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REVISITING INTRABEAM SCATTERING FOR LAMINAR BEAMS

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Abstract

Intrabeam scattering (IBS) is becoming an increasingly important effect in the design of high-brightness linear electron accelerators due to the ever-increasing transverse brightness of beams produced from radiofrequency photoinjectors. The existing theory describing the energy spread growth rate due to IBS was derived in the context of circular machines where the beam particles are frequently and randomly colliding, and therefore should only be applied to non-laminar, emittance dominated flow. This is not the case in the injector portion of a linear accelerator, where the beam is space-charge dominated and the flow is laminar. The different nature of the microscopic motion in the two cases demands a reevaluation of the applicability of IBS theory to the photoinjector. In this work, we present a simple analytic model for energy spread growth during perfectly laminar flow and show that it matches well to point-to-point multiparticle simulations. In this way we demonstrate that stochastic energy spread growth in laminar beams is more attributable to the initial random placement of the particles in the bunch rather than the traditional temperature rearrangement mechanism of IBS.

INTRODUCTION

The development of ever-brighter beam sources for linear accelerators demands precise understanding of the downstream mechanisms by which that brightness can be degraded. Of particular importance for the next generation of x-ray free-electron lasers (XFELs) is the preservation of the beam energy spread, which sets a threshold requirement for lasing in an XFEL [1, 2]. The best known way by which the energy spread can grow is the microbunching instability (μBI), which leads sufficiently cold beams to undergo space-charge amplification of initial shot noise [3]. Traditionally, μBI is suppressed with the use of a laser heater, and the minimal accepted input energy spread to mitigate μBI gain is much higher than that produced by the injector itself [4]. However, as more precise methods for controlling μBI gain are developed, the initial energy spread produced by the injector may become important.

This input energy spread is limited by yet another effect which has historically not attracted much attention in the field of linear accelerators: intrabeam scattering (IBS) [5–7]. IBS is generally thought of as the impact of stochastic, binary collisions on the beam dimensions. Because the beams produced by high-brightness beam sources are generally much colder longitudinally than transversely, IBS is thought to lead to a redistribution of thermal spread from the transverse emittance into the uncorrelated energy spread [8]. Theoretical models of this effect exist, but suffer from ambiguities from the evaluation of the Coulomb logarithm. Recently, there has been substantial interest in IBS for x-ray free-electron lasers due to the ever-increasing brightness of the beams they employ [9, 10].

Besides the difficulties making quantitative estimates resulting from the Coulomb log, there exists yet another reason to distrust the existing IBS theory in the context of linear accelerators. The picture of stochastic effects as being represented by constant and randomly distributed collisions does not make much sense in the injector portion of a linear accelerator where the beam flow is space-charge dominated and therefore quasi-laminar. This disparity between the relevant stochastic effects in injectors, and the existing models of IBS was found to have a notable impact on simulation comparisons with the theory in [11]. Indeed in this regime we expect effects which correspond more closely to the Boersch effect in electron microscopes [12]. Although there exists a theoretical description of the Boersch effect, it is best suited to the relatively low densities found in those systems, and is furthermore quite complicated [13]. With the extremely high beam densities found in XFELs, some simplifications can be found. As more machines are designed with ultra-high brightness beams in mind, a proper understanding of IBS in the context of XFELs is necessary.

In the rest of this paper we outline a simple numerical approach to evaluating slice energy spread growth induced by finite particle effects based on a simple scaling argument. We begin by laying out a basic first principles approach before deriving that the statistical width of the longitudinal space-charge field for a given three-dimensional charge distribution will in general scale with $N_e^{2/3}$, with N_e being the number of electrons in the bunch. With that fact established, the cumulative impact of the space-charge field width can be evaluated using Monte Carlo simulations with significantly fewer particles. Finally, we perform numerical benchmarks demonstrating the validity of the model, where for this paper we restrict ourselves to perfectly laminar flow scenarios. We outline in the conclusions how the method might be expanded to address more general accelerator scenarios.

