

SIMULATING ENHANCED FOCUSING EFFECTS OF ION MOTION IN ADIABATIC PLASMAS

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Abstract

The FACET-II facility offers the unique opportunity to study low emittance, GeV beams and their interactions with high density plasmas in plasma wakefield acceleration (PWFA) scenarios. One of the experiments relevant to PWFA research at FACET-II is the ion collapse experiment E-314, which aims to study how ion motion in a PWFA can produce dual-focused equilibrium. As nonlinear focusing effects due to nonuniform ion distributions have not been extensively studied; we explore the difficulties of inducing ion motion in an adiabatic plasma and examines the effect an ion column has on beam focusing. A case study is performed on a system containing a plasma lens and adiabatic PWFA. Ions in the lens section are assumed to be static, while simulations of an adiabatic matching section are modified to include the effects of ion column collapse and their nonlinear focusing fields. Using the parameters of the FACET-II beam, we find that a collapsed ion column amplifies the focusing power of a plasma without compromising emittance preservation. This led to a spot size 33% smaller than that of a simply matched beam.

INTRODUCTION

This paper investigates the potential for ion motion at the upcoming E-314 experiment at the Facility for Advanced Experimental Tests II (FACET-II). FACET-II has the capability to produce ultra-high brightness, 10 GeV beams using the linear accelerator at SLAC National Laboratory. The E-314 experiment seeks to induce ion motion in a plasma wakefield accelerator (PWFA) and examine the effects high density ion columns have on a drive beam. Ion motion arises when a long, dense beam passes by plasma ions and exerts inwardly directed Coulombic forces on them. If the beam is orders of magnitude denser than the plasma, the uniformly distributed ion column will collapse inwards, resulting in an extremely dense, nonuniform ion distribution [1]. Focusing fields from a collapsed ion column are significantly stronger than those from a uniform ion distribution; we exploit these strong fields to enhance the focusing effects of a PWFA.

There are two difficulties associated with ion motion: for ion motion to occur in a similar timescale as electron evolution, the phase advance given by Eq. (1) must

$$\Delta\phi = 2\pi\sigma_z\sqrt{\frac{r_e Z_i n_{b,0} m_e}{m_i}} \quad (1)$$

satisfy $\Delta\phi \geq 1$ for gentle ion motion to occur, and $\Delta\phi \geq \frac{\pi}{2}$ to ensure maximum collapse of the ion column [2]. The phase advance depends on the beam parameters σ_z and $n_{b,0}$, which are the longitudinal rms and core (peak) density of the beam respectively. Other terms such as atomic number (Z_i), ion mass (m_i), electron mass (m_e), and classical electron radius (r_e) are either dependent on the plasma or are physical constants. Achieving a $\Delta\phi \geq \frac{\pi}{2}$ generally requires the density of the beam core to be many orders of magnitude larger than the density of the surrounding plasma [1], and is the primary issue associated with the FACET-II beam. The second issue is that moving ions have the potential to induce large emittance growth and interfere with beam-plasma matching by scattering beam electrons during ion column collapse. Mathematically describing the beam and plasma parameters when $\Delta\phi > \frac{\pi}{2}$ is quite complicated and is explored in [3], however this paper will not address the theory of ion motion and will instead focus on ion motion in an experimentally achievable setting.

We propose using the FACET-II beam to induce ion motion in a long, high density plasma system. The main challenge is achieving a high enough $n_{b,0}$ to induce ion motion, as the unmodified FACET-II beam falls short by orders of magnitude. To remedy this, a focusing system comprising of quadrupoles, plasma lenses, and an adiabatic matching section was proposed to decrease beam spot size while incurring as little emittance growth as possible. An experimentally achievable combination of lenses and adiabatic matching sections are examined using the quasi-static particle-in-cell code QuickPIC to model beam and ion evolution. QuickPIC is our primary tool for determining if the focusing system produces the desired results.

