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MODELLING FCC-ee USING MAD-X

L. van Riesen-Haupt*, T. Persson, R. Tomás and H. Burkhardt, CERN, Geneva, Switzerland

Abstract

We present the latest developments for simulating FCC-ee using CERN's MAD-X software. Along with updated benchmark studies, we describe how the latest MAD-X updates can facilitate the simulation of FCC-ee design features, including improvements in tapering and different options for implementing a tilted solenoid.

INTRODUCTION

The Future Circular Electron-Positron Collider (FCC-ee) [1] is a proposed new collider at the European Organization for Nuclear Research (CERN) for the era beyond the Large Hadron Collider (LHC). In order to design this machine and test its feasibility, it is important to ensure that the simulation tools used for this purpose provide reliable results.

In a previous publication [2], we presented studies that explore the simulation of more general optical properties obtained using CERN's Methodical Accelerator Design (MAD-X) [3] tool. An important feature in these studies was benchmarking the results to those obtained when using the Systematic Accelerator Design software (SAD) [4], which is designed for and thoroughly tested against the largest current lepton collider, Super KEK-B, located in Japan.

In this publication, we focus on the simulation of features that are more specific to the FCC-ee, including the computation of the beam emittance using tapered lattices and the effect of a tilted experimental solenoid. Again, benchmarking the results obtained using CERN tools against SAD plays a key role in these studies. Before presenting the new results we will provide a brief summary of previous work and put it in the context of this work.

Summary of Previous Results

In [2], it was established that for simulations without radiation, MAD-X and SAD compute identical optics to a relative precision of 10^{-6}. This is also true for off-momentum particles and the momentum detuning looks very similar for both codes. These results were obtained for FCC-ee lattices that did not include the tilted experimental solenoid.

MAD-X 5.6.00 was the first version to also include a tapering functionality [5]; this is a system in which the fields of the magnets around the ring are individually adjusted in order to compensate for the change in beam rigidity due to energy losses from synchrotron radiation. The MAD-X implementation is based on a similar scheme in SAD and in [2] we showed that the optical functions and tracking behaviour with radiation in tapered MAD-X lattices were very similar to those of SAD. The publication did not explore the emittances computed using MAD-X's EMIT module for these tapered lattices and the lattices did not include the tilted solenoid.

* leon.van.riesen-haupt@cern.ch

Table 1: Value of the five radiation integrals computed using MAD-X and SAD.

Integral	SAD	MAD-X
I_1	1.441818	1.441818
I_2	5.8860828×10^{-4}	5.8860839×10^{-4}
I_3	5.4659284×10^{-8}	5.4659304×10^{-8}
I_4	$-2.2581083 \times 10^{-10}$	$-2.2581086 \times 10^{-10}$
I_5	$5.2274385 \times 10^{-11}$	$5.2261809 \times 10^{-11}$

SYNCHROTRON RADIATION

Radiation Integrals

The radiation integrals are a set of simple optics integrals that can be computed from the accelerator's Twiss functions and bending radius and can help giving quick estimates of radiative properties such as energy loss per turn or equilibrium emittances [6]. Both MAD-X and SAD can compute the five commonly used radiation integrals. These were computed for the FCC-ee ZZ lattice version 213 [7] using MAD-X and SAD and the results are shown in Table 1.

From Table 1, it can be seen that there is a very good agreement between the two codes. Their values were also checked against integrals computed directly from the twiss files using python scripts. However, it should be pointed out that these integrals were computed for an on-momentum lattice in both cases. In SAD, the fourth radiation integral is momentum dependant and captures the fact that the bending radius depends on the local momentum of the beam. Some of this behaviour can be captured by the so-called I_8 integral as explained in [6]. To this end, the I_8 integral was added to MAD-X in version 5.06.00. This update also included the implementation of the I_6 integral, which captures the energy loss in the quadrupoles and is very relevant for colliders.

Emittance of Tapered Lattices

In the first iteration of the MAD-X tapering implementation, the EMIT module was not able to find a longitudinally stable orbit for the tapered FCC-ee lattices. To fix this, MAD-X version 5.07.00 introduced a minor change to the tapering that allows users to match the radio frequency cavity phase during the tapering process in order to achieve stability. This matching can be done using the MATCH module to constrain the p_t values to zero at the locations where the beam energy is equal to the reference energy.

This improvement makes it possible to determine the beam emittance using the matrix methods in MAD-X for tapered lattices and the results can be compared to those obtained from SAD. On top of SAD, the results can be compared to results obtained when computing the emittances

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Table 2: Table showing horizontal emittances obtained from tapered lattices in MAD-X and SAD, as well as results obtained at 1 GeV in MAD-X scaled to the appropriate energies.

Lattice	Energy	MAD-X	SAD	Scaled
Z	45.6 GeV	0.27 nm	0.27 nm	0.27 nm
WW	80 GeV	0.83 nm	0.84 nm	0.83 nm
ZH	120 GeV	0.63 nm	0.63 nm	0.63 nm
tt	182.5 GeV	1.45 nm	1.46 nm	1.45 nm

computed when setting the beam energy to 1 GeV and scaling the results by the square of the energy in GeV [6]. At 1 GeV the radiation is small enough not to have a significant impact on the orbit and the optics, thereby being analogous to a tapered lattice.

