ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-MOPOTK065

MINIMISING TRANSVERSE MULTIPOLES IN ACCELERATING RF CAVITIES VIA AZIMUTHALLY MODULATED DESIGNS

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Abstract

In this paper, we build upon previous work of designing RF structures that support modes with tailored multipolar fields by applying the concept to negate the transverse multipoles in accelerating RF cavities caused by the incorporation of waveguide slots and tuning deformations. We outline a systematic method for designing structures that minimise these transverse multipoles and present analysis of simulations of two different minimisation designs.

INTRODUCTION

RF cavities designed to longitudinally accelerate beams of charged particles typically operate in fundamental, transverse magnetic TM_{010}-like modes [1–3]. The longitudinal monopolar term dominates in such modes but they also include transverse multipole terms as the addition of slots for power coupling [4] and components for tuning [5] break azimuthal symmetries. These transverse multipoles influence transverse beam dynamics and must not exceed the tolerances of the accelerator [6]. This can restrict the possible designs of the RF cavity, for example it may be necessary to incorporate a dual-slot coupler rather than a single-slot to negate dipolar components [7].

Previous work [8–11] has shown that azimuthally modulated RF cavities can be designed that support tailored modes with user-specified multipolar content of the form:

$$E_z(r,\theta,z) = \tilde{g}_0 J_0(kr) + \sum_{m=1}^{\infty} \tilde{g}_m J_m(kr)\cos(m\theta + \phi_m), \tag{1}$$

where the magnitude and orientation of the multipoles are denoted by $\tilde{g}_M$ and ϕ_M, J_M is the Bessel function of the first kind of order M and k is the wavenumber of the mode. Such modes are denoted as $TM_{\{M\}\eta 0}$ where $\{M\}$ denotes the set of integers for which $\tilde{g}_M \neq 0$, η denotes the radial order of the mode, and 0 denotes that the field is constant with z. The azimuthally modulated cavity cross-section, $r_0^{(\eta)}(\theta)$, that supports a desired $TM_{\{M\}\eta 0}$ mode is determined by solving:

$$0 = \tilde{g}_0 J_0(kr_0^{(\eta)}(\theta)) + \sum_{m=1}^{\infty} \tilde{g}_m J_m\left(kr_0^{(\eta)}(\theta)\right)\cos(m\theta + \phi_m). \tag{2}$$

We also note that the multipolar components in Eq. (1) can be explicitly calculated for a given electric field by undertaking a Helmholtz decomposition, as described in [6], and that a TM_{010}-like mode can be defined as a mode for which $\tilde{g}_0 \gg \tilde{g}_m$ for all m.

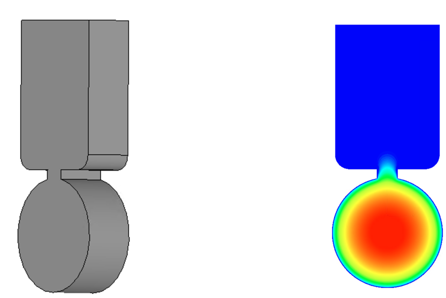
In this paper, we apply this concept to minimise the magnitude of transverse multipoles in the TM_{010}-like mode of an RF cavity coupled to a power source by a slot. We present a systematic method for doing this minimisation and give an example of two different minimisation designs. We also investigate the effect of one-way tuning pins on the transverse multipoles for both designs.

PILLBOX DESIGNS

It is well-known [12] that a perfectly sealed pillbox cavity supports a non-degenerate, fundamental TM_{010} mode that can be used for the longitudinal acceleration of charged particles as its longitudinal electric field, E_z, is of the form:

$$E_z(r,\theta,z) = \tilde{g}_0 J_0(kr). \tag{3}$$

In order to exploit this mode for use in a particle accelerator, the fundamental mode must be excited by incorporating slots into the design that couple RF power into the cavity. Figure 1a shows the design of a 3 GHz RF cavity coupled to an RF input port via a single-slot design whose width has been optimised to minimise power loss in the waveguide. The single-slot breaks the azimuthal symmetry of the pillbox and Fig. 1b shows the electric field seeps into the slot. This breaking of the azimuthal symmetry of the fundamental accelerating mode means it must contain transverse multipoles and so is TM_{010}-like.



(a) Off-axis view. (b) Contour log-plot of E_z.

Figure 1: Pillbox cavity coupled to an RF input port via a single-slot that supports a 3 GHz fundamental accelerating mode.