DEVELOPMENT OF THE PLASMA LENS

The aim of the pre-focusing lens is to uniformly reduce the rms width of the beam (σ_r) before it enters a high density adiabatic focuser. The theory laid out in [4] provides an excellent reference for the expected behavior of an electron beam after traversing a plasma lens. Most notable are the equations below,

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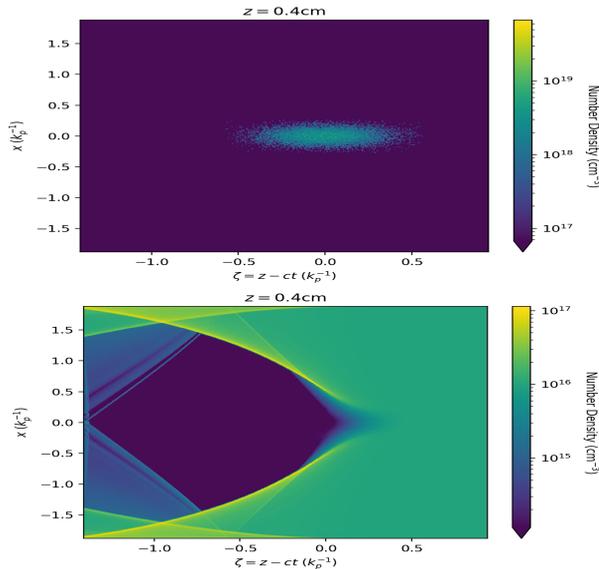


Figure 1: (Top) The reduced FACET-II beam ($k_p\sigma_r = 0.094$) while traversing a 0.5 cm plasma lens with $n_0 = 1 \times 10^{16} \text{ cm}^{-3}$. The beam produces a well defined blowout bubble (Bottom).

$$\tilde{z} = \frac{(\tilde{\beta}_0 - \tilde{\gamma}_0)\sin\tilde{L}\cos\tilde{L} + \alpha_0\cos(2\tilde{L})}{\tilde{\beta}_0\sin^2\tilde{L} + \tilde{\gamma}_0\cos^2\tilde{L} + \alpha_0\sin(2\tilde{L})} \quad (2)$$

$$\tilde{\beta} = \frac{1}{\tilde{\beta}_0\sin^2\tilde{L} + \tilde{\gamma}_0\cos^2\tilde{L} + \alpha_0\sin(2\tilde{L})} \quad (3)$$

which predict the waist location and beta function after the lens given initial beam parameters, where L is the length of the lens, α_0 , β_0 , and γ_0 represent the beam's initial Twiss parameters, and tildes denote parameters normalized by $\sqrt{K} = \sqrt{\frac{2\pi r_e n_0}{\gamma}}$ or $\frac{1}{\sqrt{K}}$ if appropriate. K depends on the classical electron radius r_e , the ambient plasma density n_0 , and the Lorentz factor of the beam γ . We assume that the beam enters the plasma lens with a Gaussian charge distribution in x , y , and z , and that it enters at a perfect waist, which occurs when $\alpha_0 = 0$ and $\gamma_0 = \frac{1}{\beta_0}$ and allows for easier manipulation of the beam with the lens. Equation (2) and Eq. (3) are valid if the beam is small compared to the plasma skin depth such that

$$k_p\sigma_r \ll 1 \quad (4)$$

and holds when the majority of the beam resides in the blowout bubble. If $k_p\sigma_r$ exceeds this condition, the beam experiences head erosion and undergoes non-uniform oscillations about its longitudinal axis. The generic FACET-II beam does not satisfy this condition for a plasma with density $n_0 = 1 \times 10^{16} \text{ cm}^{-3}$. Reducing the k_p of the plasma is not an option given the length constraints of the experiment, and instead we require σ_r to be reduced to $5 \mu\text{m}$ before it enters our lens. This could be achieved by installing a quadrupole focusing system ahead of the lens to reduce rms beam width without incurring emittance growth. While such

a quadrupole system will not be analyzed here, it is assumed that such a system exists and creates the initial beam parameters in the simulations. Using $\sigma_r = 5 \mu\text{m}$ produced results that were consistent with Eq. (2) and Eq. (3), and increased the normalized beam density \tilde{Q} [5, 6] to 84, which produced a strong blowout and enforces Eq. (4). Simulations showed the plasma lens successfully focused the beam with minimal emittance growth and head erosion. The results are given in Table 1 and a visualization of beam is shown in Fig. 1.