This was done for the FCC-ee lattice version 213 [7] for each beam energy and the results are shown in Table 2. From Table 2 it can be seen that tapered lattices in MAD-X return the expected emittances, showing that the tapering is correctly linked in the MAD-X EMIT module. It should be pointed out that the results from MAD-X agree slightly better with the scaled results than with the results from SAD. This could be due to small differences in how the emittance is computed in the two programmes or the slightly larger residual orbit and β-beating seen in the SAD tapering compared to the MAD-X implementation [2].

TILTED SOLENOID

The particle detectors around the FCC-ee interaction point (IP) require solenoids to produce a magnetic field that is constant and longitudinal within the solenoid [1]. Because the two beams collide at an angle at the IP, the solenoid field will be tilted around the y-axis in the beam's frame of reference. The tilted solenoid field results in a reduced longitudinal solenoid field of strength $B_{sol}\cos(\phi)$ and a skew dipole field of strength $B_{sol}\sin(\phi)$, where B_{sol} is the total solenoid field and ϕ is the half crossing angle.

To compensate for both the coupling due to the longitudinal field and the dispersion due to the dipole field, an anti-solenoid of equal and opposite strength and half the length of the experimental solenoid is placed at either side of the solenoid. Whilst the anti-solenoid is very effective at screening the rest of the machine from the effect of the solenoid, there are still some phenomena that take place inside the solenoid region that need to be studied carefully and simulated accurately. These effects include the vertical orbit and dispersion bumps, as well as the synchrotron radiation due to the dipole field, which have an impact on the vertical emittance. The coupling from the longitudinal field also causes the orbit bump to rotate into the horizontal plane. This effect is not fully countered by the anti-solenoid and can affect the orbit in the rest of the machine.

Implementation Methods

There are several potential ways of implementing a tilted solenoid in MAD-X. The most straight forward is to define the locations of all elements along the reference orbit, as defined by the bending dipoles and with the solenoids originally aligned to the beam. The solenoid can then be tilted by using the error module in MAD-X. This perturbation method results in a small non-zero horizontal orbit outside of solenoid region and does not take into account the additional path length due to the orbit in the solenoid. A variant of this method is to slice the solenoid into various equal length finite slices, with total integrated field that is reduced by $\cos(\phi)$ and interleave these slices with zero length bending magnets of strength $\sin(\phi)$ and a radiation length equal to that of the solenoid slices.

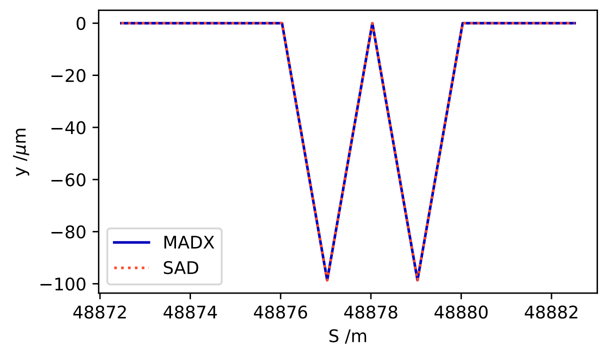

Figure 1: Vertical position Y in FCC-ee tilted solenoid around the second IP, as computed by SAD and MAD-X.

The SAD definition of the solenoid does not have the same issue of the residual orbit. This is because after it tracks the beam trajectory through the tilted solenoid it transforms the co-ordinates of the reference orbit so that it is zero at the exit. All the subsequent elements, such as quadrupoles and other magnets are then aligned on this new reference orbit. This is analogous to the convention of defining the reference orbit along the trajectory of the beam passing through a bending dipole.

The FCC-ee lattices are designed using SAD and then converted to MAD-X, therefore, in order to be able to fully simulate the optics as intended, it is important to have a means to replicate the SAD solenoid implementation in MAD-X. This can be achieved by installing a co-ordinate rotation and three translation elements, one for each dimension, at the exit of the solenoid in the MAD-X model. The initial magnitudes of these rotations and translations can be inferred directly from the SAD model in the first iteration and then adjusted using the MAD-X MATCH function to ensure the closed orbit is zero at the exit of the solenoid.

Comparison for FCC-ee

Using the implementation described above the FCC-ee ZZ MAD-X lattice version 213 [7] was modified to include a tilted 2T solenoid/anti-solenoid pair around the IP. The SAD lattice of the same version already has a tilted solenoid installed but the field is set to 0 T by default. The field was changed to 2 T and the tilt angle of 15 mrad at the IP was

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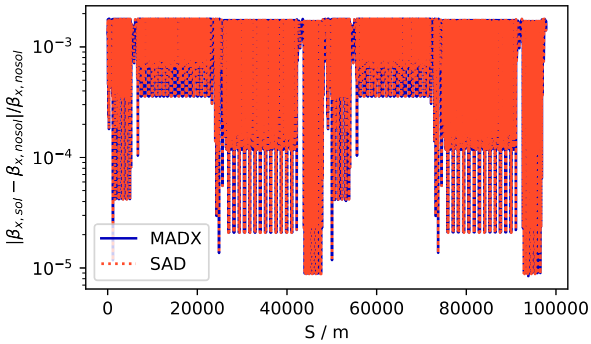

Figure 2: Horizontal β-beating from tilted solenoid as computed in MAD-X and SAD.

