

# STUDY OF COIL CONFIGURATION AND LOCAL OPTICS EFFECTS FOR THE GaToroid ION GANTRY DESIGN\*

Ewa Oponowicz<sup>†</sup>, Luca Bottura, Yann Duthiel, Ariel Haziot, CERN, Geneva, Switzerland  
 Alexander Gerbershagen<sup>1</sup>, CERN, Geneva, Switzerland

<sup>1</sup>also at Particle Therapy Research Center (PARTREC), Department of Radiation Oncology, University Medical Center Groningen, University of Groningen, The Netherlands

## Abstract

GaToroid, a novel gantry configuration for hadron therapy, is based on superconducting coils that generate a toroidal magnetic field to deliver the beam to the tumour location. Designing the complex GaToroid coils requires careful consideration of the local beam optical effects.

We present a Python-based tool for charged particle transport in complex electromagnetic fields. The code implements fast tracking in arbitrary three-dimensional field maps, and it does not require a definition of specific or regular reference trajectories, as is generally the case for accelerator physics codes. The tool was used to characterise the beam behaviour inside the GaToroid system: to automatically determine the reference trajectories in the symmetry plane and analyse three-dimensional beam dynamics around these trajectories. Beam optical parameters in the field region were compared for various magnetic configurations of GaToroid.

This paper introduces the new tracker and shows the benchmarking results. Furthermore, first-order beam optics studies for different arrangements demonstrate the main code features and serve for the design optimisation.

## INTRODUCTION

A number of well-established codes for beam tracking studies are widely available [1, 2]. However, most of them are typically designed for long magnets of small aperture, i.e., magnetic configurations generating conventional fields or superposition of such, which can be easily represented with multipole expansion theory. Beam optics studies in complex magnetic fields are either not implemented, or require complex post-processing [3].

The development of a fast and compact code for particle tracking in arbitrary magnetic (or electric) field was motivated by the beam optics studies of GaToroid, a novel gantry for hadron therapy [4, 5]. The GaToroid concept is based on an axis-symmetric toroidal fixed-field magnet which, in combination with an upstream vector magnet, is capable of delivering the beam onto the patient. Characterisation of the local beam dynamics properties of this system is required to optimise its design. This work demonstrates the transport of carbon ions ( $C^{6+}$  of kinetic energy in the range 120 – 430 MeV/u) through several examples of GaToroid configurations.

\* Project co-supported by the CERN Budget for Knowledge Transfer to Medical Applications.

<sup>†</sup> ewa.oponowicz@cern.ch

## METHOD

### Coil Geometry

The principle of GaToroid is based on two parts: the main toroidal magnet to bend the beam onto the patient (Fig. 1) and an upstream *vector magnet* (VM, modelled here as single kick) for beam steering. The angular kicks imposed at the VM are polar, depending on the beam kinetic energy, and/or azimuthal, determined by the desired irradiation direction. The azimuthal variations are discrete and limited to the number of coil pairs constituting the toroid. For a given irradiation angle, the beam passes in between a set of two coils tilted symmetrically around the toroid axis (Fig. 1a). Due to the periodic axis symmetry, beam dynamics studies concentrate on the region bounded by two coils forming the "beam channel".

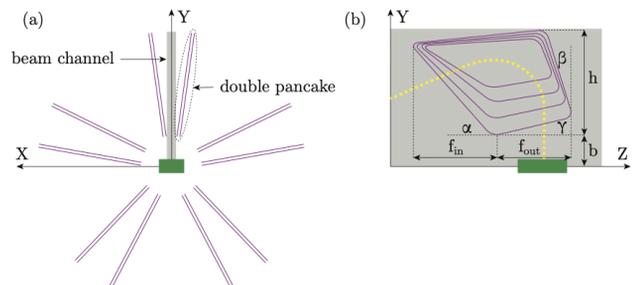


Figure 1: Schematic drawings of GaToroid with patient (green rectangle) in the axis of the toroid (treatment position), and the conductor windings (purple curves): (a) projection of a full gantry consisting of 5 beam channels; each channel is the space between two adjacent coils, (b) projection of the single coil geometry onto the bending plane together with an example beam trajectory (yellow dashed curve).  $f_{in}$  - entrance leg length,  $f_{out}$  - exit leg length,  $h$  - maximum coil height,  $b$  - bore radius,  $\alpha$  - entrance face angle,  $\beta$  - back leg angle,  $\gamma$  - exit face angle. The configuration consists of 4 coil grades.

