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BASIC RELATIONS OF LASER-PLASMA INTERACTION IN THE 3D RELATIVISTIC, NON-LINEAR REGIME

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Abstract

In the approximation where the plasma is considered as a fluid, basic relations are derived to describe the plasma wave driven by an ultra-intense laser pulse. A set of partial differential equations is obtained. It is then numerically solved to calculate the resulting 3D electric field structure that can be used as accelerating cavities for electrons. The laser strength parameter is varied to investigate regimes from weakly nonlinear up to total cavitation where all the initial electrons of the plasma are expelled.

INTRODUCTION

The high-gradient plasma-based acceleration is a growing research effort with world-wide ongoing experiences. This effort [1, 2], that can be broken down into different configurations such as laser wakefield accelerator (LWFA), plasma beat wave accelerator (PBWA) or beam-driven plasma wakefield acceleration (PWFA), is of great interest because of the extremely large acceleration gradients achievable. When this technology will be mature, it is envisioned an important reduction of size and cost of future accelerators.

Plasma-based acceleration techniques have reached a maturity level which now makes it possible to envisage laser-plasma accelerators with strong requirements for high-quality beams [3]. Massive simulations to optimize the accelerating structure, i.e. the accelerating and focusing electric field profiles, will have to be considered, as for conventional radio-frequency accelerators. PIC (Particle-in-Cell) simulations depict the acceleration physics in the most realistic manner, but ask for significant computation time. They should be oriented beforehand by rough physics considerations, even when the latter are less precise and less realistic.

In this paper, we explore how far it is possible to progress in the way to characterize the accelerating structure without having to use PIC techniques. The fluid approximation is adopted, together with the Quasi-Static Approximation (QSA), and the unchanged laser amplitude approximation. Relativistic linear, then nonlinear regimes will be studied, in 1D then 2D configurations.

This paper is organized as follows: The first section recalls basic equations of the fluid system; the second section describes the linear regime; and the third section deals with the nonlinear regime.

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STARTING EQUATIONS

The starting classical equations to be solved are, respectively, the continuity equation, the Lorentz Force equation, the electromagnetic wave equation and the Poisson equation

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0\,, \qquad (1)$$

$$\frac{\mathrm{d}\gamma m\mathbf{v}}{\mathrm{d}t} = e\nabla\Phi + e\frac{\partial \mathbf{A}}{\partial t} - e\mathbf{v}\times(\nabla\times\mathbf{A})\,, \qquad (2)$$

$$\Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = \frac{en\mathbf{v}}{c^2\epsilon_0} - \frac{1}{c^2}\frac{\partial\nabla\Phi}{\partial t}\,, \qquad (3)$$

$$\Delta\Phi = \frac{en}{\epsilon_0}\,, \qquad (4)$$

with e, m the elementary charge and mass, $n = n_0 + \delta n$ the electron plasma density, $\mathbf{v}$ the electron velocity, Φ the scalar potential of the wakefield and ϵ_0 the permittivity of vacuum. Pressure terms are ignored. The Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ is used, with $\mathbf{A}$ the vector potential associated with the laser field. In this paper, the normalized vector potential $\mathbf{a} = e\mathbf{A}/(mc)$ and the normalized scalar potential $\phi = e\Phi/(mc^2)$ are used. The QSA is used to the change of frame $t \to \tau$ and $z \to \xi + v_g t$, with z the longitudinal axis, $v_g = c\sqrt{1 - \omega_{pe}^2/\omega_0^2}$ the laser group velocity, $k_{pe} = \omega_{pe}/v_g$, ω_{pe} the plasma frequency and ω_0 the laser frequency.

LINEAR REGIME

The fluid approach is one of the first theoretical approach used to describe the LWFA. The linear theory was developed independently by two teams [4, 5] at the end of the 1980s. Both teams took advantage of the QSA to simplify the laser-plasma interaction. The wakefield (or its scalar potential) is then computed inside and after the laser pulse. The non-relativistic linear theory used in this section rests on [4].