Cavities may also require tuning post-manufacture as finite tolerances on the accuracy of machining mean the fabricated cavity may have a different resonant frequency to the ideal design. One method for tuning is to oversize the designed cavity by the upper bound of the machining tolerance, guaranteeing its resonant frequency will be lower than

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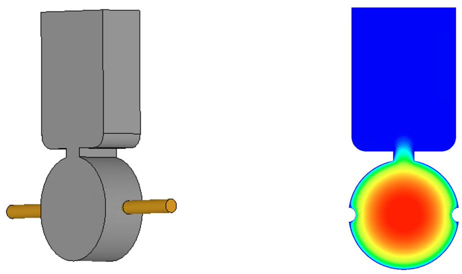
desired, and then deforming it with one-way tuning pins until the structure resonates at the desired frequency. Figure 2a shows an identical pillbox cavity to Fig. 1a but oversized by 100 µm so that it must be tuned to 3 GHz, in this example by deforming it with two, one-way tuning pins. Figure 2b shows this tuning method further breaks the azimuthal symmetry of the electric field of the fundamental accelerating mode, and thus the mode has a greater magnitude of transverse multipoles.



(a) Off-axis view. (b) Contour log-plot of E_z.

Figure 2: Pillbox structure oversized by 100 µm that supports a fundamental accelerating mode tuned to 3 GHz by two, one-way tuning pins. E_z is plotted at the longitudinal centre of the structure.

DESIGNS THAT MINIMISE TRANSVERSE MULTIPOLES

Given that slots introduce transverse multipoles, Eq. (2) can be used to design azimuthally modulated cavity shapes that negate the transverse multipoles introduced by the slot by using the following method:

1. A pillbox cavity is created that supports a TM_{010} mode that resonates at a desired frequency *f*.
2. Slots and waveguides are added to the pillbox to couple it to an RF input port. The slot widths are optimised such that maximum power is dissipated in the pillbox.
3. The slots increase the effective volume of the cavity, reducing the resonant frequency of the TM_{010}-like mode. The pillbox is scaled down until it resonates at *f*.
4. A 3D simulation is undertaken and a Helmholtz decomposition performed to determine $\tilde{g}_M$.
5. These values of $\tilde{g}_M$ are incorporated into Eq. (2) to calculate the azimuthally modulated cross-section that cancels the transverse multipoles in the TM_{010}-like mode.
6. The cross-section is incorporated into a new design. It is scaled so the TM_{010}-like mode resonates at *f* and the slot widths are re-optimised.
7. Steps 4-6 are repeated until the transverse multipoles are sufficiently minimised.

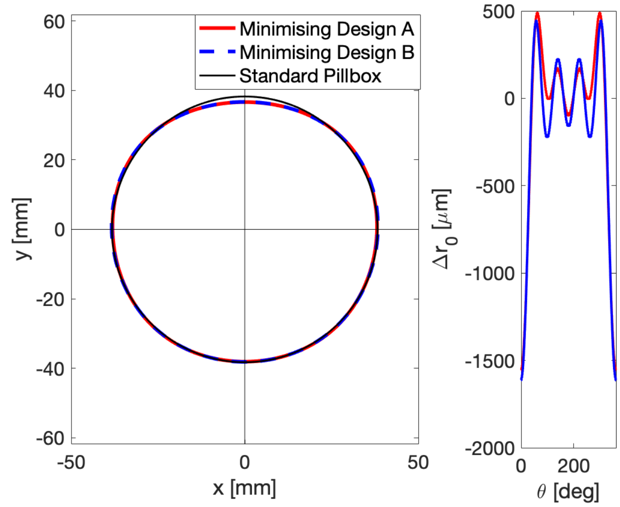


Figure 3: Comparison of the physical dimensions of the cross-sections of the three different designs (left) and the deviation of Minimising Design A [red] and Minimising Design B [blue] from the Standard Pillbox radius (right).

The effect of one-way tuning pins on the transverse multipolar content of the TM_{010}-like mode can be investigated by using the following method:

1. Assume a machine tolerance of $\pm\delta$µm and model the deformation caused by the tuning pins to be a hemisphere of radius *a* mm that encroaches a distance *d* mm into the cavity.
2. The cavity is oversized by 2δ µm.
3. The distance the hemisphere encroaches into the cavity is increased until the cavity resonates at *f*.
4. A 3D simulation is undertaken and a Helmholtz decomposition performed to determine $\tilde{g}_M$.

Note that the tuning pin deformation is longitudinally asymmetric and so will introduce longitudinally dependent multipolar terms that Eq. (1) does not capture. As the tuning pin perturbation is small, however, we ignore the calculation of the longitudinally dependent transverse multipoles and just use the average value of the multipoles along the entire structure.

EXAMPLE ANALYSIS

We applied the above methods to investigate the transverse multipoles present in the 3 GHz TM_{010}-like mode of three different, single-slot designs. We assume a ±50 µm machining error with tuning achieved by the use of two, one-way tuning pins inserted at ±90° relative to the coupling slot, as shown in Fig. 2a. The deformation caused by the pins is taken to be a hemisphere of radius 5 mm. The three designs are:

- Standard Pillbox - Circular cross-section scaled such that its TM_{010}-like mode resonates at 3 GHz.

