is off;
 $z_0 = 0, \delta_0 = 0$





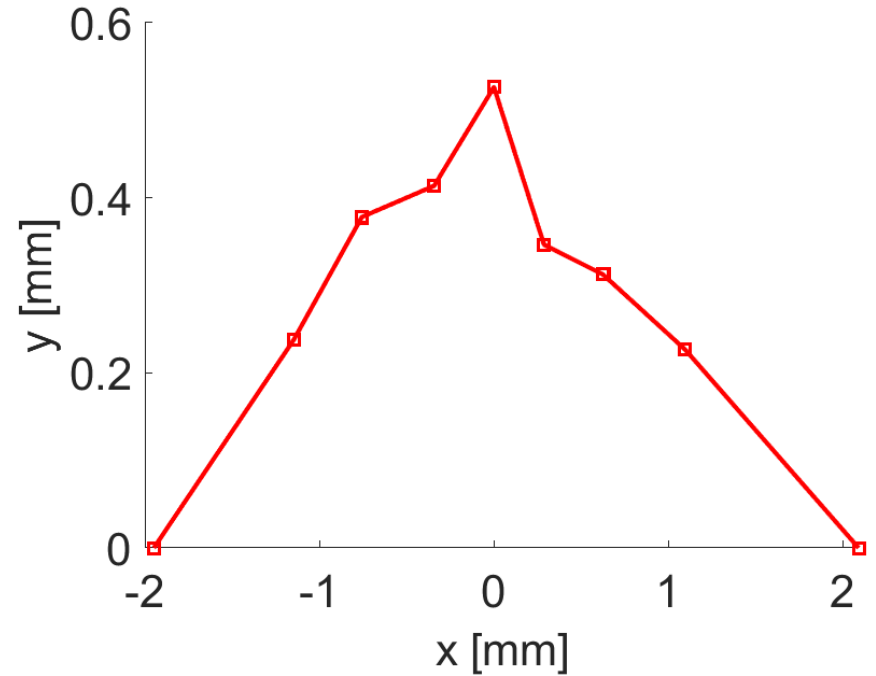
DA optimization-6D issue

COSY result



6-D DA

MOGA result



6-D DA





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Summary

- ❑ We have done the analysis for partial alpha effects in low alpha ring, and proposed a method to minimize it.
- ❑ Based on the analysis, the linear lattice is designed, bunch length under 100 nm can be maintained in the ring.
- ❑ We have done some preliminary studies on the nonlinear optimization for this kind of storage ring, the 6-D DA will be limited by T-L coupling, which need more careful studies for further optimization.





Thanks!

