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COMPARING METHODS OF RECOVERING GAMMA ENERGY DISTRIBUTIONS FROM PEDRO SPECTROMETER RESPONSES

M.H. Oruganti, B. Naranjo, J.B. Rosenzweig, UCLA, Los Angeles, CA, USA
M. Yadav, University of Liverpool, Liverpool, UK

INTRODUCTION

As high energy atomic and subatomic particles interact, they emit gamma radiation in a wide range of energies, all of which can be used to determine the nature of the original interacting particles. To calculate the energy levels of the photons emitted, the new pair spectrometer (PEDRO) channels the photons through several Beryllium nuclear fields; since the photons of interest will likely have energies between 10 MeV and 10 GeV, those photons will produce electron-positron pairs through the nuclear field interaction [1].

Under a magnetic field, the electrons and positrons bend in opposite directions (due to their opposite charges) and at different angles (due to their differences in energies). A series of spaced out cells measure the number of electrons or positrons that strike at that location, which has some relation to the energies associated with the photons that generated those electrons and positrons.

The goal of this paper is to compare several methods of reconstruction and determine which best predicts original energy distributions based on simulated spectra.

METHODS

When considering electron-positron pair production from photon interactions with nuclear fields, it is important to recognize the linear relationship between the energy distribution of photons and the spectrometer's response. This linear relationship can be modeled as follows:

$$\begin{bmatrix} \alpha_{1,1} & \alpha_{1,1} & \cdots & \alpha_{1,64} \\ \alpha_{2,1} & \alpha_{2,1} & \cdots & \alpha_{2,64} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{128,1} & \alpha_{128,1} & \cdots & \alpha_{128,64} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{64} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{128} \end{bmatrix} \quad (1)$$

In the above system of equations, and throughout the rest of this paper, the x-vector will refer to the original distribution of photons based on their energies. While the energy values will become more relevant when interpreting the model's output, the numerical values that split the energy distribution into logarithmic bins were chosen out of convenience. The y-vector will always refer to the electron-positron spectrum PEDRO outputs in response to the corresponding x-vector, or incoming photon energy distribution.

Possible Methods of Recovery

To determine the best method of recovering the x-vector given a y-vector, several methods of computation were explored and compared when applied to standardized test cases. The three methods included machine learning, combining the model with the Maximum Likelihood Estimation algorithm, and QR decomposition.

Machine Learning (ML) Using Eq. (1), training data was synthesized from creating arbitrary energy distribution (each bin containing a random number of photons between 0 and 1×10^{10}) and multiplying each x-vector by the matrix to generate the corresponding spectrum (or y-vector) that PEDRO would measure. To simulate real world noise from electrons being scattered during the pair production process, low-level noise vectors were calculated and added to the y-vectors.

Table 1 summarizes the model's architecture. The model used the Adam optimizer with a learning rate of 0.005 over 600 epochs. Mean Squared Error was chosen as the loss function for the model, which was constructed using Python's Keras library [2].

Table 1: A Summary of the ML Model to Predict Incoming Gamma Spectra Based on Positron-Electron Detection

Layer (type, bias, activation)	Output Shape	Param Num
dense (Dense, true, linear)	(None, 64)	64
dense1 (Dense, true, linear)	(None, 64)	64

Maximum Likelihood Estimation + Machine Learning (Hybrid) Maximum Likelihood Estimation (MLE) is an algorithm that, given an initial guess for the solution of an equation, iteratively converges to the nearest solution that is the most probable. It converts the issue of calculating the original energy distribution based on the PEDRO output from an analytical problem to a statistical one to estimate a series of parameters [3].

While the MLE algorithm does not require training like the ML model, it does require a guess that is sufficiently close to the true solution of Eq. (1). Given the utility of machine learning, it is possible to provide a customized guess for every y-vector: the ML model's guess. This combination serves as a hybrid approach to recovering the original energy distribution as opposed to pure ML or QR Decomposition (as will be seen in the next section).

QR Decomposition The software's goal was to effectively invert the response matrix R so given a PEDRO spectrum, the originating x-vector could be recovered. This method of recovery required no additional information beyond the matrix to be decomposed (in this case, R).

