ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOMS016

ON THE (APPARENT) PARADOX BETWEEN SPACE-CHARGE FORCES AND SPACE-CHARGE EFFECTS

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Abstract

With the advent of high-intensity linacs, space-charge forces are now well known as a major issue, causing undesirable effects on particle beam qualities like emittance growth or sudden losses. They should be stronger when there are more particles or when the latter are contained in a smaller volume. But a detailed examination of the beam along an accelerator show that space charge effects are weaker where the beam size is smaller. This article clarifies this paradox and revisits the recommendations on beam sizes in view of mitigating space charge effects.

INTRODUCTION

For high-intensity proton linacs, space charge is known as a major issue to be carefully addressed, at least in the sub-GeV energy range, as it contributes to distort the beam density profile, increasing the halo in the external parts of the beam, causing emittance growth and particle losses.

Space-charge forces are the integrated Coulomb forces of the whole beam on every witness particle within the beam. These forces are thus expected to be stronger when there are more charges or when they are closer to each other, i.e. the space occupied by them is smaller. However, when examining the equation governing the beam envelope along the accelerator, space charge effects appear to be weaker at places where the beam size is smaller.

The purpose of this paper is first to clarify the apparent paradox between space charge forces and space charge effects for a better understanding of beam behavior, and second to revisit the recommendations for mitigating harmful space charge effects when designing the beam, based on precise considerations of beam halo vs beam core.

SPACE-CHARGE FORCES

Let's consider the simplified case of an infinitely long cylinder of uniform positive charge density with a total radius R, moving at the same longitudinal speed v. According to Gauss' law, this produces an outward radial electric field, which acts as a defocusing field for the beam. At a given radius $r \le R$, it is given by:

$$E_r(r) = \frac{Ir}{2\pi\varepsilon_0 v R^2} \tag{1}$$

where I is the beam intensity, and ε_0 the vacuum permittivity.

According to Ampere's law, an azimuthal magnetic field is also induced at the same position, focusing the beam:

$$B_\theta(r) = \frac{\mu_0 I r}{2\pi R^2} \tag{2}$$

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where μ_0 is the vacuum permeability.

A witness charge q at this position will be submitted to a radial force, which is called the space-charge force

$$F_{sc}(r) = q(E_r - vB_\theta) = \frac{qI}{2\pi\varepsilon_0 v\gamma^2}\frac{r}{R^2} \tag{3}$$

where γ is the Lorentz relativistic factor. This force is linear in r inside the beam for a uniform charge distribution, and is well stronger when the beam size R is smaller as expected.

SPACE-CHARGE EFFECTS

To see the effect of this space-charge force on an accelerator beam, let's look at the above charged cylinder which is now submitted in addition to focusing forces applied by various accelerator components surrounding the beam. These external forces are generally linear in r, characterized by the focusing coefficient k:

$$F_{ext}(r) = -kr. \tag{4}$$

By adopting the following notations for time derivatives and space derivatives along the longitudinal coordinate z:

$$\dot{r} = \frac{\partial r}{\partial t} \ , \ r' = \frac{\partial r}{\partial z} \ , \quad \ddot{r} = v^2\frac{\partial^2 r}{\partial z^2} \ , \tag{5}$$

Newton Second law for a witness charge (rest mass m_0) is:

$$\gamma m_0 \ddot{r} = F_{ext} + F_{sc}. \tag{6}$$

The equation of motion for this charge can be deduced:

$$r'' + K_{ext} r - K_{sc}\frac{r}{R^2} = 0 \tag{7}$$

where

$$K_{ext} = \frac{k}{m_0 v^2 \gamma} \qquad (unit\text{: m}^{-2}) \tag{8}$$

$$K_{sc} = \frac{qI}{2\pi\varepsilon_0 m_0 v^3\gamma^3} \qquad (no\ unit). \tag{9}$$

K_{sc} is called the generalized perveance. For an arbitrary charge distribution but with elliptical symmetry, it is mentioned in [1] that Lapostolle and Sacherer independently shown that the equation of motion, Eq. (7) can be rewritten as

$$a_x'' + K_{ext,x} a_x - \frac{\varepsilon_x^2}{a_x^3} - \frac{K_{sc}}{2(a_x + a_y)} = 0 \tag{10}$$

$$a_y'' + K_{ext,y} a_y - \frac{\varepsilon_y^2}{a_y^3} - \frac{K_{sc}}{2(a_x + a_y)} = 0 \tag{11}$$

where x and y refer to the two transverse axes, a the rms beam size, ε the non-normalized beam emittance. These equations are called envelope equations.