The geometry of the coil (Fig. 1b) substantially impacts beam dynamics. In a typical toroidal configuration, the magnetic field decreases with the radius of the coil; higher energy particles require a larger kick at VM and hence experience a lower magnetic field and their bending is weaker. For treatment purposes this defocusing effect must be compensated to ensure it is an achromatic system. The field profile is modified by so called grading, i.e., introducing spacing between the coil winding in the coil for an appropriate distribution of

This paper is published with JOP the CC BY 4.0 licence (© 2022). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI

the current density. For a detailed explanation the reader is referred to [4, 5]. The main parameters defining the geometry of the coil, are: number of grades, their relative positions and reduction factors; angles ( $\alpha$  and  $\gamma$ ) and lengths ( $f_{in}$  and  $f_{out}$ ) of the entrance (vector magnet side) and exit (isocentre side) faces; maximum height, and bore size. Additionally, the angle of the back leg  $\beta$  can be optimised to reduce the amount of conductor used, in particular in the region not passed through by the beam.

### Tracking Algorithm

A tracking tool was developed in Python based on the Boris algorithm [6]. It is validated against a well-established Fortran ray tracing code Zgoubi [7]. Here a single charged particle travels through a complex static magnetic field, and the trajectories – read out from the two tracking codes – are compared. The distance between the two positions in a 3D coordinate system increases in the regions of higher magnetic field (stronger bending), but remains in a sub-millimeter range. The difference is assumed to result from the numerical approximation, since it is smaller for a finer mesh of the field map, e.g., max. 0.04 mm for a 25 mm mesh size, and max. 0.01 mm for 10 mm.

### First Order Transport Matrix

Transport of a particle from point  $A$  to point  $B$  can be described as

$$\begin{pmatrix} x_B \\ x'_B \\ y_B \\ y'_B \\ s_B \\ \left. \frac{\delta p}{p} \right|_B \end{pmatrix} = M^{AB} \begin{pmatrix} x_A \\ x'_A \\ y_A \\ y'_A \\ s_A \\ \left. \frac{\delta p}{p} \right|_A \end{pmatrix}, \quad (1)$$

where  $x_i$  - horizontal position,  $x'_i$  - horizontal angle,  $y_i$  - vertical position,  $y'_i$  - vertical angle,  $s_i$  - longitudinal position,  $\left. \frac{\delta p}{p} \right|_i$  - momentum spread,  $i = A, B$ , and  $M^{AB}$  is the 6x6 linear transport matrix. The matrix can be hence determined by tracking a set of finite (and low) number of particles. In addition to the reference particle of input parameters  $(x_0, x'_0, y_0, y'_0, t_0, \left. \frac{\delta p}{p} \right|_0)$ , at least 12 other particles must be tracked: each with only one coordinate different from the respective coordinate of the reference particle. The matrix coefficients are then calculated using the method described in [8]. This can be performed at any step of the reference trajectory, provided all the beam parameters are represented in the local reference system of the pilot particle, i.e. in the reference plane. If trajectories of all other particles include steps before and after the reference plane, and within the tracking region (here, magnetic field region), their positions and momenta are linearly interpolated to the reference plane. Coordinates of a particle which does not reach the reference plane are extrapolated under the assumption of zero field outside of the tracking region; the particle momentum remains unchanged, and the extrapolated position is a translation of

its last known position, based on the momentum vector, onto the reference plane.

## RESULTS

### Reference Trajectories

To analyse beam dynamics in the toroidal gantry, the initial step is to ensure that the beam is delivered to the patient location, i.e., isocentre. Ideally, to provide with a full energy acceptance, beams of any kinetic energy within the treatment spectrum should be deflected at the VM such that it reaches the isocentre with a sub-millimeter precision [9] and normal to the irradiation plane (except for specific cases, e.g., in the vicinity of organs at risks, dealt with by the treatment planning systems). However, the isocentre location relative to the coil geometry is arbitrary, and so is the position of the vector magnet (VM). The trajectories of reference particles of various energies, backward-tracked from an example isocentre vertically up, end up in various locations on the isocentre plane; this end position can move by as much as  $\pm 1$  m for an average distance between the isocentre and VM of around 11 m. This behaviour depends on the selected isocentre location.