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$$\hat{R} = \hat{Q} * \hat{S} \tag{2}$$

In this decomposition, $\hat{Q}$ is a square orthonormal matrix, meaning the transpose of $\hat{Q}$, $\hat{Q}^T$, is its inverse matrix. As such:

$$y = \hat{R}x = (\hat{Q}\hat{S}) * x => \hat{Q}^T y = \hat{Q}^T\hat{Q} * \hat{S}x \tag{3}$$

$$=> \hat{S}x = \hat{Q}^T y = b \tag{4}$$

Now, finding the x-vector is reduced to determining which solution will minimize the following function:

$$f(x) = ||\hat{S}x - b||^2 \tag{5}$$

Minimizing f(x) will result in a solution for Eq. (4), meaning using the least-squares optimization algorithm will provide the most likely energy distribution. Before implementing this algorithm, the values of the matrix $\hat{S}$ must be accounted for to determine whether the function is strongly convex to guarantee a unique solution. In this case, the function's second derivative did have strictly positive eigenvalues. Therefore, using the least squares optimization algorithm will avoid the issue of converging on potentially multiple solutions. After generating the QR decomposition, like the other approaches, this algorithm was tested against both discrete cases and more continuous cases.

Test Cases

To determine the efficacy of each of the approaches, there were a total of 5 standardized test cases that each method was applied to. They are listed as follows:

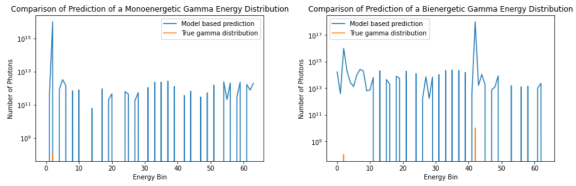
- Monoenergetic: The original spectrum contained only 10^8 photons in the 2nd bin
- Bienergetic: The original spectrum contained only 10^8 photons in the 2nd bin and 10^{10} photons in bin 42
- Smooth cases that were derived from past experiments (Non-Linear Compton Scattering, Quantum Electrodynamics, Filamentation)

RESULTS

Machine Learning

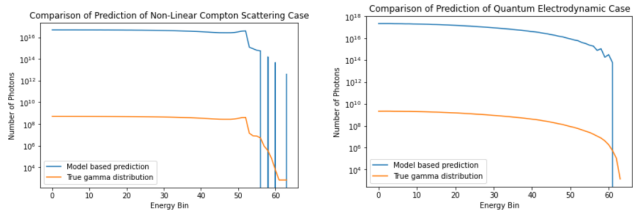
In the discrete cases in Fig. 1, the trained ML model appears to predict peaks at the correct bins where the photons were located in terms of their energies. However, the scale of the prediction is several orders of magnitude higher than the true frequency of photons. The surrounding non-peak bins are also shown to have a large number of photons (despite the spectra being monoenergetic or bienergetic respectively); these bins contain predictions that are around 1-4 orders of magnitude below the peak photon frequencies.

In the smooth cases, as shown in Fig. 2, the model failed to predict the distribution and had an error of several orders of magnitude.

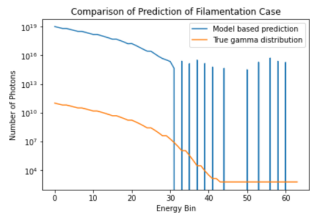


(a) Monoenergetic case. (b) Bienergetic case.

Figure 1: Discrete reconstructed gamma energy distributions using machine learning.



(a) Non-linear compton scattering case. (b) Quantum electrodynamics case.

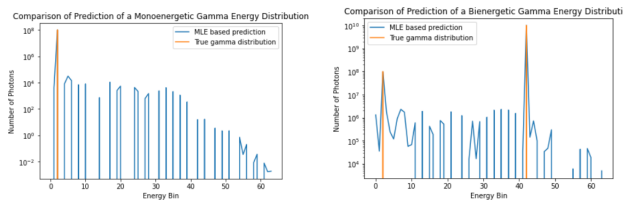


(c) Filamentation case.

Figure 2: Smooth reconstructed gamma distributions using machine learning.

Maximum Likelihood Estimation + Machine Learning (Hybrid)

In the discrete cases (see Fig. 3), the hybrid approach correctly predicts the location and heights of the peaks associated with the energy ranges of the photons. All of the other energy bins, according to the MLE prediction, contain anywhere ranging from 1 to 10^5 photons. The predicted frequency of photons approaches 0 when entering the 10 GeV range after the rightmost peak in both cases.