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In these equations, the third and fourth terms with the minus sign mean they act as defocusing forces, in opposition with the second term expressing external forces that are focusing when K_{ext} is positive.

The third term is called the emittance term. It is sometimes improperly considered as the expression of "emittance forces" but these are not physical forces. Its presence points out that due the non-zero beam emittance, the beam size can never shrink to strictly zero, whatever applied external forces. The larger the emittance, the larger external forces must be to reduce the beam size.

The fourth term is called the space-charge term. It is the expression of space charge forces. As expected, its effect taken separately is stronger when the beam size is smaller. But when compared to the emittance term we can see that its effect on the beam is relatively smaller for a smaller beam size and become negligible for enough small beam sizes. This may seem counter intuitive, but it is not if we understand that the envelope Eqs. (10) and (11), unlike Eq. (3), are not the expression of space-charge forces alone, but that of the balance between these ones and applied external forces. In other words, space charge forces can be fought by applying focusing forces, knowing that the larger the space charge, the larger must be focusing forces, and as a result, the beam size is smaller.

CONSEQUENCES

From the above basic relations, several consequences can be deduced.

(i) A little like K_{ext} in Eq. (8) that is the external force characteristic which is independent of any beam parameters (except its energy), K_{sc} in Eq. (9) is the space-charge characteristic which is independent of beam geometrical parameters like particle coordinates or beam size. Therefore, K_{sc} is the parameter to be used to compare between different accelerators when the importance of space charge is in question. When K_{sc} is larger, be prepared to have more emittance growth, more halo, more beam losses, be prepared to be forced to employ larger K_{ext}, i.e. more focusing means.

(ii) According to Eq. (9), K_{sc} is large when the beam intensity is large, which is often the case for modern linacs. Equations (1-3) indicate that K_{sc} can be huge at low energy while it is negligible at high energy when the beam reaches relativistic velocities, as focusing magnetic effects from space charge overwhelm defocusing electric effects. Space charge is really an issue only in first acceleration stages. There, acceleration alone is not possible because of the very strong defocalization induced by space charge forces. As a result, separate accelerating cavities and focusing electromagnets cannot be used in the very first part of linacs. They must be replaced by a special component allowing to accelerate and focalize particles almost simultaneously, the RFQ (Radio Frequency Quadrupole), which was invented for that. The larger the beam intensity and the longer should be the RFQ to accelerate particles up to an energy where space charge effects become manageable. We can observe that in most linacs, the RFQ generally ends when K_{sc} is decreased to well less than 10^{-5}.

(iii) For a given (low energy) linac, the combined effect of external forces and space charge forces will dictate the beam behavior, especially its emittance and density profile. The envelope equations (10) and (11) are very good indicators of these. Figures 1 and 2 (from [2-4]) illustrate well the outcome of the competition between the two forces. Whenever the space charge term is larger than the emittance term, in the x or/and y plane, we are in the space-charge dominated regime, and the emittance grows in the corresponding plane (Fig. 1). This is known as the fastest emittance growth mechanism, during which charges redistribute so as to shield the beam against external fields [1]. An equilibrium state is then reached when the density profile becomes more compact, with a large and almost flat core surrounded by a thin halo ring (bottom graphs of Fig. 2). When the emittance term is larger than the space charge term, we are in the emittance-dominated regime, the emittance and density profile remain unchanged. The most typical example of such a regime is given by the RFQ with its strong focalization by design. When the RFQ is long enough, the output density profile has a perfect Gaussian shape, exhibiting the smallest RMS size, and an indefinitely extending halo ring (top-right graph in Fig.2). Space charge effects are indeed totally 'killed' by the RFQ that has perfectly fulfilled its mission.