To determine (for a given coil configuration) the positions of the VM and of the isocentre, together with the azimuthal kick angles imparted at VM, an iterative process, restricted to a two-dimensional symmetry plane between two coils, of following steps is performed:

- (i) *Find VM position:* The pilot particles of various energies are tracked backward and vertically up from a given isocentre to the isocentre plane. As their final positions vary as a function of kinetic energy, the VM position is chosen as the average value of the positions of different-energy particles.
- (ii) *Optimise azimuthal angles:* The particles are forward-tracked from this VM location and receive azimuthal kicks. For each energy, the kick angle is optimised such that the particle arrives as close as possible to the previously fixed isocentre, and as vertically as possible.
- (iii) *Calculate particle angle at isocentre:* Based on the optimisation results, the particles are forward-tracked from the VM to calculate the angle of the particle at the isocentre.

The whole procedure is repeated for multiple isocentre positions and the final choice corresponds to the case of minimum spread of the incoming beam angles for the whole range of the treatment energies.

### Focusing Terms

Once central trajectories are known, particle tracking in a 3D field map is performed. Linear transport matrix can be extracted from the tracking data for a segment between any two tracking points of the reference trajectory. Subsequently, this segment can be modelled with a thin quadrupole approximation. The focusing term  $kl$  derived from the matrix,

© 2022, published with the final version is published with JACoW. Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI.

further normalised by the beam rigidity  $B\rho$  and the segment length  $L$ , can be expressed as quadrupole gradient:

$$g \text{ (T/m)} = \frac{kl \cdot B\rho}{L} \left( \frac{\text{m}^{-1} \cdot \text{Tm}}{\text{m}} \right). \quad (2)$$

The gradients experienced by beams of various energies (Fig. 2) give a more detailed, quantitative understanding of the beam dynamics: a strong defocusing effect in the horizontal plane over the full length of propagation. The location of strong quadrupolar behaviour follows the position of the coil grades.

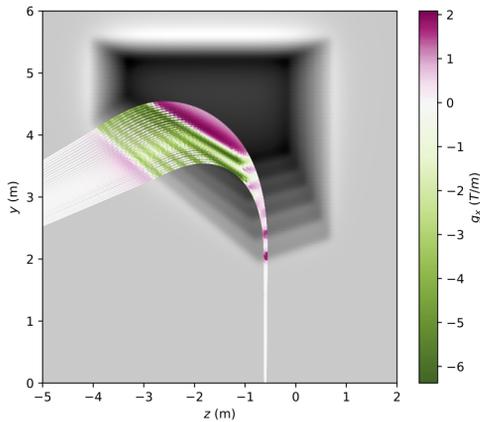


Figure 2: Color map representing the quadrupole gradient [T/m] at different steps of reference trajectories for the entire range of treatment energies, plotted on top of the main magnetic field component of a GaToroid configuration (black and white).

### Coil Exit Angle

the beam dynamics in the GaToroid system is affected by the set of geometry parameters defining the coil. To demonstrate it, we introduce modification of the exit angle  $\gamma$  (isocentre side), which resembles the edge angle in a conventional bending magnet. Varying the  $\gamma$  angle alone, while keeping the bore size fixed, leads to magnetic configurations for which the beam travelling from the same point (isocentre) experiences different bending strengths, i.e., in the hard edge approximation the magnetic field starts further from the starting location for higher tilt angles  $\gamma$ . Hence, for each magnetic configuration (tilt angle), the entire coil is shifted such that the beam starts being bent in the same location, to decouple the effects of focusing and bending. In each case, the input positions and angles of 300 MeV/u carbon ion beams are optimised such that their trajectories are comparable (Fig. 3). The focusing coefficients for the entire system (from VM to isocentre) for each configuration are presented in Fig. 4. They change slightly in the vertical plane and more substantially in the horizontal plane, decreasing with higher angles and changing into negative values (focusing) for 20°.