THEORETICAL MODEL

General Approach

The theoretical model we propose is based on a first principles consideration of the statistics of the longitudinal space-charge field. A particle in the beam experiences an instantaneous rate of energy change given by

$$\frac{d\gamma_{\mathrm{IBS}}}{ds} = \frac{q}{mc^2}\delta E_z \tag{1}$$

where $\delta E_z \equiv E_z - \langle E_z \rangle$, with E_z being the longitudinal space-charge field in the lab frame and $\langle . \rangle$ indicating an ensemble average over the beam distribution function. Specifically,

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the space-charge field is considered as a sum over discrete particles

$$E_z = \frac{q}{4\pi\epsilon_0\overline{\gamma}^2}\sum_{j=1}^{N_e}\frac{z-z_j}{\left[\frac{(x-x_j)^2+(y-y_j)^2}{\overline{\gamma}^2}+(z-z_j)^2\right]^{3/2}} \quad (2)$$

Explicit evaluation of this field during a simulation for a typical bunch charge found at XFELs is impractical, as N_e is 625 million for standard 100 pC bunches. As such, we would like to determine ways in which its evaluation can be simplified. This begins by considering the scaling of this quantity with the particle number N_e. By understanding this scaling, one can imagine computing the stochastic field for some lower particle number, then scaling the result up to the true number of electrons in the bunch.

Statistics of the Space-Charge Field

Equation 2 may be thought of as the sum of N_e identically, randomly distributed variables ξ_j defined as

$$\xi_j = \frac{z_j}{\left[\frac{x_j^2+y_j^2}{\overline{\gamma}^2}+z_j^2\right]^{3/2}} \quad (3)$$

where for now we restrict our attention to the space-charge field at the center of the bunch. We may start by asking how this variable is distributed, in particular by calculating

$$\rho_\xi = \int \rho(x,y,z)\,\delta\left(\xi - \frac{z}{\left[\frac{x^2+y^2}{\overline{\gamma}^2}+z^2\right]^{3/2}}\right)dxdydz \quad (4)$$

where ρ_ξ is the probability density function (PDF) of ξ and ρ is the PDF of the beam coordinates. For a bunch with a separable, azimuthally symmetric PDF $\rho(x,y,z) \equiv \rho_\parallel(z)\rho_\perp\left(\sqrt{x^2+y^2}\right)$, this can be simplified to the form

$$\rho_\xi = \frac{2\pi\overline{\gamma}^2}{|\xi|^{5/2}}\int_0^1 x^4\rho_\parallel\left(\frac{x^3}{|\xi|^{1/2}}\right)\rho_\perp\left(\frac{\overline{\gamma}x}{|\xi|^{1/2}}\sqrt{1-x^2}\right)dx \quad (5)$$

This integral cannot in general be evaluated analytically, however we can make some statements about the corresponding sum $\psi = \sum_{j=1}^{N_e}\xi_j$. In particular, the generalized central limit theorem tells us that, if the PDF ρ_ξ has power-law tails, i.e. if $\rho_\xi(|\xi| \gg 1) \simeq C/|\xi|^{1+\alpha}$, then the PDF of the sum converges to a stable distribution for $N_e \gg 1$ [14]. In particular, it will be a stable distribution with the PDF