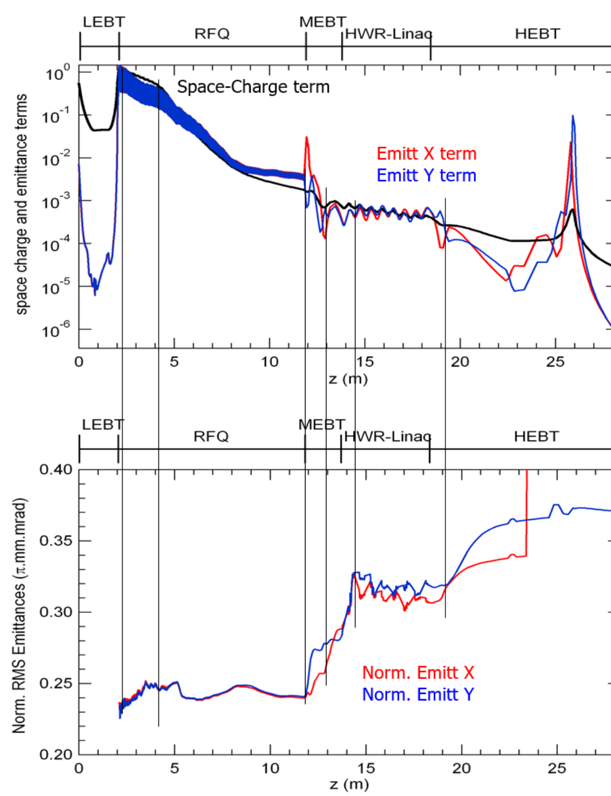

Figure 1: Evolution of the space charge term (black), the emittance term, and the beam emittance (red, blue: horiz., vert.) along the IFMIF prototype accelerator LIPAc [2].

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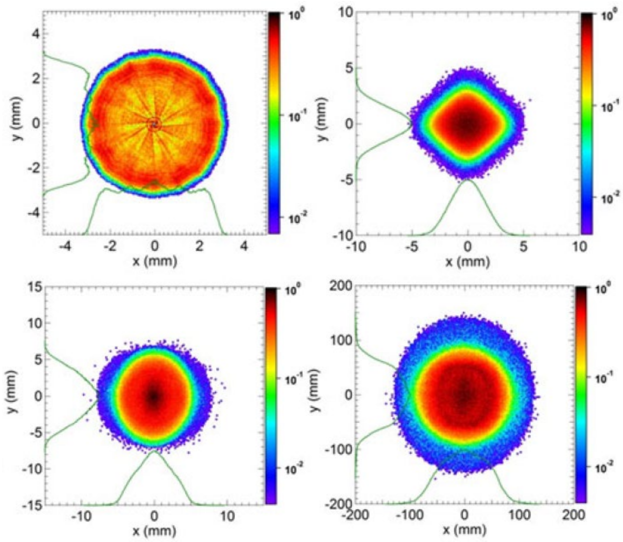

Figure 2: Density distribution with projected profiles in x and y, at the LEBT exit (top left), RFQ exit (top right), SRF-Linac exit (bottom left), and HEBT exit (bottom, right), for the IFMIF prototype accelerator LIPAc [3, 4].

CORE-HALO ANALYSIS

The above analysis based on the RMS emittance is nevertheless not so meaningful because the beam shape radically changes in the meantime. A more relevant analysis should be done accordingly to the relative importance between the beam halo and the beam core.

A method to precisely determine the core-halo limit was proposed in [5] and was proven in [6] to be consistent with the internal dynamics of a beam submitted to space charge forces. According to that, Fig. 3 [4] can be produced, showing the evolution along the linac of (i) the core size and the halo size, (ii) PHP, the percentage of particles in the halo relative to the total number of particles, and (iii) PHS, the percentage of the halo size relative to the beam total size.

We can see that the core size is the smallest in the RFQ. Then it increases downstream, in two steps, in the SRF-Linac then in the HEBT, according to the external focusing strength applied (At the linac end, the beam power reached 1.1 MW CW, and the beam size is intentionally enlarged to be spread on a beam dump). The halo evolution is not stable in the RFQ nor right at the RFQ exit, as beam losses are still substantial during and after the bunching process. Downstream the RFQ, similarly to the core, the halo increases in two steps too, for PHP and for PHS as well, but the gap in PHP is much more pronounced. Note that for this linac structure, a lot of effort has been dedicated in focusing tuning to minimize the beam halo, so as to maintain beam losses well less than the 10^{-6} range in the cryogenic SRF-Linac.

CONCLUSION

Space charge forces are very strong at low energy and are well stronger when the beam size is smaller. However, it can be efficiently fought by external focusing forces. The more these come out victorious from this competition, and the smaller the beam size. The RFQ is the strongest focusing structure allowing to defeat space charge forces, capable of completely canceling their effects. When passing from a strong focusing section to a less strong focusing one, like at the RFQ exit for example, the beam emittance will inevitably increase. But this emittance growth is not really harmful, it comes from the beam density reorganization leading to a change in its shape. A fine analysis of the halo is more meaningful, and there also, applying a relevant focusing structure allows to mitigate the halo.