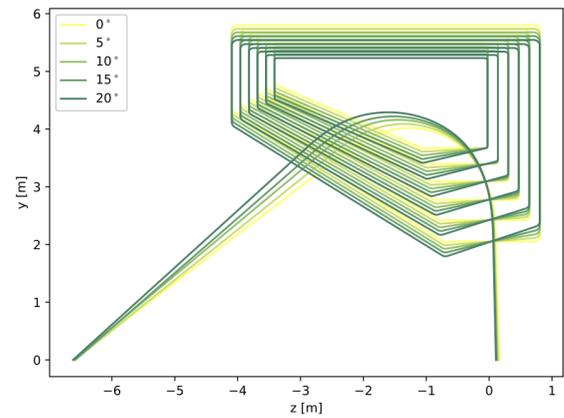


Figure 3: Configurations of the GaToroid coil for several isocentre-side angles  $\gamma$  (0 – 20°). Different configurations are shifted such that the beam arriving from the isocentre start experiencing the magnetic field at the same location. For each configuration a carbon ion beam of the same energy (300 MeV/u) is tracked such that it arrives with minimum angle to the same isocentre.

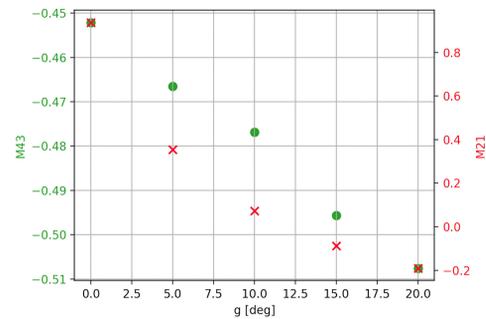


Figure 4: Focusing coefficients in the vertical plane  $M_{43}$  (green) and in the horizontal plane  $M_{21}$  (red) as a function of the tilt angle  $\gamma$  of the GaToroid isocentre side of the coil.

## CONCLUSION

A new tool for tracking particles, based on Boris algorithm, in any arbitrary three-dimensional magnetic field is presented in this work. Its implementation was motivated by the complexity of the magnetic field generated by GaToroid, a novel toroidal configuration proposed for beam delivery in hadron therapy. The features of the new tracker analyse the beam dynamics in the gantry in order to couple it to the coil design. Several examples, including tracking in two- and three-dimensional magnetic field maps of GaToroid, demonstrate the steps to be undertaken towards the ultimate goal, i.e., optimisation of the GaToroid design. Whilst this work focused on linear beam dynamics, higher orders are to be determined in the next steps. In addition, for further optimisation, GaToroid should be treated as a complete system from the accelerator extraction point to the isocentre.

## REFERENCES

- [1] D. Sagan, "DBmad: A relativistic charged particle simulation library", *Nucl. Instrum. Methods Phys. Res. Sect. A*, vol. 558, no. 1, pp. 356–359, 2006.
- [2] L. Deniau, H. Grote, G. Roy, and F. Schmidt, "The MAD-X program. User's Reference Manual", CERN, Geneva, Switzerland, 2017, <https://mad.web.cern.ch/mad/webguide/manual.html>
- [3] A. Gerbershagen, D. Meer, J. M. Schippers, and M. Seidel, "A novel beam optics concept in a particle therapy gantry utilizing the advantages of superconducting magnets", *Z. Med. Phys.*, vol. 26, no. 3, pp. 224–237, 2016.
- [4] L. Bottura, E. Felcini, G. De Rijk, and B. Dutoit, "GaToroid: a novel toroidal gantry for hadron therapy", *Nucl. Instrum. Methods Phys. Res. Sect. A*, vol. 983, p. 164588, 2020.
- [5] E. Felcini, L. Bottura, J. Van Nugteren, G. De Rijk, G. Kirby, and B. Dutoit, B, "Magnetic design of a superconducting toroidal gantry for hadron therapy", *IEEE Trans. Appl. Supercond.*, vol. 30, no. 4, pp. 1-5, 2020.
- [6] H. Qin, S. Zhang, J. Xiao, J. Liu, Y. Sun, and W. M. Tang, "Why is Boris algorithm so good?", *Phys. Plasma*, vol. 20, no. 8, p. 084503, 2013.
- [7] F. Méot, "Zgoubi user's guide". Brookhaven National Lab., Upton, NY, USA, Rep. BNL-98726-2012-IR, 2012.
- [8] M. Yoon and D. S. Robin, "Method of computing first-, second-, and third-order transfer coefficients for arbitrary fields", *IEEE Trans. Nucl. Sci.*, vol. 60, no. 5, pp. 3837–3842, 2013
- [9] S. A. Reimoser and M. Pavlovic, "Engineering design and study of the beam position accuracy in the "Riesenrad" ion gantry", *Nucl. Instrum. Methods Phys. Res. Sect. A*, vol. 456, no. 3, pp. 390–410, 2001.