Table 1: Beam Parameters at Lens and at Post-Lens Waist

Parameters	At Lens	At Waist
Charge	2 nC	2 nC
Energy	10 GeV	9.997 GeV
Energy Spread	1.4%	11.3%
ϵ_r	5 μm rad	6.67 μm rad
β_r	97.78 mm	9.47 mm
σ_z	10 μm	10 μm
n_b	$3.1 \times 10^{18} \text{ cm}^{-3}$	$2.4 \times 10^{19} \text{ cm}^{-3}$

ION MOTION IN A PLASMA FOCUSER

We expect matching to be preserved in the presence of ion motion, as the volume ion motion is confined to is small compared to the size of the beam [7]. In a standard adiabatic plasma, beam evolution is slower than the change in plasma density, which allows for a beam in quasi-equilibrium to be matched to the plasma according to the matching condition $\beta_{eq} \approx k_\beta^{-1}$, where $k_\beta = \sqrt{2\gamma}k_p^{-1}$ is the frequency of betatron oscillations. While matched, beam emittance should be preserved and the matching will hold even if plasma density increases [8, 9]. We aim to match the post-lens beam to an adiabatic plasma and slowly increase n_0 , which will reduce σ_r and increase n_b .

When calculating the phase advance inside the PWFA, we take the transverse normalized emittance ϵ_r to be constant in the adiabatic upramp. The matched σ_r then becomes $0.598 \mu\text{m}$ with a $\Delta\phi$ of 1.16, which is enough to trigger gentle ion motion. The strong transverse fields from the electron beam motivate the ion column to collapse towards the beam's z axis in a laminar manner. If we take the initial ion column to be comprised of many shells containing the same number of ions, laminar movement of ions conserves the number of shells beneath any given shell, which preserves field linearity and is effectively the same as simply increasing the ion column density [3].

We now explore the plasma system in its entirety using Fig. 2, which shows the evolution of energy, energy spread, normalized emittance, and σ_r . Examining the bottom most image, we see that simulated and computed behavior of σ_r are in close agreement during the lens and vacuum segments, while deviations occur when the beam enters the adiabatic PWFA. Once inside, energy loss and emittance growth cause the simulated beam to deviate from predicted values, and the matching degrades.