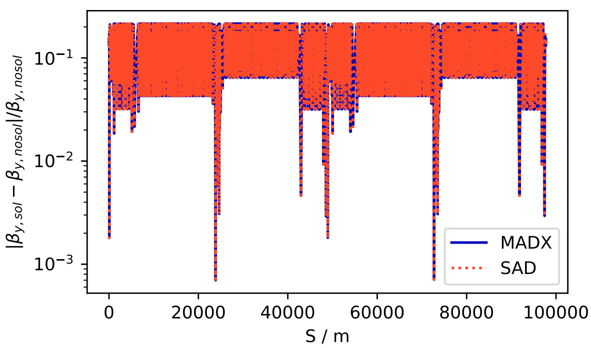

Figure 3: Vertical β-beating from tilted solenoid as computed in MAD-X and SAD.

left unchanged. The tilt in the modified MAD-X model was also set to 15 mrad.

The first thing to check in order to determine whether the application of the tilt was done correctly is the vertical dipole field effect and the co-ordinate transformations of the orbit at the edge of the solenoid. To do this, the vertical orbit around the solenoid obtained using SAD and MAD-X were plotted in Fig. 1. The plot is centred around the second interaction point, which lies around 48 km away from the first one. Outside the solenoid region, the orbit is flat and the beginning of the solenoid can clearly be identified as the location where the orbit starts to change. The orbit bump is identical in the two codes, showing that MAD-X solution can mimic the SAD solenoid implementation.

One can also inspect the β-beating induced by the tilted solenoid for the two codes. To do this, the optics was computed twice using each code, once with the solenoid field set to 2 T and once with the solenoid present but the field turned off. The difference between the β functions was computed and normalised using the β functions in the absence of a solenoid field. The results are shown in Figures 2 and 3. These figures show an identical β-beating from both codes, again showing that it is possible to achieve SAD-like results for optics with a tilted solenoid by employing the method described above. One limitation found when using this implementation was that the transformations caused

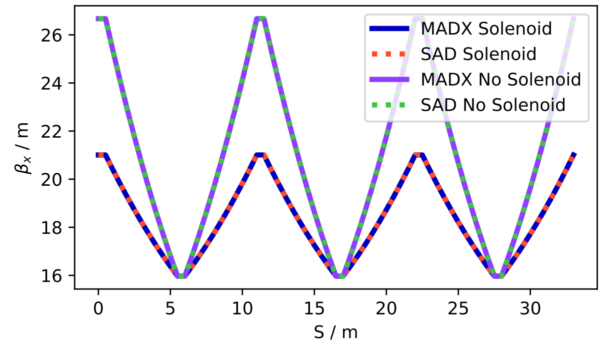

Figure 4: Horizontal β-functions in simple FODO cell with and without an overlapping solenoid in MAD-X and SAD.

some spurious vertical dispersion. The magnitude of this spurious dispersion has been decreased by implementing exact rotations in MAD-X version 5.07.00 [8], however, some dispersion of around 10^{-5} m remains. The spurious dispersion is also one of several reason why the emittance can not be reliably computed, however, this issue is being actively worked on. The most reliable way to obtain an emittance estimate for the time being is by using the sliced solenoid with interleaved bends.

Solenoid and Quadrupole Overlap

The FCC-ee also anticipates an overlapping solenoid and quadrupole field in order to bring the final focus quadrupoles as close as possible to the IP. However, currently no lattice versions with this feature exist, neither in MAD-X nor in SAD. In SAD, overlapping solenoid and quadrupole fields can be readily simulated using an accurate field map that superimposes the two types of fields. In MAD-X, such an element does not exist, however, it should be possible to simulate this by slicing the solenoid element into a number of finite slices and interleaving them with a series of thin quadrupole kicks that have the same integrated strength as the solenoid they represent [9].

This concept was tested by creating simple FODO cell in both MAD-X and SAD and computing the closed Twiss. This cell was then placed inside a solenoid in SAD, whilst slicing the quadrupoles and interleaving them with solenoid slices in MAD-X as described in the previous paragraph and again computing the closed optical functions. The resulting horizontal β functions for both cases are shown in Fig. 4, from which one can see that there is a very good agreement, observed also in the vertical plane.

CONCLUSIONS

The studies on the radiation integrals and emittance calculations in tapered lattices complement and complete the previous studies on using MAD-X to simulate FCC-ee lattices without tilted solenoids. Moreover, we have presented a method of simulating a tilted solenoid in MAD-X in a way analogous to the SAD implementation and shown how MAD-X could be effectively used in the future to simulate the overlap of solenoid and quadrupole fields.

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