(Simulations and results shown in this article have been made with the TraceWin code [7])

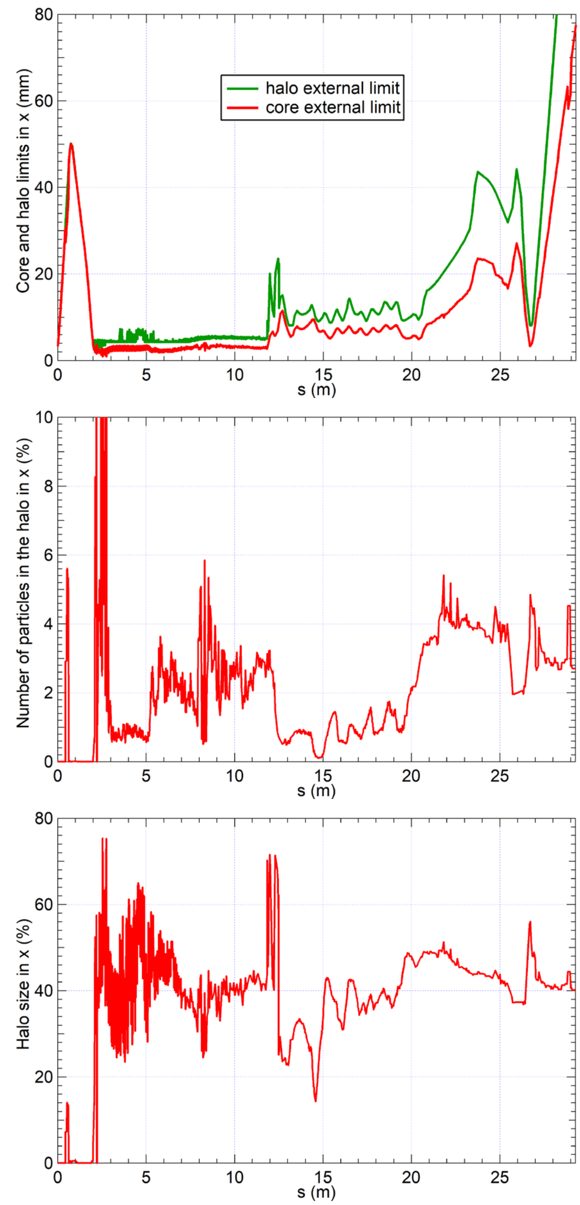

Figure 3: Evolution along the IFMIF prototype accelerator LIPAc of the core and halo external limits (top), PHP (middle), and PHS (bottom) [4].

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REFERENCES

[1] T. P. Wangler, "RF Linear Accelerators," Wiley-VCH, p. 282-306 (2008).

[2] P. A. P. Nghiem, N. Chauvin, O. Delferrière, R. Duperrier, A. Mosnier, D. Uriot, "IFMIF-EVEDA accelerators: strategies and choices for optics and beam measurements," in *Proc*. PAC'09, Vancouver, Canada, May 2009, paper TH5PFP006, pp. 3196-3198.

[3] P. A. P. Nghiem, N. Chauvin, D. Uriot, M. Valette, "A precise determination of the core-halo limit," in *Proc*. IPAC'14, Dresden, Germany, Jun. 2014, pp. 3208-3210. doi:10.18429/JACoW-IPAC2014-THPME003

[4] P. A. P. Nghiem, N. Chauvin, M. Comunian, C. Oliver, W. Simeoni Jr., D. Uriot, M. Valette, "Advanced concepts and methods for very high intensity accelerators," *Laser Part. Beams*, vol. 32, pp. 639-649, 2014.

[5] P. A. P. Nghiem, N. Chauvin, W. Simeoni Jr., and D. Uriot, "Core-halo issues for a very high intensity beam," *Appl. Phys. Lett.*, vol. 104, p. 074109, 2014.

[6] P. A. P. Nghiem, M. Valette, N. Chauvin, N. Pichoff, and D. Uriot, "Core-halo limit and internal dynamics of high intensity beams," *Phys. Plasmas*, vol. 22, p. 083115, 2015.

[7] D. Uriot and N. Pichoff, "Status of TraceWIN code," in *Proc*. IPAC'15, Richmond, VA, USA, May 2015, pp. 92-94. doi:10.18429/JACoW-IPAC2015-MOPWA008