$$\rho_\psi = \int e^{ik\psi-|ck|^\alpha}dk \quad (6)$$

where $c = (CN_e)^{1/\alpha}$ is a scale factor determining the width of the distribution. In our case, when $|\xi| \gg 1$, it is easily seen that the PDF ρ_ξ converges to $2\pi\overline{\gamma}^2\rho_\parallel(0)\rho_\perp(0)/5|\xi|^{5/2}$. Thus we identify $\alpha = 3/2$ and $C = 2\pi\overline{\gamma}^2\rho_\parallel(0)\rho_\perp(0)/5$. The scale factor c is then just $c = \left(\frac{2\pi}{5}\overline{\gamma}^2 n_0\right)^{2/3}$ where $n_0 = N_e\rho_\parallel(0)\rho_\perp(0)$ is the number density at the center of the beam. Of course, this analysis has been performed only for the center of the bunch. To confirm the scaling remains robust when considering the field at any particle within the bunch, we have computed the 90% width of the sum in equation 2 for a bunch with aspect ratio $A = \sigma_r/\gamma\sigma_z = 0.1$, typical for photoinjector conditions. The result of that study is shown in Figure 1, alongside a $N_e^{2/3}$ power law fit.

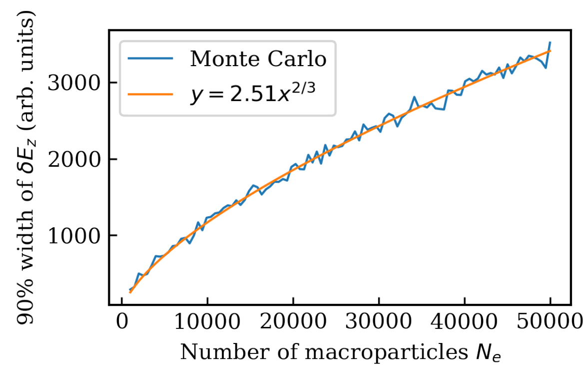

Figure 1: Scaling of the longitudinal space-charge field applied to each particle of the bunch. The width is computed at the RMS of a gaussian fit to the central 90% of energy kicks in the bunch.

Thus we find that the sum in equation 2 should scale approximately with $N_e^{2/3}$. If we define $\delta\psi = (4\pi\epsilon_0\overline{\gamma}^2/q)N_e^{-2/3}\delta E_z$, then we may rewrite equation 1 as

$$\frac{d\gamma_{\text{IBS}}}{ds} = \frac{r_e N_e^{2/3}}{\overline{\gamma}^2}\delta\psi \quad (7)$$

where now the width of the field-like term $\delta\psi$ should be independent of the number of particles used to calculate it. This fact lends itself nicely to macroparticle simulations, as we can now evaluate the width of $\delta\psi$ for much fewer particles $N_m \ll N_e$ and extrapolate the width to the true charge by the multiplicative factor $N_e^{2/3}$.

Restriction to Laminar Flow

For perfectly laminar flow, we may make the approximation that the particle coordinates scale cleanly with their respective beam dimensions. In other words, $X = x(s)/\sigma_x(s)$ can be considered constant. Thus we may very easily implement equation 1 with a Monte Carlo algorithm. To do so we sample $N_m \ll N_e$ particles from the beam distribution and evaluate the accumulation of equation 1 through predetermined envelopes. Examples of the results of this approach are shown in the next section.

NUMERICAL BENCHMARKS

In order to facilitate physically relevant numerical benchmarks in reasonable computation times, we have performed simulations of 10 fC charge beams with charge densities on the same order as those achieved in high brightness injectors.

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All simulations are performed using the General Particle Tracer (GPT) code [15]. To do this we take a standard injector configuration, for example 100 pC charge with 20 A current and 100 μm waist size, and scale the charge by a factor s, the transverse size and emittance by $s^{1/3}$, and the bunch length by $s^{1/3}$. Since the charge density scales like $Q/\sigma_z\sigma_\perp^2$, the density is preserved. Since our current focus is on laminar flow in injector-like scenarios, all of our benchmarks are performed using a beam which drifts in free space through a waist in a distance of 1 m. We show two examples here, with unscaled configuration specified as in Table 1. For simplicity, all bunch dimensions are taken to be gaussian.