The laser is assumed unperturbed, therefore, $\mathbf{A}$ is a known function in time and space. Since, in an underdense plasma, $\mathbf{A}$ varies rapidly in time compared to n, the time dynamics of A^2 is averaged to $1/2$ before applying the quasistatic approximation. The above dynamics equations are linearized at the first order, except for laser part of the velocity, which is taken to the second order (where occurs the ponderomotive force). Therefore, the motion equation becomes

$$\frac{\partial\mathbf{v}}{\partial t} = \frac{e\nabla\Phi}{m} - c^2\nabla\frac{a^2}{4}\,. \qquad (5)$$

With a small rearrangement of the three above equations, it can be shown that the evolution of Φ behave as a forced harmonic oscillator

$$\frac{\partial^2\Phi}{\partial t^2} + \omega_{pe}^2\Phi = \frac{c^2 n_0 e a^2}{4\epsilon_0}\,. \qquad (6)$$

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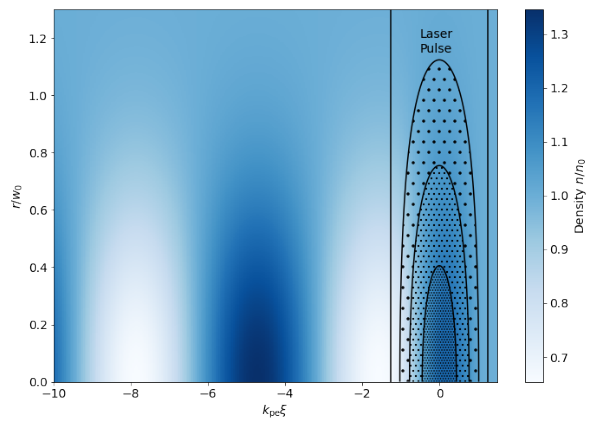

Figure 1: (Color online) Blue colormap corresponds to the 2D normalized electron plasma perturbed density $1 + \delta n/n0$ from Eq. (7). The cylindrical symmetry is assumed. The black curves corresponds to the laser intensity a^2. The dots density is a representation of the increasing laser intensity. The laser pulse goes from left to right.

The solution is known to be [4]

$$\phi(r,\xi) = -k_{\text{pe}} \int_{\xi}^{\infty} \frac{a^2(r,\xi')}{4} \sin(k_{\text{pe}}(\xi - \xi'))\, \mathrm{d}\xi' \,. \qquad (7)$$

Equation (7) is the culmination point of the linear theory, since with it, one can express both the wakefield and the electron density of the system.

The solution of Eq. (7) depends on the form of the laser pulse. For a 2D cylindrical geometry, the laser intensity $a^2(r,\xi) = \frac{a_0^2}{2}\exp(-\frac{2r^2}{w_0^2})\cos^2(\frac{\pi\xi}{L}), \quad \forall \xi \in [-L/2, L/2]$ is chosen. Figure 1 displays the electron density obtained using the Poisson equation on Φ. Figure 2 displays the resulting two components in r and ξ of the wakefield.

NONLINEAR REGIME

1D Relativistic Case

The 1D nonlinear relativistic theory of the LWFA was introduced [6–8] with an explicit solution, a few years after the linear regime. It is inspired by a more general nonlinear wave theory [9]. This section rests on [2].

The 1D equations to be solved, from Eqs. (1)-(4), are

$$\frac{\partial \delta n}{\partial t} + \frac{\partial}{\partial z}((n_0 + \delta n)v_z) = 0\,, \qquad (8)$$

$$\frac{\mathrm{d}\gamma m v_z}{\mathrm{d}t} = e\partial_z \Phi - \frac{mc^2}{2\gamma}\frac{\partial a^2}{\partial z}\,, \qquad (9)$$

$$\gamma v_x = ca\,, \qquad (10)$$

$$\frac{\partial^2 \Phi}{\partial z^2} = \frac{e\delta n}{\epsilon_0}\,. \qquad (11)$$

Using the QSA yields to $\gamma(v_g v_z/c^2 - 1) + \Phi = -1$, $\delta n/n_0 = v_z/(v_g - v_z)$ and $\gamma^2(1 - v_z^2/c^2) = 1 + a^2$. This allows us to

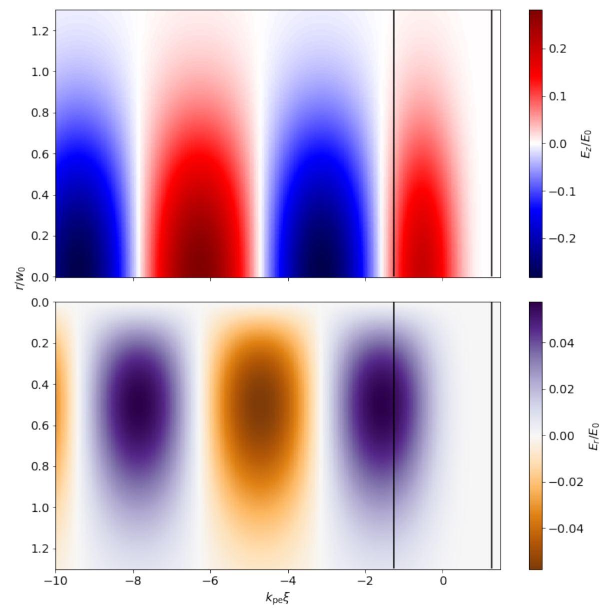

Figure 2: (Color online) Blue and red colormap (upper panel) corresponds to the 2D E_z/E_0 component of the wakefield from Eq. (7). Gold and purple colormap (lower panel) corresponds to the 2D E_r/E_0 component of the wakefield. The area where $E_z < 0$ is the accelerating location for electrons. The area where $E_r > 0$ is the focusing location for electrons. The black square is the laser influence. The laser pulse goes from left to right.