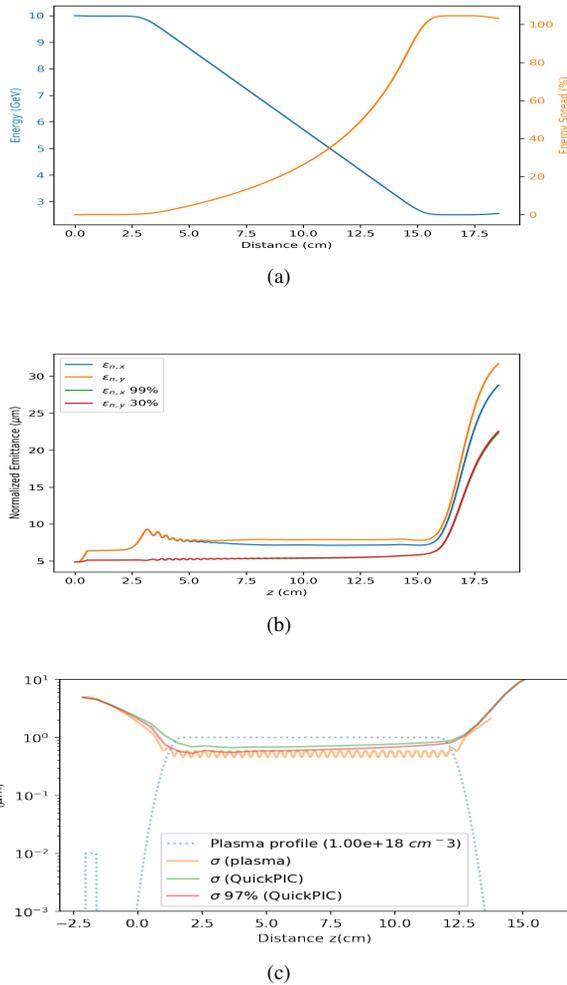


Figure 2: (a) Energy and energy spread of the beam. (b) Normalized emittance. $\epsilon_{n,y}$ 30% considers the emittance in the beam tail under the effects of ion motion. (c) Comparisons between predicted and simulated σ_r (solid lines) superimposed over plasma elements (dotted lines).

For the first part of the high density section, energy loss is sufficiently low that the phase advance calculated previously is still valid for the beam core. As the beam core moves along the ion column, the ions feel the beam's pull and collapse inwards. This results in a fully collapsed ion column in the beam tail only - limiting the enhanced focusing effects to that region, as shown in Fig. 3. Examination of the portion of the beam traversing the fully collapsed ion column reveals the superior focusing power of the ions. A parameter scan of the back 30% of the beam saw the tail section reach a minimum σ_r of 0.448 μm, 100 nm smaller than the σ_r of a beam perfectly matched to the plasma. The impact of the collapsed ion column is more apparent when the tail is compared to the rest of the beam. At $z = 6$ cm, σ_r of the core is 0.659 μm, which is larger than the matched σ_r due to minor emittance growth and energy loss in the upramp section, which were neglected in theory calculations. The approximately 33% decrease in rms width from the core

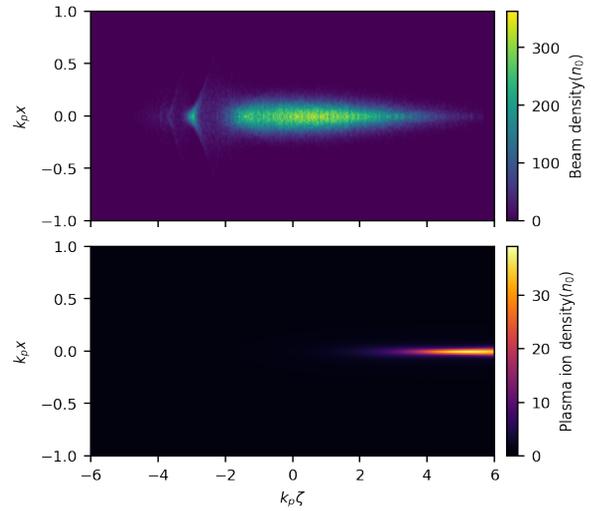


Figure 3: (Top) The high density ion column after ion column collapse (Bottom) The core of the beam is matched and the tail shrinks radially due to the collapsed ion column.

rms suggests that gentle ion motion ($1 \leq \Delta\phi \leq \frac{\pi}{2}$) is a valid regime for enhanced beam focusing.

CONCLUSION

This paper examined the potential for the FACET-II beam to self-focus using ion motion in preparation for the E-314 experiment. We examined a focusing system that would increase the FACET-II beam's density by reducing its σ_r enough for ion motion to become non-negligible. In our investigations, we assumed quadrupoles upstream of the plasma elements would reduce σ_r from 20 μm to 5 μm, which is necessary for enforcement of blowout. We simulated a plasma system that comprised of a thick plasma lens and a high density PWFA over the span of multiple centimeters. Ion motion was anticipated in the uniform portion of the PWFA, where the beam density was greatest and $\Delta\phi > 1$. In this segment, the body of the beam had a minimum σ_r of 0.659 μm, and QuickPIC simulations of this segment showed the ion column collapsing, however being in the gentle regime meant that the collapsed column formed in the tail of the beam and enhanced focusing effects were limited to that segment. Nevertheless, the tail was observed to have a σ_r as low as 0.448 μm, nearly 33% lower than the σ_r of the beam body. These effects are hoped to be verified in the upcoming E-314 experiment. Successful recreation of simulated results in a laboratory would make FACET-II a valuable tool for future study of ion motion.

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