Table 1: Unscaled beam parameters for numerical benchmarks

Parameter	Variable	Unit	Value
Charge	Q	pC	100
Energy	γmc^2	MeV	6.13
Peak current	I	A	20
Waist size	σ_x	μm	100
Norm. emittance	ϵ_n	nm rad	50 (250)

The first benchmark is shown in Figure 2 for the case of 50 nm rad unscaled normalized emittance. Here we plot the spot size evolution in the top plot and the corresponding slice energy spread evolution in the bottom, with results taken from the GPT code and from our Monte Carlo implementation of the scaling arguments. We observe an expected increase in the rate of energy spread growth in the vicinity of the waist where the beam density is very high. Furthermore, we see that our model does an excellent job predicting the energy spread growth before and during the waist, but starts to overestimate the effects after the waist. This is to be expected from the requirement of laminarity that we have imposed, as space-charge dominated waists are not perfectly laminar.

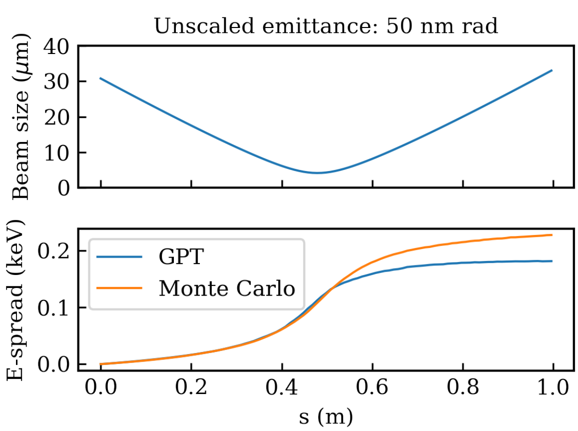

Figure 2: Beam size evolution (top) and slice energy spread growth (bottom) through a 1 m drift with 50 nm rad unscaled emittance.

Similar results hold even when the beam is not as strongly space-charge dominated, as we show in Figure 3. In this case we have taken the unscaled emittance to be five times larger, resulting in a much sharper waist. We observe a similar result, that the Monte Carlo model accurately predicts the energy spread growth up to the point of the waist, beyond which the laminarity assumption is weakened.

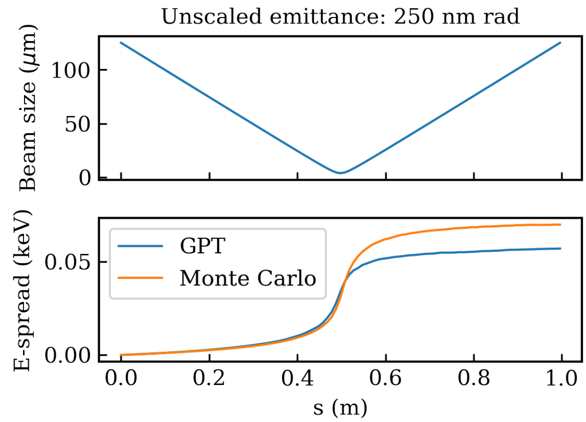

Figure 3: Beam size evolution (top) and slice energy spread growth (bottom) through a 1 m drift with 250 nm rad unscaled emittance.

CONCLUSIONS AND EXTENSIONS

We have presented a simple model for computing stochastic energy spread growth based on the fact that the space-charge field width scales as $N_e^{2/3}$ for arbitrary three-dimensional bunch distributions. We have shown through numerical benchmarks that even for a restricted Monte Carlo implementation considering only perfectly laminar flow, this simple model captures the growth of slice energy spread with a high level of accuracy. Although our implementation as presented here is restricted to laminar flow, the derivation of the field width scaling with particle number did not rely on that assumption, and thus one can imagine extending this approach to more general accelerator scenarios. Such an extension will be the subject of a future publication. This work fills an important gap in our understanding of energy spread degradation mechanisms in accelerators, agreeing much better with numerical simulations than traditional IBS theory, and without the ambiguity of the Coulomb logarithm.

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