reexpress the Poisson equation [2]

$$\frac{\partial^2 \phi}{\partial \xi^2} = k_{\text{pe}}^2 \gamma_{\text{g}}^2 \left(\frac{v_{\text{g}}}{c} \frac{(1+\phi)\gamma_{\text{g}}}{\sqrt{(1+\phi)^2\gamma_{\text{g}}^2 - a^2 - 1}} - 1 \right), \qquad (12)$$

with $\gamma_{\text{g}} = (1 - v_{\text{g}}^2/c^2)^{-1/2}$. Note that, $a^2(\xi)$ is a known function independent of ϕ. Equation (12) is the culmination point of the nonlinear theory, since with it, one can express both the wakefield and the electron density of the system. Equation (12) can be solved numerically using an iterative method with initial values. Figure 3 displays the electron plasma density, longitudinal field and scalar potential that are recovered.

2D Relativistic Case

The 2D (or 3D) nonlinear theory was investigated several times [1, 10, 11]. Our immediate goal is to solve the 2D nonlinear relativistic case, namely, to solve Eqs. (1)-(4). Two approaches are considered.

The first approach relies on a finite difference model. The system of differential equations to be solved are the continuity equation, the Lorentz Force equation, the Poisson equation and the wave equation (for the evolution of **A**). Those equations are reformulated in the 2D cylindrical geometry and with the QSA. For the sake of simplicity, the laser pulse is unperturbed. Through a Taylor expansion, the expression of first, second and mixed derivatives of each

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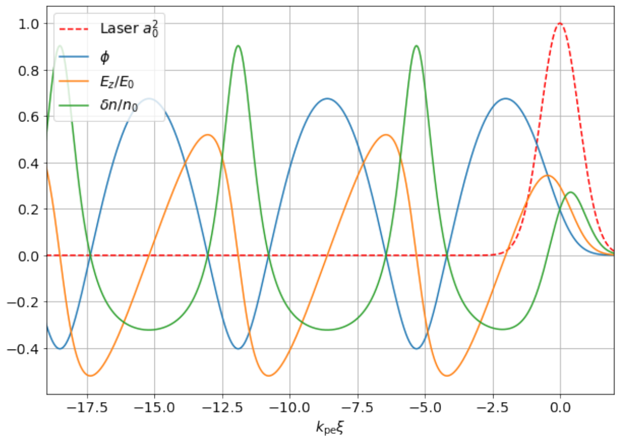

Figure 3: (Color online) Laser intensity, scalar potential, longitudinal electric field and density perturbed from Eq. (12) in the 1D nonlinear regime.

unknown are re-expressed on a 2D mesh (r, ξ). Thanks to this, the system is reframes by linear matrix equations. It uses large sparse matrices that can be solved using an iterative method under investigation. Boundary conditions are $\Phi \to 0$ when $r \to \infty$ and $\Phi = 0$ when $\xi > L$.

The second approach considered relies on semi-Lagrangian methods. The system of differential equations to be solved are the electromagnetic Vlasov equation and the Poisson equation. Those equations are reformulated in the 2D cylindrical geometry (r, ξ) and with the QSA. Since the QSA removes the time dependency of the distribution function, it is decided to solve the Vlasov equation on the ξ variable, instead of t. Thus, the discrete distribution function $f^k(r, v_r, v_\xi)$, at each $k \to \xi$, is solved using a 1D2V Vlasov algorithm based on the classical Cheng and Knorr approach. The same boundary conditions as for the first approach are used.

CONCLUSION

Within the fluid, QSA, and unchanged laser amplitude approximations, the accelerating structure can be described (i) in the linear regime by integrating a function, (ii) in the nonlinear 1D regime by solving a differential equation, and (iii) in the nonlinear 2D regime by solving a system of coupled partial differential equations. These results remain to be compared to PIC simulations where the laser diffraction and depletion, and the beam loading can be taken into account, in order to specify their limits. Based on successful preliminary results, it is expected that this approach will be fast enough to allow fast parameter analysis.

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