(a) Monoenergetic case. (b) Bienergetic case.

Figure 3: Discrete reconstructed gamma energy distributions using hybrid approach.

In these smooth cases (see Fig. 4), the hybrid approach follows a similar pattern to the ML model's predictions. Here, the hybrid approach predicts the correct pattern of distribution between bins 1 and 30. Towards the higher end, however, there are gaps in the predicted frequencies, as if it

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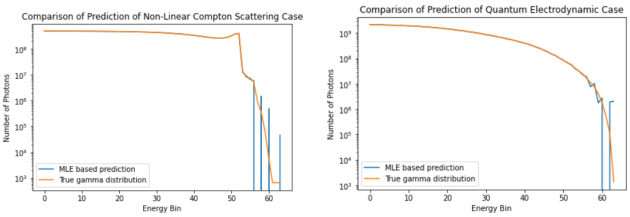
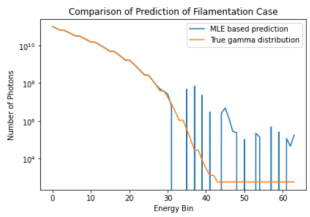


(a) Non-linear compton scattering case. (b) Quantum electrodynamics case.



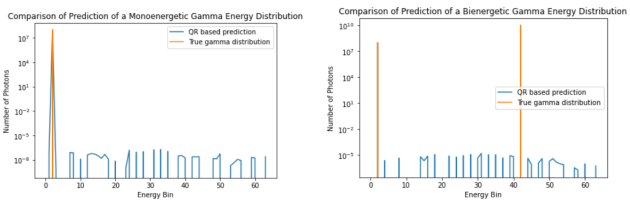
(c) Filamentation case.

Figure 4: Smooth reconstructed gamma distributions using hybrid approach.

is predicting the absence of photons in some of the energy ranges.

QR Decomposition

Looking at the predicted distributions for spectra generated from monoenergetic and bienergetic distributions in Fig. 5, the QR decomposition method predicts the peaks and their locations in photon frequency almost perfectly.



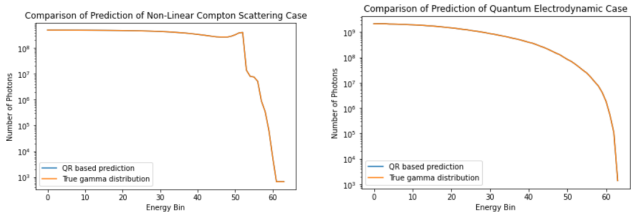
(a) Monoenergetic case. (b) Bienergetic case.

Figure 5: Discrete reconstructed gamma energy distributions using QR decomposition.

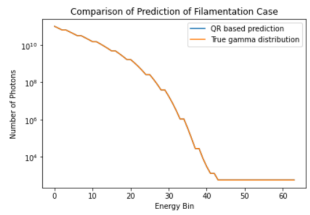
In all three of the smooth distributions (see Fig. 6), there appears to be only one graph present. The QR decomposition method generates a distribution almost perfectly overlaps with the original distribution. Across all of the energy bins, the QR decomposition method predicts the correct pattern and frequencies of photons.

ANALYSIS

Looking at the figures, the QR decomposition approach clearly provides the most accurate reconstructions of photon energy distributions from the PEDRO spectra. The other two approaches have regions of energy where the predictions differ by orders of magnitude from the true solution, making them problematic and unreliable. The ML approach showed the least promise on its own due to scaling issues, but the MLE-ML Hybrid approach helped solve some problems by correcting the predicted frequencies' order of magnitude.



(a) Non-linear compton scattering case. (b) Quantum electrodynamics case.



(c) Filamentation case.

Figure 6: Smooth reconstructed gamma distributions using QR decomposition.

The unpredictable end behavior seems to suggest the values that the model was processing may have overflowed (meaning the computer was unable to process the numbers and may have defaulted the values to 0). This indicates that while the ML approach may have been sound, computational limitations prevented its execution from reaching acceptable levels of accuracy.

The comparison of these methods and the uniqueness of the QR solution helped increase confidence in the method's ability to reliably reconstruct energy spectra, as it outperformed the other two methods. While QR decomposition worked acceptably well in this scenario, it is not guaranteed to generalize to other reconstruction situations with a linear representation.

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