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- Minimising Design A - Azimuthally modulated cross-section that minimises the transverse multipoles up to order 4 in the TM_{010}-like mode when scaled such that it resonates at 3 GHz in the absence of tuning pins.
- Minimising Design B - Azimuthally modulated cross-section that minimises the transverse multipoles up to order 4 in the TM_{010}-like mode when oversized by 50 µm and then tuned to 3 GHz by inserting two, one-way tuning pins.

Figure 3 shows the difference between the cross-sections used to make each design.

Table 1: Transverse multipolar terms calculated for the three different designs.

Design	$\tilde{g}_1/\tilde{g}_0$	$\tilde{g}_2/\tilde{g}_0$	$\tilde{g}_3/\tilde{g}_0$	$\tilde{g}_4/\tilde{g}_0$
Standard Pillbox	0.0182	0.0217	0.0464	0.139
Min-imising Design A	0.0000	-0.0000	-0.0003	-0.0002
Min-imising Design B	-0.0002	0.0003	0.0008	0.0013

Table 1 shows the transverse multipolar terms calculated from 3D CST [13] simulations of the TM_{010}-like modes of each of the designs detailed above when machined with no error. The single-slot causes dipolar, quadrupolar, sextupolar, and octupolar components to arise in the TM_{010}-like mode that can all be minimised with both Minimising Design A and B.

Table 2: Maximum transverse multipolar terms observed for three different structures operating in TM_{010}-like modes tuned to 3 GHz with two, one-way tuning pins (modelled as 5 mm hemispheres) to correct for a ±50 µm machining error.

Design	$\lvert\tilde{g}_1\rvert/\lvert\tilde{g}_0\rvert$	$\lvert\tilde{g}_2\rvert/\lvert\tilde{g}_0\rvert$	$\lvert\tilde{g}_3\rvert/\lvert\tilde{g}_0\rvert$	$\lvert\tilde{g}_4\rvert/\lvert\tilde{g}_0\rvert$
Standard Pillbox	0.0182	0.0368	0.0464	0.139
Min-imising Design A	0.0001	0.0149	0.0003	0.100
Min-imising Design B	0.0003	0.0072	0.0001	0.0511

To investigate the magnitude of the multipoles introduced by the tuning pins for the different designs, a 3D simulation and Helmholtz decomposition was also undertaken for when the Standard Pillbox and Minimising Design A are maximally oversized by 100 µm such that maximum tuning is required and when Minimising Design B was both undersized and oversized by 50 µm. Table 2 shows the absolute of the maximum transverse multipole observed across all simulations for each design. The tuning pins only introduce quadrupolar and octupolar components of significant magnitude which is expected from the symmetry of the deformation with two tuning pins placed opposite to each other. A greater deformation increases the magnitude of the quadrupolar component for both the Standard Pillbox and Minimising Design A with the maximum quadrupolar component introduced by the tuning pins approximately the same for both designs. In contrast, a greater deformation actually reduces the magnitude of the octupolar component in the case of the Standard Pillbox whilst causing an increase for Minimising Design A and also B. The maximum quadrupolar and octupolar component observed for Minimising Design B is approximately half that of A. This is expected because Minimising Design B is designed to have minimum transverse components when the tuning pins are already deforming it by approximately half the maximum deformation. Minimising Design B will also, on average, introduce lower transverse multipoles when fabricated if we assume that the machining error has a Gaussian-like distribution around the perfect machining rather than uniform.

CONCLUSION

A systematic method has been presented for designing azimuthally modulated RF cavities that minimise the transverse multipoles introduced by slots which couple the cavity to an RF input port that excites the desired, fundamental accelerating mode. Analysis of the simulation of two different example designs showed that the transverse multipoles introduced by a single-slot can be negated completely. A method for investigating the effect of tuning by deformation was also presented with an example of a design that minimises the transverse multipoles caused by tuning pins presented. Both methods could be extended to designing RF cavity systems that use n-slots and N, one-way tuning pin and support a TM_{010}-like mode with minimal transverse multipoles up to any order.

This minimisation of the transverse multipoles in the TM_{010}-like modes of accelerating RF cavities can ultimately prevent beam losses caused by unwanted transverse motion. Additionally, this systematic method for the precise cancellation of transverse multipoles could be easily extended for the precise introduction of transverse multipoles.

ACKNOWLEDGEMENTS

These studies were funded by the Royal Society under grant number RS\PhD\181200 and the CI Core Grant under ST/P002056/1.

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